Amath 351 Homework 6

Tom Trogdon

Due August 3, 2011

Exercise 1 *B&D 7.6.3*

Express the general solution of the given system of equations in terms of real-valued fuctions. Also draw a phase-plane portrait for a few of the solutions.

$$\frac{d\mathbf{y}}{dt} = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{y}, \tag{1}$$

where

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}. \tag{2}$$

Exercise 2 *B&D 7.6.8*

Express the general solution of the given system of equations in terms of real-valued fuctions.

$$\frac{d\mathbf{y}}{dt} = \begin{pmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ -2 & -1 & 0 \end{pmatrix} \mathbf{y}, \tag{1}$$

where

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}. \tag{2}$$

Exercise 3 *B&D 7.6.10*

Find the solution of the given initial value problem. What does the solution

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{1}$$

approach when $t \to +\infty$?

$$\frac{d\mathbf{y}}{dt} = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} \mathbf{y}, \tag{2}$$

where

$$\mathbf{y}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}. \tag{3}$$

Exercise 4 *B&D 7.7.5*

Find the fundamental matrix $\Phi(t)$ for the given system of equations

$$\frac{d\mathbf{y}}{dt} = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{y}, \tag{1}$$

satisfying

$$\Phi(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

Exercise 5 *B&D 7.8.3*

Find the general solution of the given system of equations

$$\frac{d\mathbf{y}}{dt} = \begin{pmatrix} -3/2 & 1 \\ -1/4 & -1/2 \end{pmatrix} \mathbf{y}, \tag{1}$$

Exercise 6 *B&D 7.9.1*

Find the general solution of the given system of equations

$$\frac{d\mathbf{y}}{dt} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{y} + \begin{pmatrix} e^t \\ t \end{pmatrix}. \tag{1}$$

Exercise 7 *B&D 7.9.7*

Find the general solution of the given system of equations

$$\frac{d\mathbf{y}}{dt} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \mathbf{y} + \begin{pmatrix} 2e^t \\ -e^t \end{pmatrix}. \tag{1}$$