AMATH 352 Homework 1

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Due Wednesday, June 27

Exercise 1

Use MATLAB to plot the functions

$$f(x) = \cos(2\pi x), \quad g(x) = \sin(3\pi x), \quad h(x) = x^2, \quad \forall x \in [0, 1].$$

Plot the three curves on the same figure. Print out the resulting figure and attach it to your homework. Upload your code to the moodle page.

Exercise 2 *For loops and if statements*

Let $f(x) = x^2$. In this problem you will code a Monte Carlo integrator to approximate

$$\int_0^1 x^2 dx = 1/3.$$

The MATLAB command `rand(1)` returns a (pseudo)random number which lies in $(0, 1)$. The following pseudo code approximates this integral:

```
start
samples = 100
total = 0

for j = 1 to samples
   x = rand(1)
   y = rand(1)
   if y < x^2 add one to total
next j

print total/samples*area of [0,1]^2

end
```

Your job is to translate this into MATLAB syntax and compute this approxmation for `samples = 100,1000,10000`. Please upload your code to moodle and turn in your output.

The following problems should be done by hand, showing the steps involved.

Exercise 3

Solve the following linear systems

(a)

$$\begin{aligned} x + 7y &= 4 \\ -2x - 9y &= 2 \end{aligned}$$

(b)

$$\begin{aligned} x - 2y + z &= 0 \\ 2y - 8z &= 0 \\ -4x + 5y + 9z &= -9 \end{aligned}$$

(c)

$$\begin{aligned} x + 4y - 2z &= 1 \\ -2x - 3z &= -7 \\ 3x - 2y + 2z &= -1 \end{aligned}$$

Exercise 4

Determine whether

(a) $\begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$ is equal to a weighted sum of $\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$ is equal to a weighted sum of $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Exercise 5

(a) Let

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix}.$$

Compute AB and BA. What do you notice?

(b) Let

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Compute AB and BA. What do you notice? Explain this in terms of the rules for scalar multiplication.

Exercise 6

Olver & Shakiban - 1.3.1c,e:

Write out the augmented matrix for the following linear systems. Then solve the system by first applying elementary row operations to place the augmented system in upper triangular form, followed by back substitution.

(c)

$$\begin{aligned} x - 2y + z &= 0 \\ 2y - 8z &= 8 \\ -4x + 5y + 9z &= -9 \end{aligned}$$

(e)

$$\begin{aligned} x_1 - 2x_3 &= -1 \\ x_2 - x_4 &= 2 \\ -3x_2 + 2x_3 &= 0 \\ -4x_1 + 7x_4 &= -5 \end{aligned}$$

Exercise 7

Olver & Shakiban - 1.3.15

Write down the elementary matrix corresponding to the following row operations on 4×4 matrices:

(a) Add the third row to the fourth row.

(b) Subtract the fourth row from the third row.

(c) Add 3 times the last row to the first row.

(d) Subtract twice the second row from the fourth row.