ME 356 Fall 2001

Solutions to Homework #1

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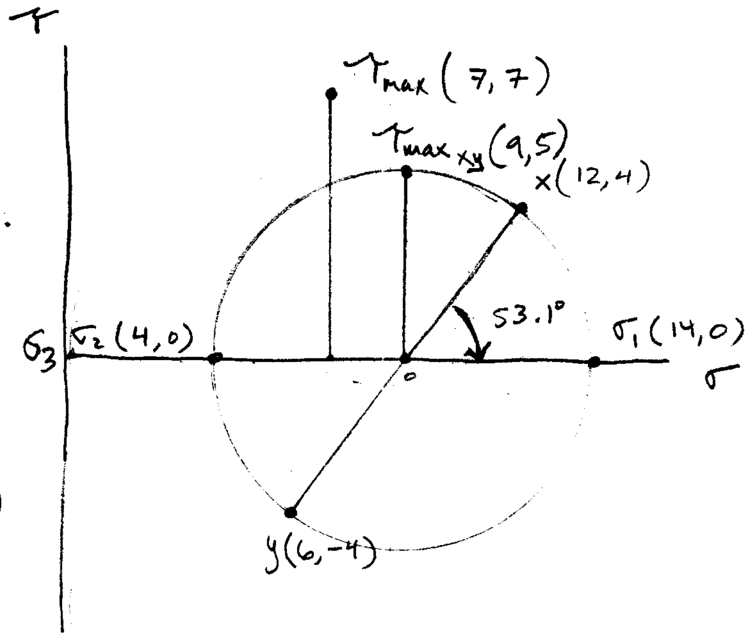
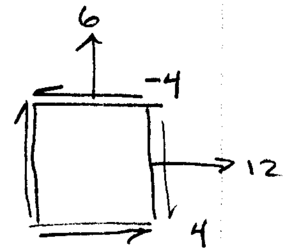
Questions, comments, gripes, and most importantly praise should be directed towards your loving TA during office hours MTF.

3.1 a) $\sigma_x = 12$, $\sigma_y = 6$, $\tau_{xy} = 4$ CW

There are many different ways to do Mohrs circle. This is one example.

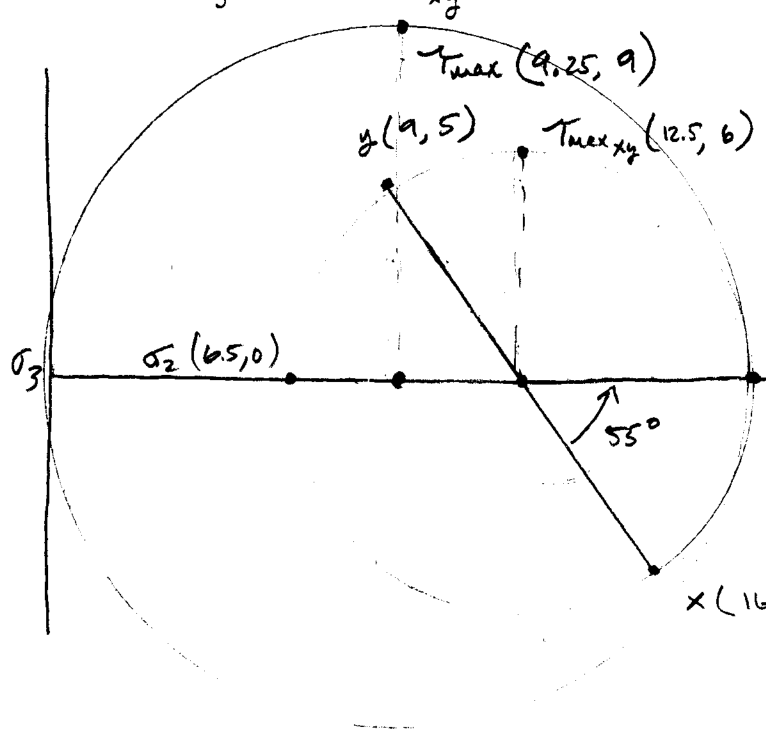
$x(\sigma_x, -\tau_{xy})$
$y(\sigma_y, \tau_{xy})$

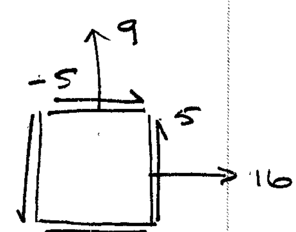




$\sigma_1 = 14$ @ $53.1/2 = 26.6°$ CW from axis
$\sigma_2 = 4$ @ $26.6° + 90° = 116.6°$ CCW from axis
$\tau_{max_{xy}} = 5$ @ $26.6° - 45° = 18.4°$ CCW from axis
$\tau_{max_{total}} = 7$

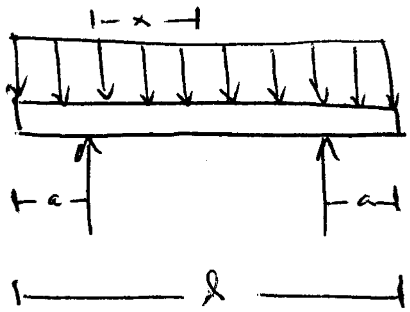
b) $\sigma_x = 16$ $\sigma_y = 9$ $\tau_{xy} = 5$ CCW

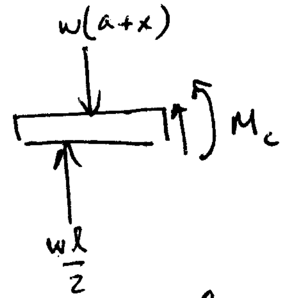




$\sigma_1 = 18.5$ @ $55/2 = 27.5°$ CCW from axis
$\sigma_2 = 6.5$ @, $27.5 = -27.5° + 90 = 62.5°$ CW from axis
$\tau_{max_{xy}} = 6$ @ $-27.5 + 45° + 90° = 17.5°$ CW from axis
$\tau_{max\ total} = 9.25$

3.13) (a)



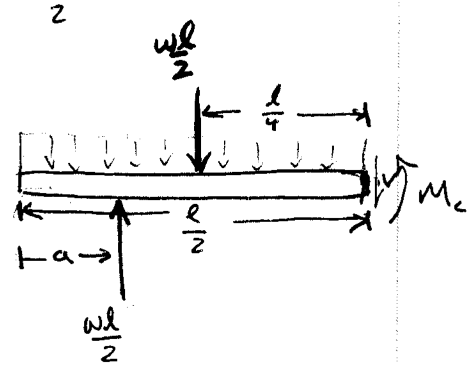


$a = 2.25$ in.

$l = 10$ in.

$w = 100$ lb/in

$$M = \frac{w}{2}\left(lx - (a+x)^2\right)$$

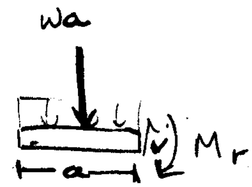


FIND MOMENT ABOUT CENTER:

$$M_c = \frac{wl}{2}\left(\frac{l}{2} - a\right) - \frac{wl}{2}\left(\frac{l}{4}\right)$$

$$= \frac{wl}{2}\left(\frac{l}{2} - a - \frac{l}{4}\right)$$

$$= \frac{wl}{2}\left(\frac{l}{4} - a\right) \Rightarrow \boxed{125\ (\text{lb}\cdot\text{in})}$$



FIND MOMENT ABOUT LEFT REACTION:

$$M_r = wa\left(\frac{a}{2}\right) = \frac{wa^2}{2} = \boxed{253.1\,(\text{lb}\cdot\text{in})}$$

(b) FIND OPTIMAL PLACEMENT OF SETBACK:

$$M_r = M_c$$

$$\frac{wa^2}{2} = \frac{wl}{2}\left(\frac{l}{4} - a\right)$$

$$\frac{\cancel{w}a^2}{\cancel{2}} = \frac{\cancel{w}l^2}{\cancel{8}\,4} - \frac{\cancel{w}la}{\cancel{2}}$$

$$a^2 = \frac{l^2}{4} - la \rightarrow \text{divide by } l^2$$

$$\frac{a^2}{l^2} = \frac{1}{4} - \frac{a}{l} \longrightarrow \frac{a^2}{l^2} + \frac{a}{l} - \frac{1}{4} = 0$$

quadratic equation:

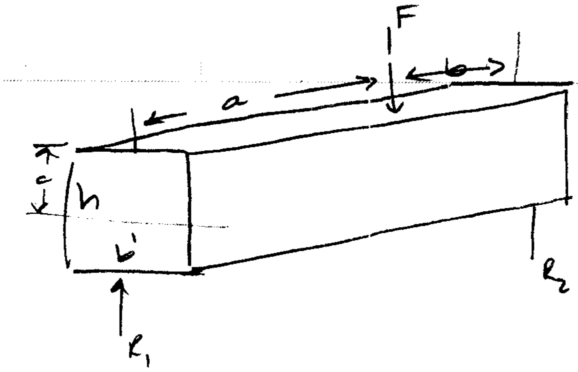
$$\frac{a}{l} = \frac{-1 \pm \sqrt{1 + 4(1)(\frac{1}{4})}}{2} = .207 \qquad a = .207\,l$$

$$\therefore \text{ since } M_{min} = \frac{wa^2}{2} = \frac{w(.207)^2 l^2}{2}$$

$$= .0214\,wl^2 = .0214(100)(10^2)$$

$$= \boxed{214\ \text{in}\cdot\text{lb}}$$

3-20)



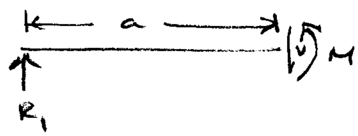
$l = a+b$

a) Show That $F = \dfrac{\sigma b' h^2 l}{6ab}$ — Make correction IN BOOK

$\sum M_{R_2} = 0$

$-R_1(a+b) + F(b) = 0$

$R_1 = \dfrac{Fb}{a+b} = \dfrac{Fb}{l}$.



$M = R_1 a = \dfrac{Fba}{l}$

$\sigma = \dfrac{Mc}{I}$ $\quad I = \dfrac{b'h^3}{12}$ → TABLE E-18 $\quad c = h/2$

$= \dfrac{M6}{b'h^2} = \dfrac{6Fba}{b'h^2 l}$

$\therefore \; F = \dfrac{b'h^2 \sigma l}{6ba}$

b) $\dfrac{F_m}{F} = \dfrac{(\sigma_m/\sigma)(b'_m/b')(h_m/h)^2(l_m/l)}{(a_m/a)(b_m/b)}$

S = scale Factor and $\sigma_m/\sigma = 1$

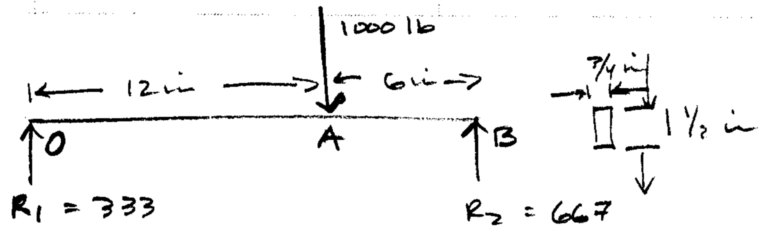
$\dfrac{(1)(s)(s)^2(s)}{(s)(s)} = s^2$

$F_m = s^2 F_m = 2^2 F_m = 4F$

ALTHOUGH WE'VE DOUBLED THE PHYSICAL DIMENSIONS WE ONLY QUADRUPLED THE LOAD CARRYING ABILITY.

3-26)

a)

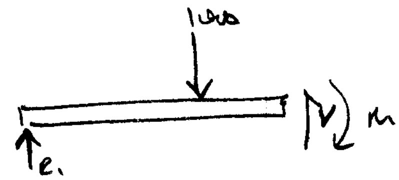


$\Sigma M_{R_2} = 0$

$-R_1(18) + 1000(6) = 0$

$R_1 = 333 \text{ lb}$

$\Sigma F = 0$



$333 - 1000 + V = 0$

$V = 667$

$\tau_{max} = \dfrac{3V}{2A} = \dfrac{3(667)}{2(1.25)} = \boxed{890 \text{ psi}}$

⇒ located on nuetral axis b/w points A and B

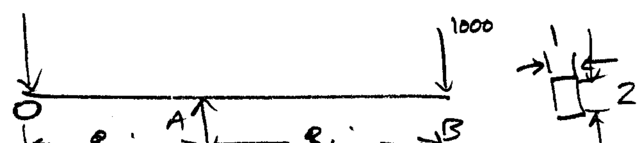
$\sigma_{max} = \dfrac{Mc}{I}$ $\quad M_A = M_{max} = 333(12) = 4000 \text{ lb·in}$

$= \dfrac{4000(.75)}{.211}$ $\quad c = h/2 = .75$

$= \boxed{14218 \text{ psi}}$ $\quad I = \dfrac{bh^3}{12} = \dfrac{3/4\,(3/2)^3}{12} = .211$

Located @ point A

b)



Use same methods as in part A for calculations

$\tau_{max} = \dfrac{3(1000)}{2(2)} = \boxed{750 \text{ psi}}$ on nuetral surface from O to B.

$\sigma_{max} = \dfrac{8000(1)}{.667} = \boxed{12{,}000 \text{ psi}}$ @ A

3-30) a) $J_{solid} = \frac{\pi d^4}{32}$

$= \frac{\pi (7)^4}{32}$

$= 236 \text{ cm}^4$

$J_{Hollow} = \frac{\pi}{32}(d_o^4 - d_i^4)$

$= \frac{\pi}{32}(7^4 - 5.8^4)$

$= 125 \text{ cm}^4$

Percent Reduction $= \frac{236 - 125}{236} \times 100$

$= \boxed{47\%}$

b) let $k = l \cdot \rho$ and $c = k\frac{\pi}{4}$

$W_{solid} = kA$

$= k\frac{\pi}{4}d^2$

$= Cd^2$

$= 49c$

$W_{Hollow} = kA$

$= k\frac{\pi}{4}(d^2 - d_i^2)$

$= c(d_o^2 - d_i^2)$

$= c(49 - 5.8^2)$

$= 15.4c$

percent Reduction $= \frac{49 + 15}{49} \times 100$

$= 68.6\%$

3-35) let $n = 5\ \text{r/min}$ and $r = 3$ in

Then, $V = \dfrac{2\pi r n}{12} = \dfrac{2\pi(3)(5)}{12} = 7.85$ ft/min

$F = \dfrac{H(33000)}{V} = \dfrac{(1)(33000)}{7.85} \cong 4200$ lb

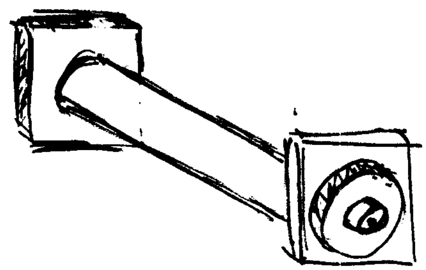
$T = F \cdot$ radius of cylinder

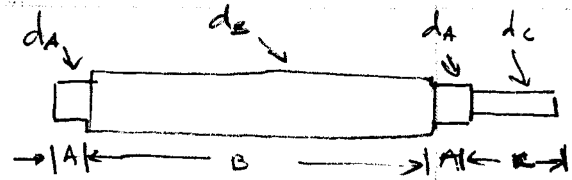
$= 4200(3) = 12600$ lb·in

$\tau = \dfrac{16T}{\pi d^3}$ } SEE Solution to 36 For derivation of this EqN.

$d = \left(\dfrac{16(12600)}{\pi(14000)}\right)^{1/3} = \boxed{1.66 \text{ in.}}$

3-36)





$$\omega = 2\pi n/60 = \frac{2\pi(8)}{60} = .838 \text{ rad/s}$$

$$T = \frac{H}{\omega} = \frac{1000}{.838} = 1190 \text{ N}\cdot\text{m}$$

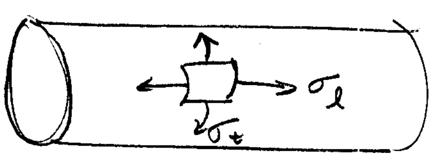
$$\tau_{max} = \frac{Tr}{J} \qquad r = \frac{1}{2}d \quad J = \frac{\pi}{32}(d^4)$$

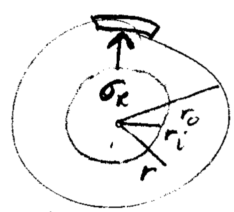
$$= \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} \Rightarrow d = \left(16T/\pi\tau\right)^{1/3} = \left(16(1190)/\pi(75)\right)^{1/3}$$

$$= \boxed{4.32 \text{ cm}}$$

3-41) determine whether pipe is thin or thick walled: $\frac{\text{radius}}{t} \geq 20 \Rightarrow$ thin walled

$\frac{3}{\frac{1}{4}} = 12$; so the cylinder is thick walled.





$r_i = 2.75$
$r_o = 3$

$\sigma_R, \sigma_t, \sigma_l$ are all principle stresses

$$\sigma_t = P\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = +11.52P$$

$$\sigma_l = \frac{Pr_i^2}{r_o^2 - r_i^2} = 5.26P$$

$$\sigma_R = P\frac{(r_i^2 - r_o^2)}{r_o^2 - r_i^2} = -P$$

Next page →

3-41 continued)

$\therefore \quad \tau_{max} = \dfrac{\sigma_t - \sigma_R}{2} = \dfrac{(11.52 + 1)P}{2} = 6.26P$

$\tau_{max} = 4000$ psi

$\dfrac{4000}{6.26} = P = \boxed{639 \text{ psi}}$

3-44) Thick walled vessel

$r_i = .875$
$r_o = 1.1875$

$\sigma_t = \dfrac{P(r_o^2 + r_i^2)}{r_o^2 - r_i^2} = 3.38P$

$\sigma_R = \dfrac{P(r_i^2 - r_o^2)}{r_o^2 - r_i^2} = -P$

$\sigma_t > \sigma_\ell > \sigma_R$

$\sigma_\ell = \dfrac{Pr_i^2}{r_o^2 - r_i^2} = 1.18P$

$\therefore \quad \sigma_t = \sigma_{max}$

$P = \dfrac{.8(57000)}{3.38} = \boxed{13491 \text{ psi}}$

USE TABLE E-20 TO get yield stress of MATerial

$S_y = 57$ kpsi