AMATH 351 Summer 2011
Midterm

Friday, July 22

Name: _____

Problem	Points	Score
1	15	
2	20	
3	15	
4	15	
5	20	
6	15	

Total	

1. **(15)** Solve the initial-value problem

$$x\frac{dy}{dx} = x + 2y, \qquad y(1) = 0. \qquad (x > 0)$$

2. **(20)** Solve the initial-value problem

$$(y')^2 - 2y' + 1 = y - x, \qquad y(1) = 1,$$

using the substitution $v = y - x$.
Hint: Factor first!

3. **(15)** Consider $\left(4y^3 - nx^3y\right) y'(x) + (-6x^2y^2 + 12x^2 + 1) = 0$.

(a) *(5)* For what value of n is this an *exact* equation?

(b) *(10)* Using the value of n that you found in part (a), solve this equation. Write it in the form $f(x, y) = c$.

4. **(15)** Consider the DE $y''(t) - 3y'(t) - 4y(t) = 5e^{4t}$.

   (a) *(10)* Find the general solution.

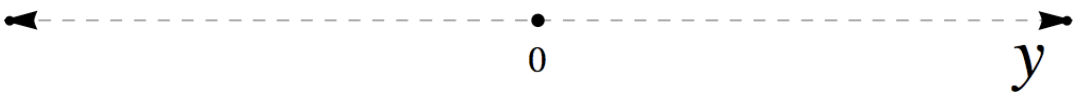
   (b) *(5)* Even though this is the general solution, what can you say about $\lim_{t\to\infty} y(t)$?

5. **(20)** Consider the equation

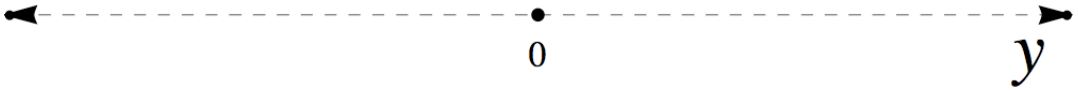
$$\frac{dy}{dt} = y(y-a).$$

(a) *(15)* Sketch the phase-line (labeling the equilibrium points, if any) below in the cases when $a > 0$, $a = 0$ and $a < 0$.

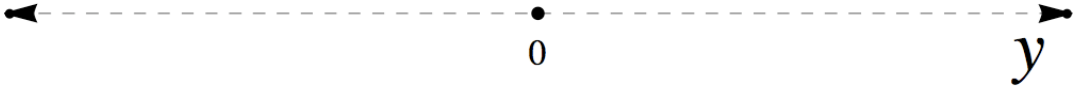
($a > 0$)



($a = 0$)



($a < 0$)



(b) *(5)* For each case above, classify the equilibria as asymptotically stable, semi-stable or unstable.

6. **(15)** Consider the equation

$$\frac{1}{2}y'' + xy' - x^2 y = 0.$$

Assume y_1 and y_2 are solutions such that

$$\begin{aligned} y_1(0) &= 1, \\ y_1'(0) &= 0, \\ y_2(0) &= 0, \\ y_2'(0) &= 1. \end{aligned}$$

Determine $W(y_1, y_2)$ so that no unknown constants are present.