The Complexity of Channel Scheduling in Multi-Radio Multi-Channel Wireless Networks

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***Abstract*—The complexity of channel scheduling in Multi-Radio Multi-Channel (MR-MC) wireless networks is an open research topic. This problem asks for the set of edges that can support maximum amount of simultaneous traffic over orthogonal channels under a certain interference model. There exist two major interference models for channel scheduling, with one under the physical distance constraint, and one under the hop distance constraint. The complexity of channel scheduling under these two interference models serves as the foundation for many problems related to network throughput maximization. However, channel scheduling was proved to be NP-Hard only under the hop distance constraint for SR-SC wireless networks. In this paper, we fill the void by proving that channel scheduling is NP-Hard under both models in MR-MC wireless networks. In addition, we propose a polynomial-time approximation scheme (PTAS) framework that is applicable to channel scheduling under both interference models in MR-MC wireless networks. Furthermore, we conduct a comparison study on the two interference models and identify conditions under which these two models are equivalent for channel scheduling.**

***Index Terms*—Complexity; Channel Scheduling; Multi-Radio Multi-Channel Wireless Networks; Maximum weight channel scheduling; NP-Hard; Polynomial time approximation scheme; PTAS**

I. Introduction

Multi-Radio Multi-Channel (MR-MC) Wireless Networks have attracted increasing interest in recent years. Equipped with multiple radios, nodes can communicate with multiple neighbors simultaneously over orthogonal channels. These concurrent transmissions can significantly improve the network throughput [9]. According to the IEEE 802.11 standard, 3 of the 11 specified channels in 802.11b/g are orthogonal and 802.11a provides 13 orthogonal channels [14].

To enable the concurrent transmissions via multiple radios transmitting over orthogonal channels simultaneously, the key problem is the channel scheduling, which aims to maximize the concurrent traffics without interfering each other. Channel scheduling in MR-MC wireless networks has been investigated under the assumption that the communication range equals the interference range [19]. This assumption can't be satisfied since in reality, a node's interference range, denoted by $d_{interference}$, is usually larger than its communication range, denoted by d_{com}. This means that two nodes might interfere with each other although they can not communicate with each other. In this paper, we study the complexity of channel scheduling in MR-MC networks when $d_{interference} \geq d_{com}$.

Without loss of generalities, we set $d_{interference} = P \times d_{com}$, where $P \geq 1$. We call this the P *interference-free model*. Then, if two nodes want to launch bidirectional communications, any other node whose minimum distance to the two nodes is $\leqslant d_{interference}$ must keep silent. This indicates that the distance between any two interference-free communications should be $> d_{interference}$. A channel scheduling that supports only interference-free concurrent communications under the P interference-free model is called P *interference-free* channel scheduling.

In this paper, we will study the complexity of interference-free channel scheduling in MR-MC wireless networks. Intuitively, the channel scheduling in MR-MC networks should be harder than that in single-radio single-channel (SR-SC) networks. Channel scheduling is always dependent on the traffic demands. Generally speaking, the traffic demands among nodes, which are based on the underlying routing strategy, vary from time to time. At any given moment, the traffic demands together with the network topology form a *demand graph*, denoted by $G(V, E)$, where each edge in E is associated with a weight corresponding to the traffic demand. Given an edge-weighted graph $G(V, E)$, a channel scheduling will compute $E' \subseteqq E$, such that all the edges in E' can be activated concurrently without interference. An *Optimal Weighted Channel Scheduling under the Physical distance constraint (OWCS/P)*, can be defined as follows: *Given an edge-weighted demand graph $G(V, E)$ representing an MR-MC wireless network, compute an optimal channel scheduling $O(G) \in E$, such that $O(G)$ is P interference-free and the weight of $O(G)$ is maximized under the P interference-free model.*

Two interfering nodes might not realize the existence of each other when $P > 1$ if no position information is available. Therefore the OWCS/P relying on the physical distance constraint is not practical. This is the reason why most existing channel scheduling schemes employ hop distance constraint instead of the physical distance constraint. In the *hop interference-free model*, two nodes interfere with each other if and only if they are within H hops. Correspondingly, we can define the problem of *Optimal Weighted Channel Scheduling under Hop distance constraint (OWCS/H)*. However, these two interference models are not the same as shown in Fig. 1. Both the physical interference-free model and the hop interference-free model are popular but their relations have never been addressed in literature. In this paper, we will investigate the conditions under which these two interference models are equivalent.

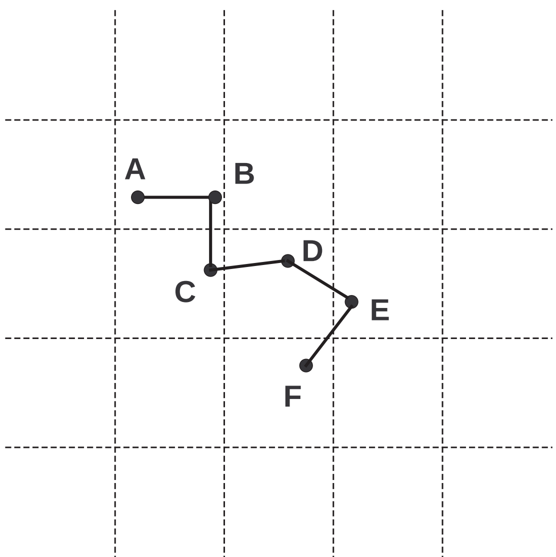

Fig. 1. The difference between physical distance constraint and hop distance constraint. The hop distances from A to C and from F to C are 2 and 3, respectively. The physical distance from A to C and from F to C are larger than 1 but smaller than 2. Assume $d_{com} = 1$ and $d_{interference} = 2$. Then, C is interference-free with both A and F under 1 hop constraint, but C does interfere with them.

This paper also tackles the following two open problems. Note that from now on we use OWCS/H$_{\geq 1}$ (OWCS/P$_{\geq 1}$) to represent the OWCS/H (OWCS/P) problem when $H \geq 1$ ($P \geq 1$).

1) It has been shown that OWCS/H$_{\geq 1}$ is NP-Hard in SR-SC wireless networks [18]. Since SR-SC is a special case of MR-MC, OWCS/H$_{\geq 1}$ is NP-Hard too in MR-MC networks. However, whether OWCS/P$_{\geq 1}$ is NP-hard or not is still open. In Section IV, we close this question by proving the NP-Hardness of OWCS/P$_{\geq 1}$ in MR-MC networks.
2) As both OWCS/P$_{\geq 1}$ and OWCS/H$_{\geq 1}$ are NP-Hard, it is impossible to find out an optimal solution in polynomial time. We are interested in the following question: *is there a Polynomial Time Approximation Scheme (PTAS) for OWCS/P$_{\geq 1}$ and OWCS/H$_{\geq 1}$ in MR-MC wireless networks?* For a maximization optimization problem such as OWCS/P$_{\geq 1}$ or OWCS/H$_{\geq 1}$, a PTAS can find a solution that is at least $(1 - \epsilon)$ times of the optimal in polynomial time for an infinitesimally small real number ϵ. Whether a PTAS exists for OWCS/P$_{\geq 1}$ or OWCS/H$_{\geq 1}$ in MR-MC wireless networks is open. Answering this question can facilitate scheduling algorithm design and cross-layer optimization. Motivated by the observations that many NP-Hard problems admit PTAS when restricted to geometric graphs [1], and that OWCS/H$_{\geq 1}$ in SR-SC wireless networks [17] has a PTAS, we design a PTAS framework that is applicable to both OWCS/P$_{\geq 1}$ and OWCS/H$_{\geq 1}$ in Section V.

The major contributions of the paper can be summarized as follows:

- We study the complexity of OWCS/P$_{\geq 1}$, in both SR-SC and MR-MC wireless networks, and prove that OWCS/P$_{\geq 1}$ is NP-Hard for both types of networks.
- We construct a PTAS for both OWCS/P$_{\geq 1}$ and OWCS/H$_{\geq 1}$ in MR-MC wireless networks.
- We obtain the conditions under which OWCS/P$_{\geq 1}$ and OWCS/H$_{\geq 1}$ are equivalent. To be specific, OWCS/P$_{=1}$ and OWCS/H$_{=1}$ are equivalent; and OWCS/P$_{\geq 1}$ and OWCS/H$_{=1}$ are equivalent under a polynomial time transformation.
- The results in this paper fill the void of complexity study for channel scheduling in wireless networks.

The rest of the paper is organized as follows: Section II discusses the related work focusing on channel scheduling in MR-MC networks and complexity study in SR-SC wireless networks. In Section III, we present our network model and assumptions. Section IV analyzes the complexity of OWCS/P$_{\geq 1}$ in MR-MC wireless networks. Section V proposes a PTAS framework for OWCS/P$_{\geq 1}$ in MR-MC wireless networks. Section VI argus that the PTAS framework proposed for OWCS/P$_{\geq 1}$ is applicable to OWCS/H$_{\geq 1}$. Section VII conducts a comparison analysis on the physical and hop interference-free models. Finally, Section VIII summarizes and concludes the paper.

II. Related Work

In this section, we survey the most related research on channel scheduling for MR-MC networks and on complexity analysis for SR-SC networks.

The benefits of using multiple radios and channels have been theoretically studied in [9], [12] and [8]. A topology control approach jointly considering channel scheduling was studied in [13]. By considering two types of traffic demands, [20] proposed both dynamic and static channel scheduling and link scheduling methods. Ref. [21] jointly considered the scheduling problem with routing in MR-MC wireless networks. This work proposed a column based approach to decompose the original problem into sub-problems and solves them iteratively.

The concept of employing control (or primary) channels and communication (or secondary) channels was investigated in many works. In [11], the multiple radios at each node are divided into two groups, with one assigned fixed channels for packet reception and connectivity maintenance, and the other assigned switchable channels for capacity increasing. A common default channel is introduced in [7], [15] and [6] to handle the network partition caused by dynamic channel assignment, and to facilitate channel negotiation for data communications. Note that the primary channel does not have to be the same for all nodes in the network [11], [10] and [14].

Another important category of related research is code based channel assignment. The CDMA code assignment problem is considered in [4] and [2]. In [19], we proposed a s-disjunct code based localized channel assignment scheme for MR-MC wireless networks.

The complexity of channel scheduling in MR-MC wireless networks under the physical distance constraint is still open. There exist several works targeting the scheduling complexity of SR-SC wireless networks under the hop distance constraint [5] [18] [16] [17]. Ref. [5] introduced a polynomial time link scheduling algorithm under the node exclusive interference model where no two edges incident on the same node. This work proved that the optimal channel scheduling is reducible to the classical maximum matching problem and can be solved in polynomial time. Ref. [3] extended the problem by considering edge weight and proved that the maximum weighted matching problem can be solved in polynomial time. Ref. [18] showed that OWCS/H$_{=1}$ in SR-SC wireless networks is NP-Hard. The OWCS/H$_{\geq 2}$ was shown to be NP-Hard in [16] for SR-SC networks. Since SR-SC is a special case of MR-MC, the OWCS/H$_{\geq 1}$ is NP-Hard in MR-MC wireless networks. A PTAS for OWCS/H$_{\geq 1}$ in SR-SC wireless networks was proposed in [17]. The existence of the PTAS for both OWCS/P and OWCS/H in MR-MC networks is open.

Table I summarizes the current result on the complexity of OWCS/P and OWCS/H, where each entry indicates whether the problem is known to be NP-Hard; and whether it is known to admit a PTAS.

$OWCS$/-	SR-SC Networks	MR-MC Networks
$P \geq 1$	Unknown ; Unknown	Unknown ; Unknown
$H \geq 1$	NP-Hard ; PTAS	NP-Hard ; Unknown

TABLE I
CURRENT OWCS COMPLEXITY RESULTS.

In this paper, we will prove that OWCS/P$_{\geq 1}$ is NP-Hard in both SR-SC and MR-MC networks, and propose a PTAS framework that is applicable to both OWCS/P$_{\geq 1}$ and OWCS/H$_{\geq 1}$ in MR-MC wireless networks. These results complete the complexity study for channel scheduling in wireless networks, i.e. this paper will give answers to all the "Unknown"s listed in Table I.

III. NETWORK MODEL AND DEFINITIONS

The network with traffic demands is modeled by an edge-weighted graph $G(V,E)$, where $|V| = n$ denotes the total number of nodes. An edge weight indicates the amount of traffic delivered along that edge. There exists a set of $C = \{c_1, c_2, \cdots, c_k\}$ orthogonal channels. For $\forall$ node $i \in V$, $1 \leq i \leq n$, it is equipped with r_i radios and can access a set of $C_i \subseteq C$ channels, where $|C_i| = k_i$. For example, C can be the orthogonal channels in 802.11a wireless networks.

In this paper, we consider the OWCS/P and OWCS/H problems based on geometric graphs. These problems looks for the set of edges that can support maximum amount of simultaneous traffic under the physical or hop interference-free model. Let d_{com} be the uniform communication range for all nodes. Usually, $d_{com} \leq d_{interfere}$, where $d_{interference}$ is the uniform interference range. Without loss of generality, we set $d_{com} = 1$ and set $d_{interfere} = P \times d_{com} = P$. Under the physical interference-free model, two nodes interfere with each other if and only if their physical distance is $\leq d_{interference}$. Under the hop interference-model, two nodes interfere with each other if and only if their hop distance is $\leq H$.

Given an edge-weighted network $G(V,E)$, let $d(u,v)$ denote the distance between node u and v, where $u, v \in V$.

Definition 3.1: ***Edge-physical-distance***: the physical distance between edge (u_1, u_2) and (v_1, v_2) equals to $\mathbf{min}\{d(u_i, v_j)\}$, where $i, j \in \{1, 2\}$.

If $d(u,v) < d_{com}$, u and v can reach each other in 1 hop, where $u, v \in V$ and $u \neq v$. Let $h(u,v)$ denotes the minimum hop distance, which is the number of hops between node u and v, where $u, v \in V$.

Definition 3.2: ***Edge-hop-distance***: the hop distance between edge (u_1, u_2) and (v_1, v_2) equals to $\mathbf{min}\{h(u_i, v_j)\}$, where $i, j \in \{1, 2\}$.

The classical matching problem is to find a subgraph of G, such that any two edges in the subgraph do not share a common node. Next, we define two induced matching problems for OWCS/P and OWCS/H, respectively.

Definition 3.3: A ***P interference-free matching*** G' is a subgraph of G, such that the edge-physical-distance between any two edges in $G' > P$.

Definition 3.4: A ***H-hop interference-free matching*** G' is a subgraph of G, such that the edge-hop-distance between any two edges in $G' > H$.

Let $w(e)$ denote the weight of an edge $e \in E$.

Definition 3.5: The ***Weight of a graph*** G, denoted by $W(G)$, is the sum of the weights of all the edges in G.

Definition 3.6: The ***optimal P interference-free matching*** $O(G)$ is a P interference-free matching of G, such that $W(O(G))$ is the maximum among all the P interference-free matchings of G.

Definition 3.7: The ***optimal H-hop interference-free matching*** $O(G)$ is a H-hop interference-free matching of G, such that $W(O(G))$ is the maximum among all the H-hop interference free matchings of G.

Defs. 3.7 and 3.6 give the formal definitions of OWCS/P and OWCS/H. In the following sections, we use subscript to indicate the value or range of values of P and H. For example, OWCS/P$_{=1}$ is the problem constrained by $P = 1$.

IV. The Complexity of OWCS/P

In Section II, we have mentioned that the complexity of OWCS/P is still open. In this section, we will first prove that OWCS/P is NP-Hard in SR-SC wireless networks. This result can be directly generalized to MR-MC networks as SR-SC is a special type of MR-MC networks.

To prove the NP-hardness of OWCS/P in SR-SC networks, we consider the complexity of OWCS/P$_{=1}$ first. We claim that OWCS/P$_{=1}$ is equivalent to OWCS/H$_{=1}$ in SR-SC networks.

Lemma 4.1: The problems OWCS/P$_{=1}$ and OWCS/H$_{=1}$ are equivalent in SR-SC wireless networks.

Proof: Let OPT/P$_{=1}$ be the set consisting of all the edges of any optimal solution to the OWCS/P$_{=1}$ problem for a SR-SC network $G(V, E)$. According to the definition of the OWCS/P$_{=1}$ problem, for $\forall e_1, e_2 \in$ OPT/P$_{=1}$, their edge-physical-distance is > 1. This indicates that the two incident nodes of e_1 can not communicate with the two incident nodes of e_2, which means that the edge-hop-distance of e_1 and e_2 is > 1. Therefore OPT/P$_{=1}$ is a feasible solution to OWCS/H$_{=1}$ for G.

For $\forall e_3 \in E$ but $\notin$ OPT/P$_{=1}$, there exists an edge $e_4 \in$ OPT/P$_{=1}$ such that the edge-physical-distance between e_3 and e_4 is at most 1, which means that at least one incident node of e_3 has a direct edge connecting to one of the incident nodes of e_4 in G. Therefore the edge-hop-distance between e_3 and e_4 is at most one hop, which means that e_3 could not be placed in OPT/P$_{=1}$ under the hop-distance interference model. Thus OPT/P$_{=1}$ is an optimal solution to OWCS/H$_{=1}$.

Based on a similar argument, an optimal solution to OWCS/H$_{=1}$ for network G is also an optimal solution to OWCS/P$_{=1}$ for the same G. Therefore, The problems OWCS/P$_{=1}$ and OWCS/H$_{=1}$ are equivalent for any SR-SC wireless network. ■

We can also prove Lemma 4.1 based on the concept of *interference graphs*. Given an edge-weighted graph $G(V, E)$, the interference graph is a node-weighted graph $G_I(V_I, E_I)$, where $V_I = \{e \mid e \in V\}$, $E_I = \{(e_1, e_2) \mid e_1$ and e_2 interfere with each other in $G\}$, and the weight of node e in G_I equals the weight of the corresponding edge e in G. When $P = 1$ and $H = 1$, the corresponding interference graphs are the same for the same network G. Therefore the optimal scheduling, which is equivalent to computing a maximum weight independent set in G_I for SR-SC wireless networks, will be the same.

Since OWCS/H$_{=1}$ is NP-Hard in SR-SC wireless networks [18], we can derive the following theorem from Lemma 4.1:

Theorem 4.1: OWCS/P$_{=1}$ is NP-Hard in SR-SC wireless networks.

As SR-SC wireless networks is a special case of MR-MC wireless networks, we conclude that OWCS/P$_{=1}$ is NP-Hard in MR-MC wireless networks. This is summarized by the following theorem:

Theorem 4.2: OWCS/P$_{=1}$ is NP-Hard in MR-MC wireless networks.

Now we have proved that the OWCS/P problem is NP-hard when $P = 1$ for both SR-SC and MR-MC networks. In the following, we will investigate the complexity of OWCS/P$_{>1}$.

Theorem 4.3: OWCS/P$_{>1}$ is NP-Hard in SR-SC wireless networks.

Proof: To prove the NP-hardness of OWCS/P$_{>1}$ in SR-SC wireless networks, we will show that OWCS/P$_{>1}$ is polynomial-time reducible to OWCS/P$_{=1}$ in SR-SC wireless networks.

Let $G(V, E)$ be an edge-weighted graph for an instance of OWCS/P$_{>1}$. We can construct $G'(V', E')$, a problem instance of OWCS/P$_{=1}$, in the following way: For each node $u \in V$, we create a corresponding node u' in G'. All u''s form V'. An edge (u', v') exists in E' if and only if $d(u, v) \leq P$ in G. The weight of an edge (u', v') in G' is set to $-\infty$ if $(u', v') \notin E$; otherwise, its weight remains the same as in the original grant G. Let G' be an instance of OWCS/P$_{=1}$. It is obvious that this construction procedure takes polynomial time. An example illustrating the transformation from G to G' is given in Figs. 2 and 3.

For any optimal solution OPT to OWCS/P$_{=1}$ in G', it is also an optimal solution to OWCS/P$_{>1}$ in G as G is a subgraph of G' and $G' \setminus G$ contains only the edges with the weight $-\infty$. None of these edges could be included in OPT that signals maximum weight scheduling.

Similarly we can argue that any optimal solution to OWCS/P$_{>1}$ in G is also an optimal solution to OWCS/P$_{=1}$ in G'. This completes the proof. ■

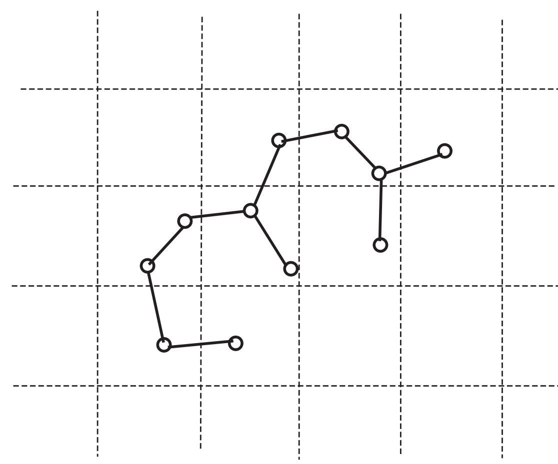

Fig. 2. A Traffic Demand Graph G with edge weight not shown. Here $P = 2$ ($d_{com} = 1$ and $d_{interference} = 2d_{com}$).

Since SR-SC is a special case of MR-MC, we obtain the following theorem:

Theorem 4.4: OWCS/P$_{>1}$ is NP-Hard in MR-MC wireless networks.

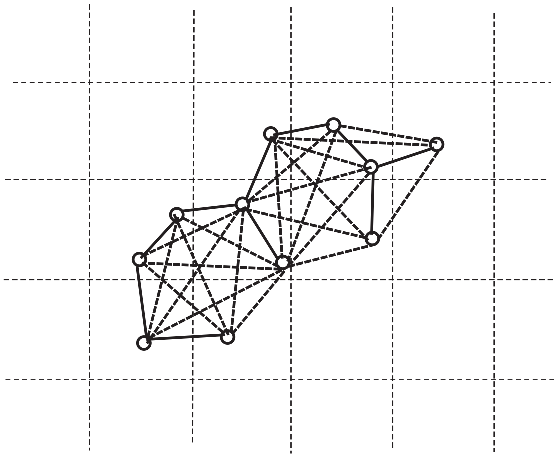

Fig. 3. The transformed graph G', where the dash lines are the new edges whose length is greater than 1 and ≤ 2 in the original graph G. The weight of each sold edge remains the same as in G and the weight of each dashed edge is $-\infty$. In G', $P = 1$.

By combining the results of Theorems 4.2 and 4.4, we conclude that OWCS/P$_{\geq 1}$ is NP-Hard in MR-MC wireless networks.

Note that we could not construct an interference graph and then compute a maximum-weight independent set for MR-MC networks due to the special constraints resulted from the number of radios per node: a node can transmit on all radios over different orthogonal channels simultaneously.

V. A Polynomial Time Approximation Scheme for P-Interference-Free Scheduling in MR-MC Wireless networks

In Section IV, we have proved that OWCS/P$_{\geq 1}$ in MR-MC wireless networks is NP-Hard. In this section we will construct a Polynomial-Time Approximation Scheme (PTAS) for this problem. In other words, we will propose a procedure that can compute a polynomial-time approximate solution with a performance ratio $(1-\epsilon)$ for an arbitrarily small positive number ϵ. Let $P_{tas}(G)$ denote the solution given by the PTAS procedure and $O(G)$ the optimal solution for the OWCS/P$_{\geq 1}$ problem in a MR-MC network G. We will prove that $W(P_{tas}(G)) \geq (1-\epsilon)W(O(G))$.

A. Summary of the PTAS Construction

The PTAS construction procedure is summarized by the following four steps. We will detail steps 3 and 4 in the following subsections.

1) **Griding:** Let $\mathcal{A}$ be the smallest square aligned along the x and y axes that can cover the network G. Partition $\mathcal{A}$ into small grids with each having a size of $(P+2) \times (P+2)$. For simplicity, we assume that the size of $\mathcal{A}$ is a multiple of $(P+2)$. Now we label each grid by (a, b), where $a, b = 0, 1, \cdots, N-1$, with N the total number of grids at each row or column. Then the id of the grid at the lower-left corner can be denoted by $(0, 0)$. We also denote the ith row and the jth column of the grids by Row_i and Col_j, respectively.
2) **Shifted Dissection:** Partition vertically the area $\mathcal{A}$ by columns of the grids Col_j, where $j = k_1, k_1 + m + 1, k_1 + 2(m+1), \cdots$, and $k_1 = 0, 1, \cdots, m$. Then partition horizontally $\mathcal{A}$ by rows of the grids Row_i, where $i = k_2, k_2 + m + 1, k_2 + 2(m+1), \cdots$, and $k_2 = 0, 1, \cdots, m$. We obtain a number of super-grids with each containing exactly $m \times m$ grids if it resides in the inner area of $\mathcal{A}$, or less than $m \times m$ grids if it is on the boundary. Since each of k_1 and k_2 has $m+1$ choices, we will have in total $(m+1)^2$ number of different partitions. Each partition can be treated as shifting right the leftmost column of grids (column Col_0) k_1 positions[1] and/or shifting up the lowest row of grids (Row_0) k_2 positions. Therefore we call this partition procedure *shifted dissection*. Denote each dissection by $P_{a,b}$, where a, b indicate that $P_{a,b}$ is obtained by shifting Col_0 to column b and Row_0 to row a. Figs. 4, 5, and 6 illustrate $P_{0,0}$, $P_{1,0}$, and $P_{0,1}$, respectively.

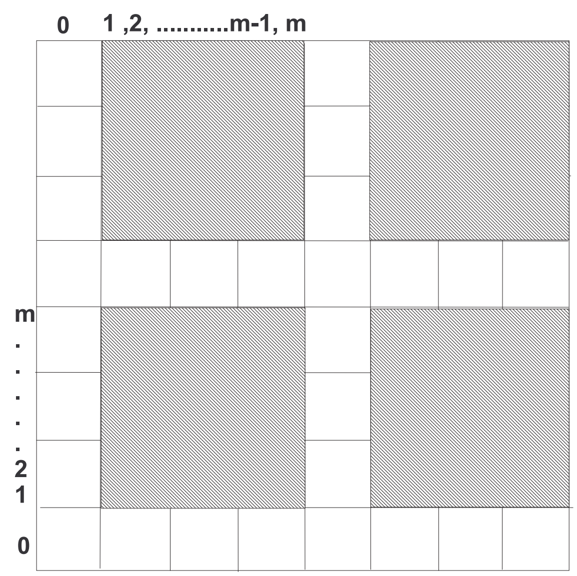

Fig. 4. Partition $P_{0,0}$.

3) **Computation and Performance Analysis:** Consider a specific partition $P_{a,b}$. For each super-grid B in $P_{a,b}$, denote by G_B the subgraph induced by all edges incident to at least one node residing in B. Now compute an maximum weight channel scheduling S_B for G_B. The union of all S_B's generated from all super-grids in $P_{a,b}$, denoted by $S_{a,b}$, form an approximate solution to the original OWCS/P problem given G. We repeat this procedure for each partition and output the result that have the maximum weight, i.e. $W(P_{tas}(G)) = \textbf{argmax}_{a,b}\{W(S_{a,b}) \mid a, b = 0, 1, \cdots, m\}$. We will prove that $W(P_{tas}(G)) \geq (1-\epsilon)W(O(G))$ for a given small positive real ϵ in Subsection V-B.

[1]Each position corresponds to one grid size.

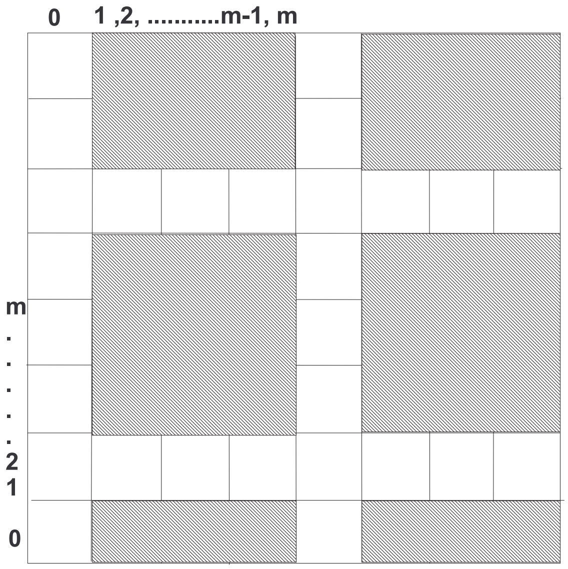

Fig. 5. Partition $P_{1,0}$.

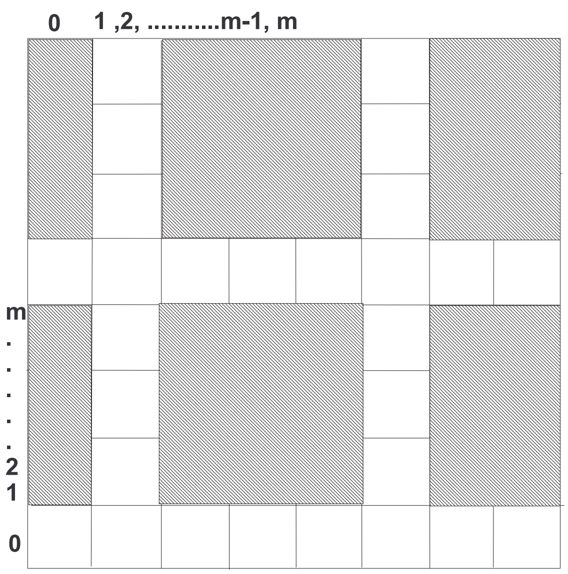

Fig. 6. Partition $P_{0,1}$.

4) **Complexity Analysis:** In Subsection V-C, we will prove that the procedure proposed above takes polynomial time.

B. Computation and Performance Analysis

In this subsection, we elaborate the procedure of computing the PTAS in detail. The parameters and their definitions are summarized in Table II.

After shifted dissection, we have $(m+1)^2$ different partitions. Consider a partition $P_{a,b}$. Let B be any super-grid in $P_{a,b}$. Then B has at most 8 neighboring super-grids, denoted by B_i for $i = 1, 2, \cdots, 8$, as shown in Fig. 7. We have the following observations:

$P_{a,b}$	A shifted partition
B	A super-grid in $P_{a,b}$
G_B	The subgraph induced by all edges incident to at least one node residing in B
S_B	The set of edges with maximum total weight that support simultaneous transmission in the network G_B
$G_{a,b}$	The subgraph induced by all edges incident to at least one node residing in any super-grid in $P_{a,b}$
$S_{a,b}$	The union of all S_Bs for $P_{a,b}$
$\overline{G}_{a,b}$	The subgraph induced by all edges with both incident nodes not residing in any super-grid in $P_{a,b}$
$O(G)$	An optimal (maximum weight channel scheduling) solution for G
$P_{tas}(G)$	The solution given by the PTAS procedure

TABLE II
PARAMETERS AND THEIR DEFINITIONS.

1) Simultaneous communications in G_{B_i} and G_{B_j} do not interfere with each other when $i \neq j$ since the separating grid size is $P+2$. Therefore the union of S_{B_i} and S_{B_j} is an optimal solution to the union of G_{B_i} and G_{B_j}, which implies that $S_{a,b}$ is an optimal solution to $G_{a,b}$ (see Fig. 7).
2) A maximum weight channel scheduling for any G_{B_i} can be computed in polynomial time (see Subsection V-C).
3) There are in total $O(\frac{N^2}{m^2})$ number of super-grids in $P_{a,b}$.
4) A grid will NOT be included in any super-grid among all $(m+1)^2$ partitions for $2m+1$ times.

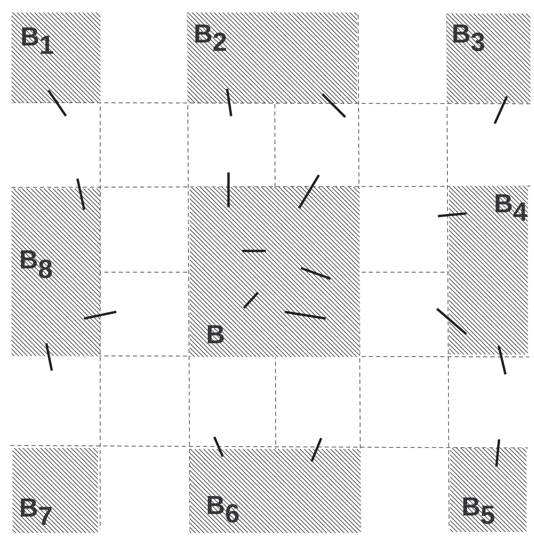

Fig. 7. $G_{a,b}$ for partition $P_{a,b}$.

Since
$$E_{a,b} \cup \overline{E}_{a,b} = E,$$
where $\overline{E}_{a,b}$ is the set of edges in $\overline{G}_{a,b}$. We have
$$(O(G) \bigcap E_{a,b}) \cup (O(G) \bigcap \overline{E}_{a,b}) = O(G) \bigcap E. \quad (1)$$
Define $E^r_{a,b} = \overline{E}_{a,b} \bigcap O(G)$, i.e. $E^r_{a,b}$ is the set of edges in the optimal scheduling $O(G)$ but are removed from $G_{a,b}$. We obtain
$$(O(G) \bigcap E_{a,b}) \cup E^r_{a,b} = O(G).$$

Since $S_{a,b}$ is an optimal solution to $G_{a,b}$ (see Observation 1 mentioned in the above),

$$W(S_{a,b}) \geq W(O(G) \bigcap E_{a,b}).$$

Therefore,

$$W(S_{a,b}) + W(E^r_{a,b}) \geq W(O(G)), \tag{2}$$

which yields

$$W(S_{a,b}) \geq W(O(G)) - W(E^r_{a,b}). \tag{3}$$

Now consider all the $(m+1)^2$ partitions. Since a grid will not be included in any super-grid for $2m+1$ times in total (Observation 4), it will be excluded from the $S_{a,b}$ computation for $2m+1$ times. Therefore we have

$$\sum_{a,b} W(E^r_{a,b}) \leq (2m+1) W(O(G)), \tag{4}$$

where $a, b = 0, 1, 2, \cdots, m$.

Based on the Pigeon hole principle and Eq. (4), we obtain

$$\mathbf{argmin}_{a,b}\{W(E^r_{a,b})\} \leq \frac{(2m+1)}{(m+1)^2} W(O(G)) \tag{5}$$

Let $P^{min}_{a,b}$ be the partition that yields $\mathbf{argmin}_{a,b}\{W(E^r_{a,b})\}$ for $a, b = 0, 1, \cdots, m$. Then, from Eq. (3) and Eq. (5), we have

$$\begin{aligned} W(O(S^{min}_{a,b})) &\geq W(O(G)) - W(E^{r\prime}_{a,b}) & (6) \\ &\geq \frac{m^2}{(m+1)^2} W(O(G)) & (7) \end{aligned}$$

where $E^{r\prime}_{a,b} = \overline{E}^{min}_{a,b} \bigcap O(G)$.

Note that in our algorithm we will examine all the $(m+1)^2$ partitions and output the $S_{a,b}$ that has the maximum weight. Denote this $S_{a,b}$ by $P_{tas}(G)$. Based on Eq. (6), we can obtain Lemma. 5.1 easily.

Lemma 5.1: $W(P_{tas}(G)) \geq \frac{m^2}{(m+1)^2} W(O(G))$

Now it is the time to compute m. Given an error parameter ϵ, we require that

$$\frac{m^2}{(m+1)^2} \geq 1 - \epsilon \tag{8}$$

which yields $m = \lceil f(\epsilon) \rceil$ where

$$f(\epsilon) = \frac{\sqrt{1-\epsilon}}{1 - \sqrt{1-\epsilon}}. \tag{9}$$

Summarizing the above analysis, we obtain the following procedure to compute a PTAS for the maximum weight P-interference-free channel scheduling in MR-MC wireless networks:

Algorithm 1 A PTAS For P-Interference-Free Channel Scheduling

Input: An edge-weighted graph $G(V, E)$ and a positive small real number ϵ.
Output: $P_{tas}(G)$, a set of edges that can support simultaneous communications in G over orthogonal channels..

1: **function** $P_{tas}(G)$=**PTAS**(G, ϵ)
2:     Compute $m = \lceil f(\epsilon) \rceil$ based on Eq. (9).
3:     Do shifted dissection to obtain the $(m+1)^2$ partitions.
4:     **for** each partition $P_{a,b}$, where $a, b = 0, 1, \cdots, m$ **do**
5:         Compute $S_{a,b}$
6:     **end for**
7:     $P_{tas}(G) = S_{a',b'}$ such that $W(S_{a',b'}) = \mathbf{argmax}_{a,b}\{W(S_{a,b})\}$
8:     Output $P_{tas}(G)$
9: **end function**

C. Complexity analysis

Algorithm 1 summarizes the procedure to compute a PTAS for the OWCS/P$_{\geq 1}$ problem in MR-MC networks. In this subsection, we will analyze its time complexity.

From Algorithm 1, we realize that the dominating component in the time complexity analysis is the loop containing line 5, which computes an optimal solution for the subgraph $G_{a,b}$ for a partition $P_{a,b}$. We will first prove that computing an optimal S_B for any super-grid B takes polynomial time.

Since the side of a grid is $P+2$, the area of a super-grid is $< (m(P+2)+2)^2$. Let C_i denote the set of orthogonal channels available for node i, and r_i the number of radios node i has. Let $k = max\{|C_i|\}$ and $r = max\{r_i\}$ for $i = 1, 2, \cdots, n$, where n is the total number nodes in G.

Consider any super-grid B and the corresponding graph $G_B(V_B, E_B)$, the induced graph of all edges incident to at least one node residing in B. Construct G^*_B by replacing each edge $(u, v) \in E_B$ with **min** $\{|C_u \bigcap C_v|, r_u, r_v\}$ parallel edges. We have $|E^*_B| \leq$ (**max** $\{k, r\})|E_B|$. Let S_B be the set of edges corresponding to an optimal solution for G_B. Since the area of B is at most $(m(P+2)+2)^2$, the total number of edges received the same channel in the schedule S_B is $O(m^2)$. Therefore there exist $|E^*_B|^{O(m^2)}$ candidate sets of edges for each channel. Considering all channels, we have in total $k \cdot |E^*_B|^{O(m^2)}$ candidate sets of edges. Therefore we can employ brute-force to find out the optimal scheduling for G_B by trying all $|E^*_B|^{k \cdot O(m^2)}$ combinations. In summary, we have

Lemma 5.2: It takes polynomial time to find out S_B for each super-grid B.

Since each partition contains $O(\frac{N^2}{m^2})$ number of super-grids (Observation 3 in Subsection V-B), we have

Lemma 5.3: It takes polynomial time to find out $S_{a,b}$ for any partition $P_{a,b}$.

From all the above analysis, we can obtain the following Theorem:

Theorem 5.1: A PTAS for P interference-free channel scheduling in MR-MC wireless networks does exist. Given ϵ, the PTAS can be computed in $|E|^{O(m^2)}$, where $m = \lceil f(\epsilon) \rceil$.

VI. A PTAS for H-hop interference-free matching in MR-MC wireless networks

In Section V, we have proposed a PTAS for OWCS/P$_{\geq 1}$ in MR-MC wireless networks. In this section, we will propose a PTAS for OWCS/H$_{\geq 1}$ in MR-MC wireless networks. Actually the whole PTAS procedure and its analysis in Section V can be transplanted directly to compute a polynomial time approximation scheme for the OWCS/H$_{\geq 1}$ problem in MR-MC wireless networks.

First, the idea of constructing a PTAS is the same as that illustrated in Subsection V-A, except that we set $P = H$ and replace the concept of P interference-free matching with H-hop interference-free matching. Note that the shifted dissection and the procedure to compute an optimal scheduling for each super-grid remain unchanged. Furthermore, the performance analysis in Subsection V-B is also applicable under the H-hop interference-free model. Thus the function $f(\epsilon)$ is the same as the one shown in Eq. (9), and Lemma 5.1 also holds true for the H-hop interference model.

As the area of a super-grid is $< (m(H+2)+2)^2$, the maximum number of edges in $O(G_B)$ in one channel is $O(m^2)$. Therefore, with a similar argument as that of Lemma 5.2, we have:

Lemma 6.1: The optimal H-hop interference-free matching for one super-grid can be found in polynomial time

In conclusion, according to Lemmas 5.1 and 6.1, we can obtain the following theorem:

Theorem 6.1: The PTAS for H-hop interference-free channel scheduling in MR-MC wireless networks does exist. Given ϵ, the time complexity of the PTAS procedure is $|E|^{O(m^2)}$, where $m = \lceil f(\epsilon) \rceil$.

From the above analysis, we can conclude that the proposed PTAS for OWCS/P$_{\geq 1}$ is a framework that is applicable to the OWCS/H$_{\geq 1}$ problem in MR-MC wireless networks.

VII. Discussion OWCS/P and OWCS/H

In this paper, we have studied the complexity of OWCS/P, and proposed PTASs for both OWCS/P$_{\geq 1}$ and OWCS/H$_{\geq 1}$ in MR-MC wireless networks. We have considered two interference models: the Physical interference-free model and the Hop interference-free model.

In the physical interference-free model, two nodes interfere each other if and only if their distance is $\leq P \times d_{com}$, where P can be any real number that is ≥ 1. In the hop interference-free model, two nodes interfere with each other when they are within H hops. Thus, H is an integer that is ≥ 1. Therefore, in common sense, the physical interference-free model is more precise than the hop interference-free model. However, we have shown that the two models are equivalent when $P = H = 1$ in Lemma 4.1, and any instance of OWCS/P$_{>1}$ is polynomial time reducible to an instance in OWCS/P$_{=1}$ in the proof of Theorem. 4.3. Therefore, we can conclude that OWCS/H$_{=1}$ is equivalent to OWCS/P$_{\geq 1}$ under the polynomial transformation. Thus in theory, focusing on the OWCS/H$_{=1}$ problem suffices to obtain the results for channel scheduling under either model.

However, it is interesting to observe that the hop interference-free model is not as precise as the physical interference-free model, as shown in Fig. 1. A careful study indicates that the foundation supporting the polynomial transformation in the proof of Theorem. 4.3 is the position information. Without position information, a node can not distinguish all the other nodes that might interfere with it. Nevertheless, given the position information, the two models are equivalent in practice. However, when no position information is available, hop interference-free model can be employed as the hop-distance does not require the physical position information.

In conclusion, the physical interference-free model represents the channel scheduling problem more clearer. However, the hop interference-free model is easier to study and implement in practice.

VIII. Summary and Future Work

In this paper, we have solved the following two open problems:

1) Is the OWCS/P$_{\geq 1}$ problem NP-Hard in MR-MC wireless networks?
2) Are there polynomial time approximation schemes for OWCS/P$_{\geq 1}$ and OWCS/H$_{\geq 1}$ in MR-MC wireless networks?

We have proved that OWCS/P$_{\geq 1}$ is NP-Hard and proposed PTASs for OWCS/P$_{\geq 1}$ and OWCS/H$_{\geq 1}$ in MR-MC wireless mesh networks. The major contributions of this paper are summarized (boldfaced) in Table III.

OWCS/-	SR-SC Networks	MR-MC Networks
$P \geq 1$	***NP-Hard ; PTAS***	***NP-Hard ; PTAS***
$H \geq 1$	NP-Hard w/PTAS	NP-Hard ***; PTAS***

TABLE III
OWCS Complexity Results.

By comparing the physical interference-free model and the hop interference-free model, we found that they are both suitable for studying the scheduling problem theoretically. In practice, when position information is unavailable, the hop interference-free model is applicable though it becomes less precise. Nevertheless, our results indicate that a theoretical study on channel scheduling under the hop interference-free model suffices.

This paper has filled the void of the complexity study for channel scheduling in MR-MC wireless networks. We will seek simple approximate algorithms that have guaranteed performance in our future research.

Acknowledgments

The research was partially supported by the US National Science Foundation under grants CNS-0347674, CNS-0721669, and CNS-0831852, and partially supported by the National Basic Research Program of China (973) under Grant No. 2005CB321801 and the National Natural Science Foundation of China under Grant No. 60673167.

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