# AMATH 352 Homework 4

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Due Wednesday, July 18

## Exercise 1

Consider the $\lambda$-dependent linear system

$$(I - \lambda^{-1} H_{100})\mathbf{x} = \mathbf{b}, \quad \lambda > 0,$$

where $H_{100}$ is the $100 \times 100$ Hilbert matrix (see page 58 of the text). In MATLAB `hilb(100)` will return this matrix. For large enough $\lambda$ it can be shown that

$$\|\mathbf{x}_n - \mathbf{x}\|_2 \to 0 \text{ as } n \to \infty \text{ where } \mathbf{x}_n = \mathbf{b} + \sum_{i=1}^{n} (\lambda^{-1} H_{100})^i \mathbf{b}.$$

We can approximate the solution $\mathbf{x}$ by $\mathbf{x}_n$ and this type of behavior is called *convergence in norm*.

1. With $\lambda = 5$ and

   $$\mathbf{b} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T,$$

   evaluate $\|\mathbf{x} - \mathbf{x}_{10}\|_2$ (in MATLAB `norm(x)` returns the 2-norm of `x`).

2. How large does $n$ need to be so that $\|\mathbf{x} - \mathbf{x}_n\|_2 < 10^{-10}$?

3. If $\lambda$ is too small $\mathbf{x}_n$ will not converge in norm. By restricting $\lambda$ to the integers, find the smallest value of $\lambda$ such that $\mathbf{x}_n$ still converges in norm. Note: to investigate this you'll need to vary $n$.

Please upload the main algorthm needed to compute $\mathbf{x}_n$ for this problem. You don't need to include every detail in your uploaded code.

**The following exercises should be done by hand, showing all steps.**

## Exercise 2

Olver & Shakiban— 2.5.14

## Exercise 3

Olver & Shakiban— 2.5.21b

## Exercise 4

Olver & Shakiban— 2.5.30

## Exercise 5

Olver & Shakiban— 3.1.7

## Exercise 6

Olver & Shakiban— 3.2.6

## Exercise 7

Olver & Shakiban— 3.3.11