# Amath 351 Homework 7

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## Exercise 1 *B&D 6.2*

Find $\mathcal{L}\{f(t)\}$ for the following functions:

1. $f(t) = t$
2. $f(t) = t^2$
3. $f(t) = t^n$, $n$ a positive integer.
4. $f(t) = \cos bt$
   Note: Use $\cos bt = (e^{ibt} + e^{-ibt})/2$ and you may assume that elementary integration formulas carry over to complex-valued functions.
5. $f(t) = \cosh bt$
   Note: $\cosh bt = (e^{bt} + e^{-bt})/2$ and it is easiest to obtain this from $\mathcal{L}\{\cos bt\}$ by making a re-definition of $b$.

## Exercise 2 *B&D 6.2*

**The Gamma Function.** The gamma function is denoted by $\Gamma(p)$ and is defined by the integral

$$\Gamma(p+1) = \int_0^\infty e^{-x} x^p dx.$$

The integral converges as $x \to \infty$ for all $p$. For $p < 0$ it is also improper because the integrand become unbounded as $x \to 0$. However, the integral can be shown to converge at $x = 0$ for $p > -1$.

(a) Show that for $p > 0$

$$\Gamma(p+1) = p\Gamma(p).$$

(b) Show that $\Gamma(1) = 1$.

(c) If $p$ is a positive integer $n$, show that

$$\Gamma(n+1) = n!.$$

Since $\Gamma(p)$ is also defined when $p$ is not an integer this function provides an extension of the factorial function to nonintegral values of the independent variable. Note that it is also consistent to define $0! = 1$

## Exercise 3 *B&D 6.2*

Solve the following initial value problems with Laplace transforms.

(a) $y'' - y' - 6y = 0, \;\; y(0) = 1, \;\; y'(0) = -1.$

(b) $y'' - 2y' + 2y = 0, \;\; y(0) = 1, \;\; y'(0) = 0.$

## Exercise 4 *B&D 6.4-5*

Solve each of the following initial value problems.

(a)

$$y'' + y = f(t), \;\; y(0) = 0, \;\; y'(0) = 1, \;\; f(t) = \begin{cases} 1 & \text{if } 0 \le t < \pi/2 \\ 0 & \text{if } \pi/2 \le t < \infty. \end{cases}$$

(b)

$$2y'' + y' + 6y = \delta(t - \pi/6)\sin t, \;\; y(0) = 0, \;\; y'(0) = 0,$$

## Exercise 5 *B&D 6.6*

Express the solution to of the given intital value problem in terms of a convolution integral.

$$y'' + 4y' + 4y = g(t), \;\; y(0) = 2, \;\; y'(0) = -3.$$

Assume that $G(s) = \mathcal{L}\{g(t)\}$ exists.