ME 356 AQ01 KUMAR

SOLUTION TO MIDTERM # 2.

THINGS TO LEARN FROM THIS TEST

- Problem # 1.

  To my surprise, most students ignored $f = 0.9$, and made the calculations more involved. But generally everyone knew how to go about estimating fatigue strength and expected life in the finite-life part of the S-N curve.

- Problem # 2.

  The main thing to note here is that we have a case of biaxial stresses: σ_1 and σ_2 are both present. These need to be "combined" via calculation of a Von Mises stress σ'. The alternating Von Mises stress σ_a' and the steady Von Mises stress σ_m' have to be determined before we go into the Goodman fatigue criteria.

- Problem # 3. See Text Example 7-14 for an identical problem.

MK.

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MK

#1. a) $S_e' = .506\ S_{ut} = (.506)(100) = 50.6$ ksi.

$$k_a = a\,S_{ut}^{\,b} = (2.67)(100)^{-.265} = .788$$

$$k_b = .874\,(1)^{-.107} = .879$$

assume remaining k's $= 1.0$

$$S_e = (.788)(.879)(50.6) = \underline{\underline{35.05 \text{ ksi}}}$$ Ans.

b) $S_f = a N^b$ $\qquad a = \dfrac{f^2 S_{ut}^2}{S_e}$

$f = 0.9$ (given) $\qquad b = -\frac{1}{3} \log \dfrac{f\,S_{ut}}{S_e}$

$$a = \frac{(.9)^2(100)^2}{35.05} = 231.1 \text{ ksi.}$$

$$b = -\frac{1}{3}\log\frac{(.9)(100)}{35.05} = -0.137$$

$$S_f = (231.1)(50{,}000)^{-.137} = \underline{\underline{52.5 \text{ ksi}}}$$ Ans.

c) $$N = \left(\frac{\sigma_a}{a}\right)^{1/b} = \left(\frac{65}{231.1}\right)^{\frac{1}{-0.137}} = \underline{\underline{10{,}464 \text{ Cycles}}}$$ Ans.

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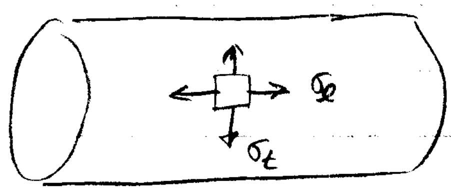
Midterm Prob # 2.

Assume no stress conc.

AISI 1018 Cold drawn.

$S_{ut} = 440$ MPa

$S_y = 370$ MPa



Thin-walled:

$$\sigma_t = \frac{pd}{2t} = \sigma_1$$

$$\sigma_\ell = \frac{pd}{4t} = \sigma_2$$

$$\sigma_r = -p = \sigma_3$$

note: σ_t, σ_ℓ and σ_r are also *principal* Stresses

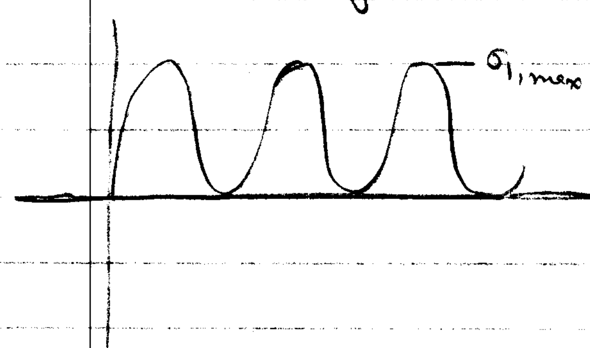
$d = 500$ mm $\qquad t = 5$ mm

$$\frac{d}{t} \sim \frac{500}{5} \sim 100$$

($\rightarrow \sigma_t, \sigma_\ell \gg p$ $\rightarrow$ neglect σ_r)

$$\sigma_{1,max} = \frac{pd}{2t} \quad (p = p_{max}) \quad \sigma_1$$

$$\sigma_{1,min} = 0$$



$$\sigma_{1,a} = \frac{\sigma_{1,max} - \sigma_{1,min}}{2} = \frac{p_{max}\, d}{4t}$$

$$\sigma_{1,m} = \frac{\sigma_{1,max} + \sigma_{1,min}}{2} = \frac{p_{max}\, d}{4t}$$

let us drop the subscript 'max', and say

$$\sigma_a = \frac{pd}{4t} \qquad \sigma_m = \frac{pd}{4t}$$

where p is the design pressure.

Similarly,

$$\sigma_{2a} = \frac{\sigma_{2\,max}}{2} = \frac{pd}{8t}$$

$$\sigma_{2m} = \frac{pd}{8t}$$

σ_1 and σ_2 are axial (tensile) stresses.
we will multiply the alternating components of σ_1 and σ_2 by the factor $\frac{1}{k_{c,ax}}$

$\Rightarrow$

$$\sigma_{1a} = \frac{pd}{k_c 4t} \qquad \sigma_{1m} = \frac{pd}{4t}$$

$$\sigma_{2a} = \frac{pd}{k_c 8t} \qquad \sigma_{2m} = \frac{pd}{8t}$$

$$\sigma_{3a} = \sigma_{3m} = 0$$

$$\sigma_a' = \left[\frac{(\sigma_{1a} - \sigma_{2a})^2 + (\sigma_{2a} - \sigma_{3a})^2 + (\sigma_{3a} - \sigma_{1a})^2}{2}\right]^{1/2}$$

$$2\sigma_a'^2 = \left(\frac{pd}{4k_c t} - \frac{pd}{8k_c t}\right)^2 + \left(\frac{pd}{8k_c t}\right)^2 + \left(\frac{pd}{4k_c t}\right)^2$$

$$= \left(\frac{2pd - pd}{8k_c t}\right)^2 + \left(\frac{pd}{8k_c t}\right)^2 + \left(\frac{pd}{4k_c t}\right)^2$$

$$= \left(\frac{pd}{8k_c t}\right)^2 + \left(\frac{pd}{8k_c t}\right)^2 + \left(\frac{pd}{4k_c t}\right)^2$$

$$= \left(\frac{pd}{k_c t}\right)^2\left[\frac{1}{8^2} + \frac{1}{8^2} + \frac{1}{4^2}\right] = \left(\frac{pd}{k_c t}\right)^2\left[\frac{1}{32} + \frac{1}{16}\right]$$

$$\sigma_a'^2 = \left(\frac{pd}{k_c t}\right)^2\left(\frac{3}{32}\right)\cdot\frac{1}{2} = \frac{3}{64}\left(\frac{pd}{k_c t}\right)^2$$

$$\frac{1+2}{32}$$

$$\sigma_a' = \frac{\sqrt{3}}{8}\left(\frac{pd}{k_c t}\right)$$

Similarly,

$$\sigma_m' = \frac{\sqrt{3}}{8}\left(\frac{pd}{t}\right)$$

for fatigue failure: $(n_f = 1.0)$ (Goodman)

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = 1$$

→ Solve for p.

$$S_{ut}\,\sigma_a' + S_e\,\sigma_m' = S_e S_{ut}$$

$$S_{ut}\left[\frac{\sqrt{3}}{8}\frac{pd}{k_c t}\right] + S_e\left[\frac{\sqrt{3}}{8}\frac{pd}{t}\right] = S_e S_{ut}$$

$$p\left[\frac{\sqrt{3}}{8}\frac{d}{t}\right]\left\{\frac{S_{ut}}{k_c} + S_e\right\} = S_e S_{ut}$$

$$\therefore\ p = \frac{S_e S_{ut}}{\left(\frac{\sqrt{3}}{8}\frac{d}{t}\right)\left(S_{ut}/k_c + S_e\right)}$$

For our problem:

$d/t = \frac{500}{5} = 100$; $S_e' = .506(440) = 223$ MPa

$k_{surface} = .887$ assume all other k's $= 1.0$

$\therefore$ $S_e = (.887)(223) = 197.8$ MPa.

For axial loads $k_c = 0.85$

$$p = \frac{(197.8)(440)}{\left(\frac{\sqrt{3}}{8}\right)(100)\left(\frac{440}{.85} + 197.8\right)} = 5.62 \text{ MPa}.$$

Thus, a pressure of 5.62 MPa will eventually cause a fatigue failure. This pressure can be thought of as a "fatigue strength of this pressure vessel."

#3.

AISI 1018 Steel $S_{ut} = 440$ MPa

$S_y = 370$ MPa

Fully reversed $T = 100$ N.m , $M = 200$ N.m

a) $S_e' = (.506)(440) = 223$ MPa

$k_a = a\, S_{ut}^{\,b} = (4.45)(440)^{-.265} = 0.89$

$k_b = 1.24\, d^{-.107} = (1.24)(25)^{-.107} = 0.88$

Rest of the k's = 1.0

$S_e = (.89)(.88)(223) = \underline{\underline{175 \text{ MPa}}}$

b) Factor of Safety guarding against static failure (yielding in the first cycle).

$n = \dfrac{S_y}{\sigma'}$ when σ' = max Von Mises Stress in the first cycle

$Z_{net} = \dfrac{\pi(.68)(25)^3}{32} = 1043 \text{ mm}^3$

$\sigma_o = \dfrac{M}{Z_{net}} = \dfrac{200 \text{ N.m}}{1.043 \times 10^{-9} \text{ m}^3} = 191.8$ MPa

$\tau_o = \dfrac{Td}{2J_{net}}$ $J_{net} = \dfrac{\pi(.82)(25)^4}{32} = 31447 \text{ mm}^4$

$\tau_o = \dfrac{(100 \text{ N.m})(25 \times 10^{-3} \text{ m})}{2 \times 31447 \times 10^{-12} \text{ m}^4} = 39.7$ MPa

$K_f = 1 + q(K_t - 1) = 1 + .76(2.07 - 1) = 1.81$ (bending)

$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + .93(1.58 - 1) = 1.54$ (torsion)

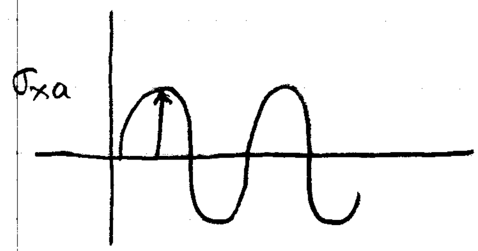
$\sigma_{xm} = \tau_{xy,m} = 0$ (fully reversed stresses)

$\sigma_{xa} = K_f \, \sigma_o = (1.81)(191.8) = 347.2 \text{ MPa}$

$\tau_{xy,a} = K_{fs} \, \tau_o = 1.54 \, (39.7) = 61.2 \text{ MPa}$

Von-Mises stress

$\sigma_a' = \sqrt{\sigma_{xa}^2 + 3 \, \tau_{xy,a}^2}$



$= \sqrt{347.2^2 + 3 \, (61.2)^2}$

$\sigma_a' = 363 \text{ MPa}$

Now for yielding in first cycle, $\sigma_{max}' = \sigma_a'$

$n = \dfrac{S_y}{\sigma_{max}'} = \dfrac{370}{363} = \underline{\underline{1.02}}$ Ans

(c) for fatigue failure

$n_f = \dfrac{S_e}{\sigma_a'} = \dfrac{175}{363} = \underline{\underline{0.48}}$ Ans

Since $n_f < 1.0$, or $\sigma_a' > S_e$, this shaft can not have infinite life.

(d) we estimate N: $\sigma_a' = S_f = a N^b$

Assuming $f = 0.9$, $a = \dfrac{(.9 \, S_{ut})^2}{S_e} = \dfrac{(.9)^2 (440)^2}{175} = 896 \text{ MPa}$

$b = -\dfrac{1}{3} \log \left(\dfrac{(.9) \times 440}{175} \right) = -0.118$

$N = \left(\dfrac{363}{896} \right)^{\frac{1}{-0.118}} = \underline{\underline{2060 \text{ Cycles}}}$ Ans