AMATH 351 Homework 4

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Exercise 1 *Reduction of order, B&D 3.5*

For problems a and b, use the method of reduction of order to find a second solution y_2 of the given differential equation when you've already been given one solution y_1. Remember to rewrite the equations in standard forms if necessary (make the coefficient of the 2nd order derivative equal to 1).

(a) $t^2y'' + 3ty' + y = 0$, $t > 0$; $y_1(t) = t^{-1}$. (Note that $p(t) = 3t/t^2$.)

(b) $(x-1)y'' - xy' + y = 0$, $x > 1$; $y_1(x) = e^x$. (Note that $p(x) = -x/(x-1)$.)

Exercise 2 *Euler Equations, B&D 3.3*

As we discussed in class, an equation of the form

$$t^2\frac{d^2y}{dt^2} + \alpha t\frac{dy}{dt} + \beta y = 0, \quad t > 0, \tag{1}$$

where α and β are real constants, is call an Euler equation.

(a) Let $x = \ln t$ and calculate dy/dt and d^2y/dt^2 in terms of dy/dx and d^2y/dx^2.

(b) Use the results of part (a) to transform (1) into

$$\frac{d^2y}{dx^2} + (\alpha - 1)\frac{dy}{dx} + \beta y = 0. \tag{2}$$

This shows that if $y(x)$ is a solution of (2) then $y(\ln t)$ is a solution of (1). Furthermore, we already know how to solve (2) in terms of exponentials. This is an alternate way of deriving the results in the notes.

Exercise 3 *Euler Equations, B&D 3.3*

For the following DEs, find the general solution

(a) $t^2y'' + ty' + y = 0$.

(b) $t^2y'' - ty' + 5y = 0$.

Exercise 4 *Method of undetermined coefficients, B&D 3.6*

Note: don't forget to add your particular solution y_p to the homogeneous solution y_h.

(a) Find the general solution of the given differential equation

$$y'' - 2y' - 3y = 3e^{2t}. \tag{1}$$

(b) Find the general solution of the differential equation

$$y'' + 2y' + 5y = 3\sin 2t. \tag{2}$$

Exercise 5 *Variation of parameters, B&D 3.7*

Find the general solution of the given differential equations.

(a) $4y'' - 4y' + y = 16e^{t/2}$.
Hint: don't forget to write the quation in the standard form (dividing both sides by 4).

(b) $y'' - 2y' + y = e^t/(1 + t^2)$.