AMATH 351 Homework 1

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Due Wednesday, June 29

Exercise 1 *Classification of differential equations*

For the following differential equations, find out **(a)** its order, **(b)** whether it's linear or nonlinear and **(c)** if it's nonlinear, what the terms are that show it's nonlinear.

1. $y''' + 2y' + y + e^x = 0$.

2. $2\frac{dx}{dy} + xy = 0$.

3. $yy'' + ty' + y = 0$.

4. $y'' + e^y + y + x = 0$.

Exercise 2 *Direction Fields*

(a) For the equation $y' = y - 2$ for a function $y(x)$, draw its direction field with x on the horizontal axis. (Hint: you need to divide the plane into at least three regions.) When x approaches $+\infty$, what does y approach?

(b) For the equation $y' = -y - 2$ for a function $y(x)$, draw its direction field with x on the horizontal axis. When x approaches $+\infty$, what does y approach?

(c) Find the equilibrium solution(s) of the equation $y' = y(y - 3)$.

Exercise 3 *Math Modeling*

Consider the equation

$$\frac{dp}{dt} = 0.5p - 450. \tag{1}$$

This equation has been used as a model of a field mouse population.

(a) Find the time at which the population becomes extinct if $p(0) = 850$.

(b) Find the time of extinction if $p(0) = p_0$, where p_0 is a number between 0 and 900.

(c) Find the initial population p_0 if the population is to become extinct when $t = 12$.

Exercise 4 *Verify solutions*

(a) For $y'''' + 4y''' + 3y = t$, verify that $y_1(t) = t/3$ and $y_2(t) = e^{-t} + t/3$ are solutions.

(b) For $ty' - y = t^2$, verify that $y(t) = 3t + t^2$ is a solution.

Exercise 5 *Separable Equations*

(a) Solve equation

$$\frac{dy}{dx} = \frac{\sqrt{y^2 - a^2}}{a},$$

with the initial condition

$$y(0) = a.$$

Your answer should be a function of x with the parameter a still present.

(b) Solve the initial value problem

$$\frac{dr}{d\theta} = \frac{r^2}{\theta}, \quad r(1) = 2.$$

(c) Solve the initial value problem

$$y' = \frac{3x^2}{3y^2 - 4}, \quad y(1) = 0.$$

(d) Solve the initial value problem

$$y' = \frac{e^{-x} + e^x}{3 + 4y}, \quad y(0) = 1.$$