ME 356 Fall 2001

Solutions to Homework #2 and #3

By: Nick Silverman

Good Luck on Monday!

Questions, comments, gripes, and most importantly praise should be directed towards your loving TA during office hours MTF.

4.2) $k = \frac{F}{y}$; The total spring rate (k) is a function of the cantilever spring rate and the torsional spring rate.

Cantilever spring rate calculations:

$$k_c = \frac{F}{\delta} \quad ; \quad \delta = \frac{F}{k_c}$$

Torsional spring rate calculations:

$$k_T = \frac{T}{\Theta} = \frac{F\ell}{\Theta} \quad ; \quad \Theta = \frac{F\ell}{k_T}$$

Total spring rate calculations:

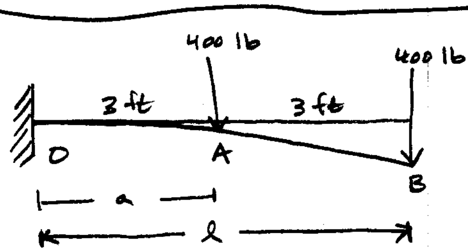
$$k = \frac{F}{y} \quad ; \quad y = \frac{F}{k} \quad ; \quad y = \underline{\ell\Theta} + \delta$$

$$\therefore \frac{F}{k} = \ell\Theta + \delta$$

$$= \ell\left(\frac{F\ell}{k_T}\right) + \frac{F}{k_c} = \frac{\cancel{F}\ell^2}{k_T} + \frac{\cancel{F}}{k_c}$$

$$\boxed{k = \frac{1}{\frac{\ell^2}{k_T} + \frac{1}{k_c}}}$$

4.9)



$I = 13 \text{ in}^4$ $\quad E = 30$ Mpsi

$a = 36$ in $\quad \ell = 72$ in

Equation from table E-9:

$$y = \frac{Fx^2}{6EI}(x - 3\ell)$$

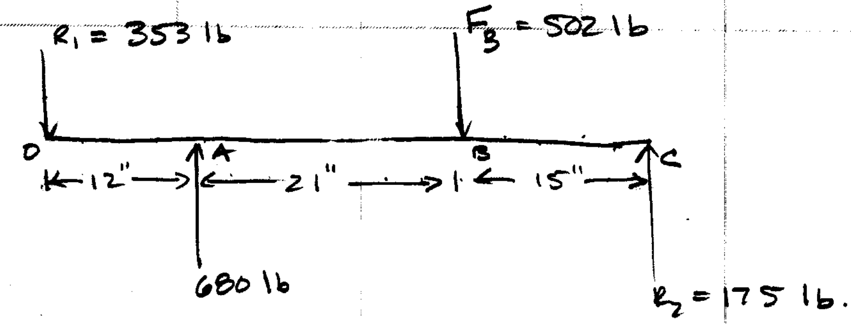
Through superposition we get:

$$y = \frac{F_1 a^2}{6EI}(a - 3\ell) + \frac{F_2 \ell^2}{6EI}(\ell - 3\ell)$$

$$= \frac{F_1 a^2}{6EI}(a - 3\ell) - \frac{F_2 \ell^3}{3EI} = \frac{(400)(36)^2}{6(30)(10^6)(13)}(36 - 3(72)) - \frac{400(72)^3}{3(30)(10^6)(13)}$$

$$= \boxed{-.167 \text{ in}}$$

4.13)



If in equilibrium, then $T_A = T_B$:

$$T_A = \frac{600-80}{2}(9) = 2340 \text{ lb}\cdot\text{in}$$

$$T_B = \frac{(T_2 - T_1)}{2}(12) = T_2\frac{(1-0.125)}{2}12 = T_A$$

So $T_2(1-0.125)6 = 2340$

$T_2 = 446$ lb and $T_1 = 446(.125) = 56$ lb

$\therefore \boxed{F_B = 502 \text{ lb}}$

To find R_2 :

$$\Sigma M_O = 12(680) - 33(502) + 48R_2 = 0$$

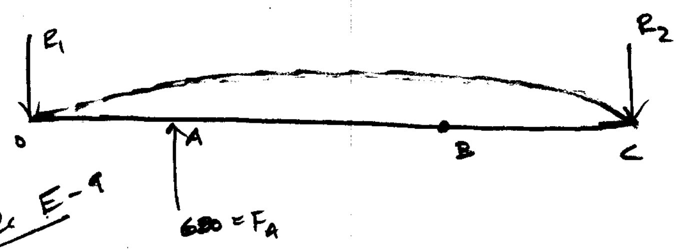
$\boxed{R_2 = 175 \text{ lb}}$

To find R_1 :

$$\Sigma F = 175 - 502 + 680 - R_1 = 0$$

$\boxed{R_1 = 353 \text{ lb}}$

Now we must treat the problem as two separate problems and then sum the results.



From Table E-9

$$y_A = \frac{Fbx}{6EIl}(x^2+b^2-l^2)$$

$x = 12"$ $l = 48"$ $F = 680$ lb $E = 30$ Mpsi $b = 36"$

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.5)^4}{64} = .249 \text{ in}^4$$

why do we use I for circle and not eyelet?

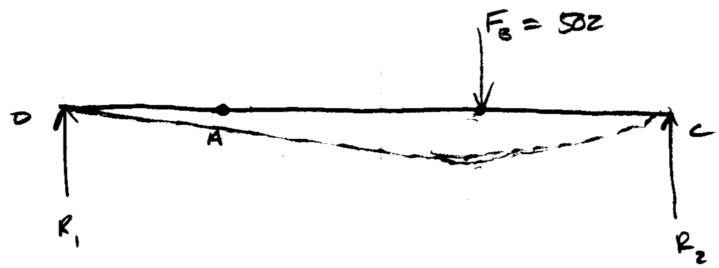
$$y_A = \frac{680(36)(12)(12^2+36^2-48^2)}{6(30)(10^6)(.249)(48)} = \boxed{.11798 \text{ in}}$$

$$y_B = -\frac{Fa(l-x)}{6EIl}(x^2+a^2-2lx)$$

$a = 12 \quad x = 21 + 12 = 33$

$$y_B = -\frac{680(12)(48-33)(33^2+12^2-2(48)(33))}{6(30)(10^6)(.249)(48} = \boxed{.11009 \text{ in}}$$

Now consider other reaction:



$$y_A = \frac{Fbx}{6EIl}(x^2+b^2-l^2)$$

$b = 15'' \quad x = 12'' \quad l = 48'' \quad I = .249 \text{ in}^4$

$$= \frac{502(15)(12)(12^2+15^2-48^2)}{6(30)(10^6)(.249)(48)} = \boxed{-.08127 \text{ in}}$$

$$y_B = \frac{Fbx}{6EIl}(x^2+b^2-l^2) \qquad x = 33''$$

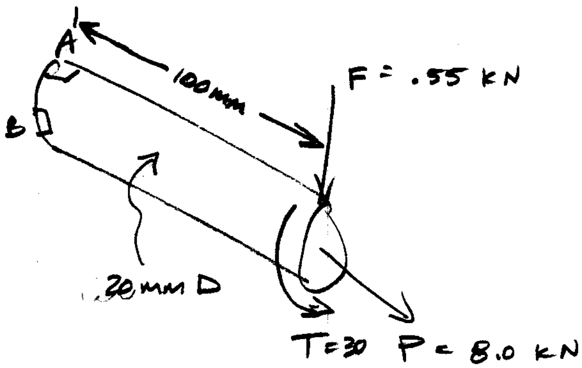
$$= \frac{502(15)(33)(33^2+15^2-48^2)}{6(30)(10^6)(.249)(48)} = \boxed{-.11435 \text{ in}}$$

By superposition we get:

$$y_A = .11798 \text{ in} - .08127 \text{ in} = \boxed{.03671 \text{ in}}$$

$$y_B = .11009 \text{ in} - .11435 \text{ in} = \boxed{-.00426 \text{ in}}$$

6.6) AISI 1006 is a ductile material, therefore we use the distortion energy hypothesis.



@ A: Axial stress due to P + Bending stress:

$$\sigma_x = \frac{F}{A} + \frac{Mc}{I} = \frac{4P}{\pi d^2} + \frac{64F\ell c}{\pi d^4} = 9.55 \times 10^7 \text{ Pa}$$

Shear stress due to due to torsion T:

$$\tau_{xy} = \frac{32T}{\pi d^3} = 3.82 \times 10^7 \text{ Pa}$$

$$\sigma' = [\sigma_x^2 + 3\tau_{xy}^2]^{1/2} = \boxed{1.16 \times 10^6 \text{ Pa}}$$

$= 116$ MPa

$$n = \frac{S_y}{\sigma'} = \frac{280 \text{ MPa}}{116 \text{ MPa}} = \boxed{2.41}$$

@ B: Normal stress due to force P:

$$\sigma_x = \frac{4P}{\pi d^2} = 2.55 \times 10^7 \text{ Pa}$$

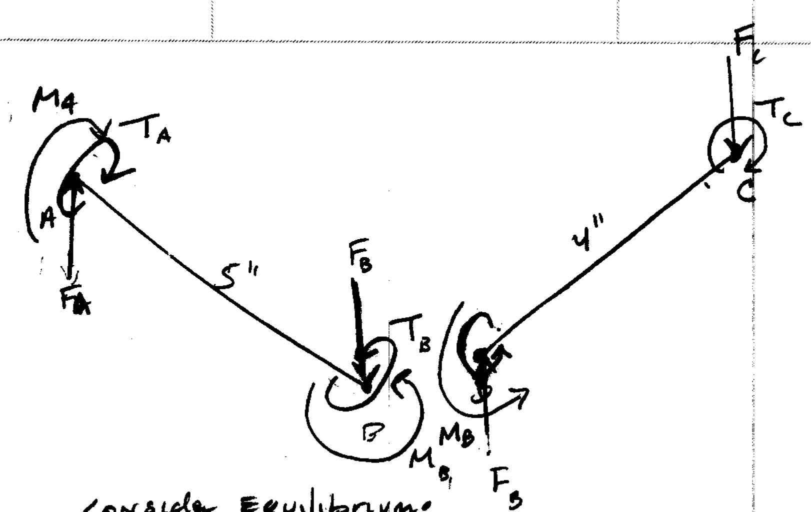
Shear stress due to torque + F:

$$\tau_{xy} = \frac{4}{3}\frac{4F}{\pi d^2} + \frac{32T}{\pi d^3} = 4.05 \times 10^7 \text{ Pa}$$

$$\sigma' = [\sigma_x^2 + 3\tau^2]^{1/2} = \boxed{74.6 \text{ MPa}}$$

$$n = \frac{S_y}{\sigma'} = \frac{280 \text{ MPa}}{74.6 \text{ MPa}} = \boxed{3.75}$$

6.8)



Consider Equilibrium:

$F_c = 300 \text{ lb} = F_B \quad , \quad F_A = F_B = 300 \text{ lb}$

$T_c = 1(300) = 300 \text{ lb}\cdot\text{in}$

$T_B = 4(300) = 1200 \text{ lb}\cdot\text{in} = T_A$

$M_{B_1} = 1(300) = 300 \text{ lb-in}$

$M_A = (5+1)(300) = 1800 \text{ lb}\cdot\text{in}$

@ A:

$\sigma_x = \frac{Mc}{I} = \frac{32 M_A}{\pi d^3} = \frac{1800(32)}{\pi(.75)^3} = \boxed{43459 \text{ psi}}$

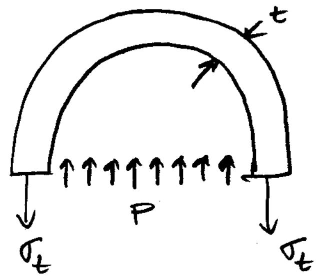
$\tau_{xz} = \frac{T_A c}{J} = \frac{32 T_A c}{\pi d^4} = \frac{32(1200)(.375)}{\pi(.75)^4} = \boxed{14486 \text{ psi}}$

$S_y = .5(32) = 16 \text{ kpsi}$

$\tau_{max} = \left((\sigma_x/2)^2 + (\tau_{xz})^2\right)^{1/2} = \left((43.5/2)^2 + (14.5)^2\right)^{1/2}$
$= \boxed{26.1 \text{ kpsi}}$

$n = \frac{S_{sy}}{\tau_{max}} = \frac{16}{26.1} = .613$ Failure

6.9)



$\Sigma F = 0$

SINCE t IS SO SMALL: $d_i = d_i + t$

$$\sigma_t \pi (d_i + t) t = p \frac{\pi d_i^2}{4}$$

$$\sigma_t = \frac{p \cancel{\pi} d_i^{\cancel{2}}}{4 \cancel{\pi} (d_i \cancel{+t}) t} = \frac{P(d_i + t)}{4t}$$

SINCE THE PRESSURE VESSEL IS SPHERICAL, WE KNOW:

$$\sigma_t = \sigma_\ell = \frac{P(d_i + t)}{4t} \quad \text{and} \quad \sigma_R = -P$$

USE THE DISTORTION ENERGY THEORY EQUATION:

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_t - \sigma_\ell)^2 + (\sigma_\ell - \sigma_R)^2 + (\sigma_R - \sigma_t)^2\right]^{1/2}$$

WHEN SUBSTITUTING IN PRINCIPLE STRESSES WE GET:

$$\sigma' = \sigma_t - \sigma_R = S_y = \frac{P(d_i + t)}{4t} + P = P\left[\frac{d_i + t}{4t} + 1\right]$$

$$P = \frac{S_y}{\frac{d_i + t}{4t} + 1} \qquad S_y = 54000 \qquad t = .05 \text{ in}$$

$$\boxed{P = \frac{54000}{\frac{8 + .05}{4(.05)} + 1} = 13.09 \text{ psi}}$$

WE DO NOT HAVE TOOLS TO ESTIMATE PRESSURE AT RUPTURE!

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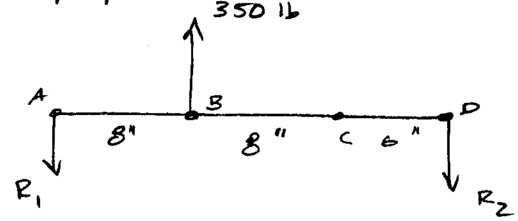
6.14)

XY - PLANE:



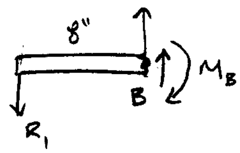
$\Sigma M_D = 0$

$-350(14) + R_1(22) = 0$

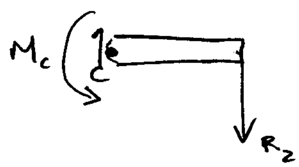
$R_1 = 223$ lb

$\Sigma F = 0 \rightarrow R_2 = 127$ lb

$T_B = (300-50)\,8 = 1000$ lb·in $\quad T_c = (360-27)(3) = 1000$ lb·in

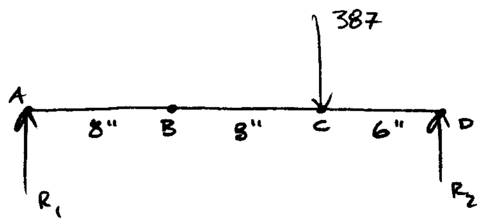


$M_B = R_1 8'' = 1784$



$M_c = R_2(6) = 762$

XZ PLANE:

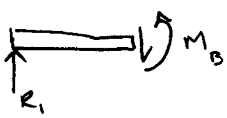


$\Sigma M_D = 0$

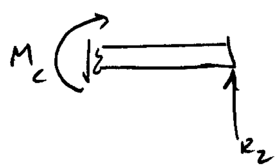
$387(6) - R_1(22) = 0$

$R_1 = 105$ lb

$\Sigma F = 0 \rightarrow R_2 = 282$ lb.



$M_B = R_1(8'') = 840$ lb·in



$M_c = R_2(6'') = 1692$ lb·in

THE RESULTANTS ARE:

$$M_B = \left[(840)^2 + (1784)^2\right]^{1/2} = 1972 \text{ lb}\cdot\text{in}$$

$$M_c = \left[(1692)^2 + (762)^2\right]^{1/2} = 1889 \text{ lb}\cdot\text{in}$$

$\therefore M_B$ IS MAXIMUM MOMENT

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi(d^3)} \approx \frac{5100}{d^3}$$

$$\sigma_x = \frac{M_B c}{I} = \frac{32 M_B}{\pi d^3} = \frac{32(1972)}{\pi d^3} \approx \frac{20100}{d^3}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x}{2} \pm \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^{\,2}\right]^{1/2} \qquad [\text{EQ } 6-27]$$

$$= \frac{20100}{2d^3} \pm \left[\left(\frac{20100}{2d^3}\right)^2 + \left(\frac{5100}{d^3}\right)^2\right]^{1/2} = \frac{10500}{d^3} \pm \frac{11270}{d^3}$$

$$\sigma_1 = \frac{10500}{d^3} + \frac{11270}{d^3} = \frac{21320}{d^3} \text{ psi}$$

$$\sigma_2 = \frac{10500}{d^3} - \frac{11270}{d^3} = -\frac{1220}{d^3} \text{ psi}$$

FROM TABLE E-24:

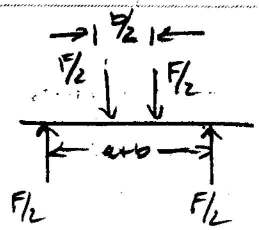
$S_{UT} = 26$ kpsi $\quad S_{UC} = 97$ kpsi

TO FIND d USE EQUATION 6.16b:

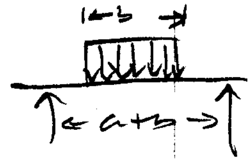
$$\frac{\sigma_1}{S_{UT}} - \frac{\sigma_2}{S_{UC}} = \frac{1}{n} \Rightarrow \frac{\frac{21320}{d^3}}{26000} + \frac{\frac{1220}{d^3}}{97000} = \frac{1}{2.8}$$

$$\boxed{d = 1.34 \text{ in}}$$

6.16)

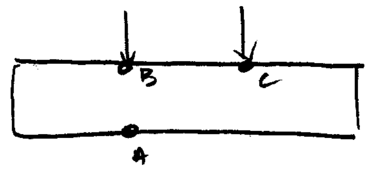


(c)



Compare factor of safetys for assumed loading:

(c)



$$M_{max} = \frac{F}{2}\left(\frac{a}{2} + \frac{b}{4}\right) = 23.1 \text{ Nm}$$

consider pt A:

$$\sigma_x = \frac{Mc}{I} = \frac{32M}{\pi d^3} = 136.2 \text{ MPa}$$

$$\sigma_y = 0$$

$\tau_{xy} = 0$ b/c on outer surface fiber

$$\tau_{max} = \frac{\sigma_x}{2} = 68.1 \text{ MPa}$$

$$S_{sy} = 110 \text{ MPa}$$

$$\boxed{n = \frac{S_{sy}}{\tau_{max}} = 1.61}$$

Now consider pt. B!

pt. B ~~has~~ compressive and contact stress so,

$$\sigma_x = -136.2 = \sigma_3 \qquad \sigma_z = 0 = \sigma_1$$

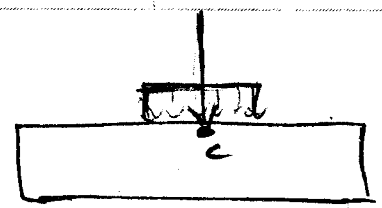
$$\sigma_y = \frac{F}{A} = -20.4 = \sigma_2$$

$\tau_{xy} = 0$ ∴ σ_x and σ_y are principle stresses

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{0 + 136.2}{2} = 68. \text{ MPa}$$

$$\therefore \boxed{n = \frac{S_{sy}}{\tau_{max}} = 1.61}$$

(d)



M_{max} is the same $\therefore$ σ_x is same

$\therefore$ τ_{max} is the same $\therefore$ n is the same.

6.17) use distortion energy theory

PT. A

c) $\sigma' = \left[\sigma_x^2 - \cancel{\sigma_x \sigma_y}^{0} + \cancel{\sigma_y^2}^{0} + \cancel{3\tau_{xy}^2}^{0}\right]^{1/2}$

$\sigma' = \sigma_x$ $\quad \boxed{n = \dfrac{S_y}{\sigma'} = \dfrac{220}{136.2} = 1.61}$

PT. B

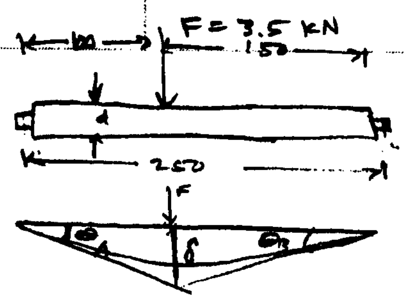
$\sigma' = \left[\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + \cancel{3\tau_{xy}^2}^{0}\right]^{1/2}$

$= 125.6 \text{ MPa}$

$\boxed{n = \dfrac{220}{125.6} = 1.75}$

d) same as above since σ_x is the same.

4.21)



$n = .001 \ mm/mm$

$E = 207 \ GPa$

$\theta_A > \theta_B$

m = slope = .001
n = design factor = 1.28

USE EQUATION:

$$\boxed{\frac{Fb\,(L^2 - b^2)}{6LEI} = \frac{m}{n}}$$

And solve for d

$$\frac{(3500)(.15)(.25^2 - .15^2)}{6(.25)(207)(10^9)\frac{\pi d^4}{64}} = \frac{.001}{1.28}$$

$$d = \left(\frac{3500(.15)(.25^2 - .15^2)(1.28)64}{6(.25)(207)(10^9)\pi(.001)}\right)^{1/4}$$

$$\boxed{d = .0364 \ m}$$

IF THE FORMULA FOR SLOPE @ A is NOT AVAILABLE:

$\tan\theta_A = \frac{\delta}{a}$ WHERE δ = DEFLECTION UNDER F.

$$\delta(x) = \frac{Fbx}{6EIL}(x^2 + b^2 - l^2)$$

$$\delta = \frac{Fba}{6EIL}(a^2 + b^2 - l^2)$$

So Now

$$\frac{\delta}{a} = \frac{m}{n} \Rightarrow \delta = \frac{100(.001)}{1.28} = .078125 \ mm$$

plug this into the above equation and solve: $\boxed{d = .03392 \ m}$