AMATH 352 Homework 6

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Due Friday, August 3

Exercise 1 *Data fitting*

Consider fitting the data

$$\{(0,1),(1,2),(2,-1),(3,4),(4,12),(5,10),(6,20),(7,40),(8,30),(9,60),(10,99)\},$$

with a quadratic polynomial. To do this we assume $p(x) = ax^2 + bx + c$ and setup the system

$$\begin{aligned}
(0,1) &\mapsto p(0) = 0 + 0 + c = 1,\\
(1,2) &\mapsto p(1) = a + b + c = 2,\\
&\vdots\\
(10,99) &\mapsto p(10) = 100a + 10b + c = 99.
\end{aligned}$$

This system is over determined as we have 11 equations and just 3 unknowns. Write this system in the form

$$A\mathbf{c} = \mathbf{y}.$$

Use MATLAB to solve the system in the least-squares sense to find the 'best' a, b and c. Plot the original data and your fit on the same graph. Upload your code and turn in your plot.

Exercise 2

Use the algorithm set out on the top of page 234 to code the algorithm for Gram-Schmidt. Note that when we did this in class we took the coding convention and ignored the superscripts. This is how you should code it. You start with vectors $\{\mathbf{w}_1, \ldots, \mathbf{w}_n\}$ and modify them so that they become orthogonal.

$\mathbf{w}_1 = \mathbf{w}_1/\|\mathbf{w}_1\|$ `% Start`
for $j = 2, \ldots, n$
  for $k = j, \ldots, n$
    $\mathbf{w}_k = \mathbf{w}_k - \langle \mathbf{w}_k, \mathbf{w}_{j-1} \rangle \mathbf{w}_{j-1}$
  end
  $\mathbf{w}_j = \mathbf{w}_j/\|\mathbf{w}_j\|$
end

It is probably easiest to start with all $\mathbf{w}_j$'s as columns of a matrix W so that you can refer to $\mathbf{w}_1$ with `W(:,1)`, $\mathbf{w}_2$ with `W(:,2)`, etc.

(a) Use your code to perform Gram-Schmidt on the vectors

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{w}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{w}_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

Upload your code on the moodle page and turn in the last vector, $\mathbf{w}_5$, after this process.

(b) Consider adding another vector

$$\mathbf{w}_6 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

How does the fact that we have a linearly dependent set of vectors manifest itself in the algorithm? Hint: Print out $\mathbf{w}_j$ before dividing by its norm.

The following exercises should be done by hand, showing all steps.

Exercise 3

Olver & Shakiban— 5.1.1b,d,f

Exercise 4

Olver & Shakiban— 5.1.5

Exercise 5

Olver & Shakiban— 5.2.9c

Exercise 6

Olver & Shakiban— 5.2.10

Exercise 7 *EXTRA CREDIT — 5 points*

Olver & Shakiban— 5.3.18 — for the second question restrict to 2×2 matrices — an extra 2 pts without this restriction