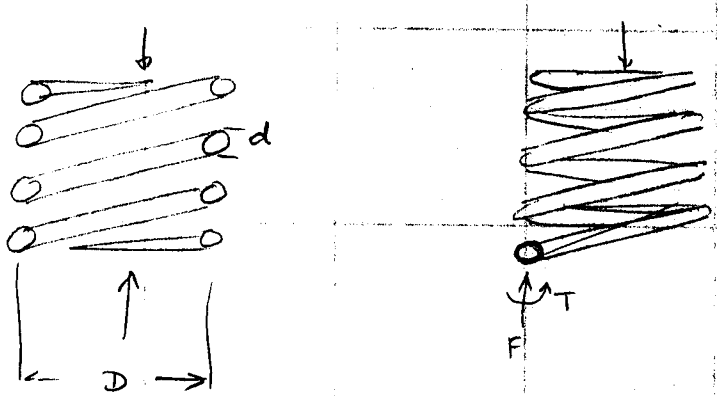


Read
10.1 – 10.8
Example 10.1

$$T = F\frac{D}{2}$$

$$\tau_{max} = \underbrace{\frac{Tr}{J}}_{\text{Torsion}} + \underbrace{\frac{F}{A}}_{\text{direct (non-flexural) shear}} \quad \text{(at inside fiber)}$$

$$T = \frac{FD}{2} \;; \quad r = \frac{d}{2} \;, \quad J = \frac{\pi d^4}{32} \;, \quad A = \frac{\pi d^2}{4}$$

$$\Rightarrow \quad \tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Spring Index $C = \dfrac{D}{d}$, a measure of coil curvature

$$\tau = K_s \frac{8FD}{\pi d^3}$$

K_s : Shear Stress correction factor.

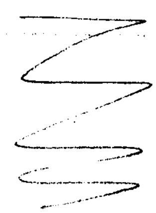
$$K_s = \frac{2C+1}{2C}$$

Curvature Effect.

Curvature of wire increases the stress on the inside of the spring. Highly localized – much like a stress concentration – important for fatigue.

$$K_c = \frac{2C\,(4C+2)}{(4C-3)\,(2C+1)}$$

Deflection of Helical Springs.

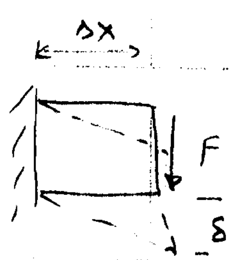


$l = \pi D N$

N: # of coils.

WE USE CASTIGLIANO'S THEOREM TO FIND THE DEFLECTION δ DUE TO F :

$$U_{TOTAL} = U_{TORSION} + U_{SHEAR}$$



Shear strain $\gamma = \dfrac{\delta}{\Delta x}$

$$\gamma = \frac{\tau}{G} = \frac{F}{AG}$$

$$dU = F\frac{\delta}{2} = \frac{F}{2}\cdot\gamma\,\Delta x$$

$$dU = \frac{F}{2}\cdot\frac{F}{AG}\,\Delta x$$

integrating over length l (constant F)

STRAIN ENERGY $U_S = \dfrac{F^2}{2AG}\,l$ for direct shear.

Similarly, $U_T = \dfrac{T^2 l}{2GJ}$ = Torsional Strain Energy.

deflection of a helical spring

$$U = U_T + U_S$$

$$= \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

$$= \frac{(F\,D/2)^2\,(\pi D N)}{2G\,(\pi d^4/32)} + \frac{F^2\,(\pi D N)}{2\,G\,(\pi d^2/4)}$$

$$= \frac{4F^2D^3N}{G\,d^4} + \frac{2F^2DN}{G d^2}$$

$$\delta = \frac{\partial U}{\partial F} = \frac{8FD^3N}{G d^4} + \frac{4FDN}{Gd^2}\left(\frac{d^2}{d^2}\right)\left(\frac{D^2}{D^2}\right)\left(\frac{2}{2}\right)$$

$$C = \frac{D}{d}$$

$$\delta = \frac{8FD^3N}{d^4 G}\left(1 + \frac{d^2}{2D^2}\right)$$

$$\delta = \frac{8FD^3N}{d^4 G}\left(1 + \frac{1}{2C^2}\right)$$

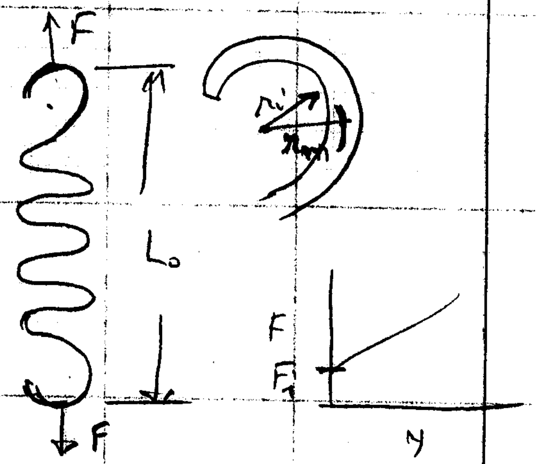
$$C \sim 6 \text{ to } 12 \rightarrow \frac{1}{2C^2} \ll 1$$

$$\Rightarrow \boxed{\delta = \frac{8FD^3N}{d^4 G}}$$

Now $\boxed{k = \frac{F}{\delta} = \frac{d^4 G}{8D^3N}}$

Note: Here N is the number of active coils, N_a, that participate in storing energy.

Extension springs



Stress conc. in the hook:

$$K = \frac{r_m}{r_i}$$

initial tension created to hold the free length more accurately.

Compression springs.

Note: ends can be plain, squared or ground.

SPRING Stability.

$$y_{cr} = L_o C_1 \left[1 - \left(1 - \frac{C_2}{\lambda_{eff}^2} \right)^{1/2} \right]$$

C_1, C_2 are elastic constants.

y_{cr} = critical deflection (onset of buckling)

$$\lambda_{eff} = \alpha \frac{L_o}{D}$$

α = end-condition constant (see Table 10-3.)

Steels: $\boxed{L_o < 2.63 \frac{D}{\alpha}}$ (eq. 10-16)

Nomenclature – See Table 10-2

end coils	N_e
Total coils	N_t
Free length	L_o
Solid length	L_s
pitch	p

N_a = active coils

Spring Materials

tensile strength S_{ut} varies with wire dia.

$$S_{ut} = \frac{A}{d^m} \quad , \quad A, m \rightarrow \text{Table 10-5}$$

Torsional yield strength

$$.35\, S_{ut} \le S_{sy} \le .52\, S_{ut}$$

(see eq. 10-18)
and eq. 10-28)

for music wire

$$S_{sy} = .45\, S_{ut}$$

Fatigue loading

$$F_a = \frac{F_{max} - F_{min}}{2}$$

$$F_m = \frac{F_{max} + F_{min}}{2}$$

Static

$$\tau = K_s \cdot K_c \frac{8 F D}{\pi d^3}$$

fatigue

$$\tau_a = K_s \cdot K_c \cdot \frac{8 F_a D}{\pi d^3}$$

a stress increase factor due to curvature of spring; treated as a stress conc. factor.

$$K_s \cdot K_c = K_B \quad (\text{Bergstrasser factor})$$

$$\tau_m = K_B \frac{8 F_m D}{\pi d^3} \qquad \begin{cases} K_s = \dfrac{2C+1}{2C} \\ K_B = \dfrac{4C+2}{4C-3} \end{cases}$$

Data: Size, material, and tensile strength do not affect the endurance limits (for ∞ life) for wire sizes under 10 mm.

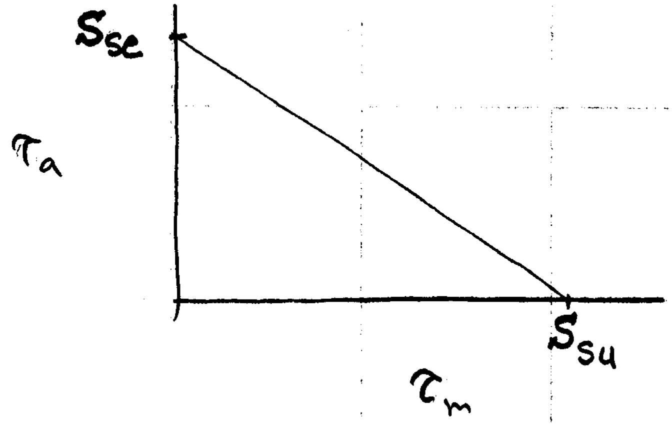
$$S_{se} = \underset{\text{surf. finish}}{k_a}\ \underset{\text{size}}{k_b}\ \underset{\text{loading}}{k_c}\ S_e' = 45 \text{ kpsi } (310 \text{ MPa})$$

(unpeened springs)

$S_{se} = 67.5$ kpsi. (465 MPa) for peened springs).

(not corrected for temp. or misc. effects).

Now we construct a Goodman diagram



S_{se} = torsional endurence limit

S_{su} = torsional rupture strength

use $S_{su} = 0.67\ S_{ut}$

Now $$\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} = \frac{1}{n}$$

EXAMPLE

(#10.3) Helical Comp. Sp. is wound using 0.105 in dia music wire OD = 1.225 in with plain Ground ends, 12 total Coils.
find (a) L_o ; (b) F_s (c) Sp. rate , (d) check buckling.

F_s = Force needed to compress to solid length.

Table 10-5 $m = 0.145$, $A = 201$ kpsi.

$$S_{ut} = \frac{201}{(.105)^{.145}} = 278.4 \text{ ksi} \qquad \text{(eq. 10-17)}$$

$$S_{sy} = .45(278.4) = 125.4 \text{ ksi} \qquad \text{(eq. 10-28)}$$

$$D = 1.225 - .105 = 1.120 \text{ in.} \qquad \text{(Mean spring dia)}$$

$$C = D/d = 10.67$$

$$K_B = \frac{4(10.67)+2}{4(10.67)-3} = 1.126 \qquad (10-6)$$

Table 10-2 : $N_a = 12 - 1 = 11$

$$L_s = .105(12) = 1.26 \text{ in.}$$

$$F_s = \frac{\pi d^3 S_{sy}}{8 K_s D} \qquad (10.3)$$

$$= \underline{45.2 \text{ lb}}$$

$$k = \frac{d^4 G}{8 D^3 N_a} = 11.5 \text{ lb/in.}$$

$$L_o = F_s/k + L_s = 5.2 \text{ in.}$$

Assuming $\alpha = 0.5$

(Eq. 10-16) $\frac{2.63 D}{\alpha} = 5.89$ in. → no buckling if both ends are fixed. ($L_o < 5.89$)