AMATH 351 Homework 3

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Substitutions

Exercise 1

Consider the differential equation

$$y' = xy\ln(xy) - y/x.$$

Use the substitution $v = \ln(xy)$ to reduce this to a separable equation and find the general solution.

Optional Exercises

The next two exercises are purely optional. Use the subsitution $v = y/x$ to solve the following DEs:

(a) $xyy' + 4x^2 + y^2 = 0$

(b) $xy' = y(\ln x - \ln y)$.

Homogeneous Equations with Constant Coefficients, Two Different Roots (B&D 3.1)

Exercise 2

For the following equations, find the characteristic equation and general solution.

(a)
$$y'' + 3y' + 2y = 0.$$

(b)
$$2y'' - 3y' + y = 0.$$

Exercise 3

(a) Find the solution of the given initial value problems and describe its behavior as $t \to +\infty$.

$$6y'' - 5y' + y = 0,\ y(0) = 4,\ y'(0) = 0. \tag{1}$$

(b) Solve the initial value problem

$$4y'' - y = 0,\ y(0) = 2,\ y'(0) = \beta. \tag{2}$$

Then find β so that the solution approaches zero as $t \to +\infty$.

Wronskian and linear independence (B&D 3.2, 3.3)

Exercise 4

Find out if the given pair of functions are linearly independent. (Hint: calculate the Wronskian.)

(a) e^{2t} and e^{-t}

(b) e^{-2t} and te^{-2t}

(c) $e^t \sin t$ and $e^t \cos t$.

Exercise 5

Find the Wronskian of two solutions of the given differential equation

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0 \tag{1}$$

without solving the equation. (Hint: use Abel's theorem.)

Repeated roots of the characteristic equation (B&D 3.4)

Exercise 6

(a) Find the general solution of the differential equation

$$4y'' + 12y' + 9y = 0. \tag{1}$$

(b) Solve the given initial value problem. What does the solution y approach when $t \to +\infty$?

$$y'' - 6y' + 9y = 0,\ y(0) = 0,\ y'(0) = 2 \tag{2}$$

Complex roots of the characteristic equation (B&D 3.3)

Exercise 7

Find the general solution of the differential equations

(a)

$$y'' + 2y' + 2y = 0$$

(b)

$$4y'' + 9y = 0$$

Exercise 8

Solve the initial value problem. Is the amplitude of the solution's oscillation increasing/decreasing/constant?

$$y'' + 4y = 0,\ y(0) = 0,\ y'(0) = 1 \tag{1}$$