Promoting large, compact mature forest patches in harvest scheduling models

(Short Title: Compact mature forest patches in harvest scheduling)

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Abstract:

Spatially-explicit harvest scheduling models that can promote the development of dynamic mature forest patches have been proposed in the past. This paper introduces a formulation that extends these models by allowing the total perimeter of the patches to be constrained or minimized. Test run results suggest that the proposed model can produce solutions with fewer, larger, and more compact patches. In addition, patches are more likely to be temporally connected with this formulation. Methods for identifying the tradeoffs between the net present value of the forest and the size and perimeter of the evolving patches are demonstrated for a hypothetical forest.

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1. Introduction

Harvest scheduling models can aid forest management decisions that involve both timber and non-timber objectives. They have been used in the United States for over 40 years to help identify the most profitable management alternatives for a forest and to address a variety of sustainability concerns. These models were initially formulated as linear programs (LP) (e.g., [8]) with limited capability to address spatial objectives. Non-timber objectives or constraints that are fundamentally spatial in nature, such as conservation of mature forest habitat patches, were cumbersome or impossible to model.

Recently, spatially-explicit harvest scheduling models have been introduced that are beginning to address these shortcomings. These models use binary (0-1) variables to represent the decisions whether or not a particular treatment regime should be applied to a specific forest management unit. These 0-1 variables provide more spatial control for incorporating certain non-timber objectives. Spatially-explicit models have been studied extensively in the context of applying adjacency constraints, whose function is to limit the size of clearcuts. Adjacency constraints have been promoted as contributing to many non-timber objectives (e.g. [1, 9, 15-17, 24-27, 30]). However, they tend to disperse harvests across the landscape, which is counterproductive if the management goals include preserving or fostering the development of large patches of mature forest [3, 6].

To mitigate this dispersion effect, Rebain and McDill [23] proposed a model formulation capable of promoting or enforcing the development of large mature forest patches – patches that consist of management units with forest stands that are older than a certain age and larger than a certain size. The patches created by this model are not fixed and may shift to different locations on the landscape over time. This way, large areas of mature forest habitat will exist at some location on the landscape at any given point in time, but species dependent on large areas of mature forest habitat might need to find new, suitable habitats when their original habitat is lost (i.e., harvested). In fact, the model does not guarantee that the mature habitat that exists in one period will be anywhere close to the mature habitat that existed in the previous or following periods. Although this is not a perfect solution for every species, especially for those with limited capabilities for dispersal, this ecosystem management strategy might still be a useful option in landscapes that lack reserves [28] or to supplement a reserve system.

Another key limitation of the Rebain and McDill [23] model is that it only requires mature forest patches to meet age and size requirements, but the shape and other spatial attributes may also be important. Mature forest patches of the same size but different shapes can provide very different habitats for wildlife. For example, due to edge effects, large and mature forest patches do not necessarily provide interior habitat if their shape is too elongated [5]. Patches with complex shapes have proportionately more edge habitat than those of similar area but roughly circular shape [3]. Modern managed forest ecosystems tend to have more edge habitat and less interior habitat than pristine ecosystems because harvesting creates a lot of edge. The model proposed in this paper addresses this issue.

Forest edges – defined as “abrupt transitions between two relatively homogeneous ecosystems, at least one of which is a forest” [12] – generate unique habitats. Edge habitats, if numerous, can have a profound impact on the overall integrity of the forest ecosystem. On the negative side, they have been shown to reduce biodiversity in forests (e.g. [4, 21, 36]), and, under certain circumstances, they attract predators and therefore might become “ecological traps” [4]. Additionally, they often provide habitat that favors invasive, potentially harmful, plant species [12]. On the other hand, edge can also benefit some native terrestrial vertebrates, such as the New England cottontail (*Sylvilagus transitionalis*) or its predator, the bobcat (*Felis rufus*), whose early successional forest habitat is in decline in the northeastern United States due to forest maturation [10]. In landscapes where the negative impacts of edges outweigh their positive impacts, Matlack and Litvaitis [12] recommend designing harvest layouts that minimize the length of edges relative to interior habitat. This work follows this recommendation in a context where edge is defined as a transition between mature forest habitat patches and everything else.

The proportion of edge habitat within a patch is determined by the shape and size of the patch, assuming that edge width is constant for the entire patch. Circular patches are the most compact and will have the largest interior-to-edge ratio for a given size of patch. The perimeter-area ratio (PAR) of a patch is correlated to the proportion of edge habitat; the lower the PAR of a patch, the more compact it is and the less edge habitat it tends to contain relative to the total area of the patch. This metric also ensures that larger habitat patches receive better scores for compactness than smaller ones with the same shape. In forest planning problems with an objective of promoting interior mature forest habitat, a similar way to promote compactness is to minimize the edge length of the mature forest patches while keeping their area above a certain limit. Minimizing the perimeter while keeping the area of mature forest constant would be equivalent to minimizing the PAR. However, the models used in this study minimize the perimeter while requiring the area of mature forest to be at or above a certain value, which is similar to minimizing PAR but not exactly equivalent. Because it is easier to model, the edge length is minimized in the integer programs used in this study, and edge length is used as a surrogate of PAR. PAR was used as a post-optimization measure to evaluate the impact of the approach on compactness, but it was not part of the integer programming formulation.

Similar perimeter minimizing approaches have been followed by a number of studies in the optimal reserve selection and the land allocation literature. The optimal reserve selection problem, a variant of the set covering problem, seeks to minimize the cost of representing a set of species within a network of candidate reserves or to maximize the species representation under budget constraints. Spatial attributes of the network, such as the relative proximity, the connectedness, or the compactness of the reserves, have been extensively modeled (e.g., [14, 18, 32, 33]). For a recent comprehensive survey, see Williams et al. [34]. The following studies addressed the issue of compactness.

Williams and Revelle [32, 33] incorporated the minimum perimeter concern into their model by requiring buffers of unit width to surround the core reserve areas. As cost minimization favors the selection of as few buffers as possible, the model indirectly promotes compactness and contiguity. Their assumption that the reserve parcels form a regular grid network, where the parcels are squares of unit area, made the integer programming (IP) formulation simple and tight. However, their approach is less attractive in the case of harvest scheduling, where the shapes of the cutting units are typically predefined and irregular, which would make it difficult to control the width of the buffer. Furthermore, requiring a buffer around each mature forest patch may be unnecessary and costly. Marianov et al. [11] also takes advantage of a regular grid structure by predefining the most compact aggregates of square-shaped parcels. Wright et al. [35] were the first to model the perimeter explicitly. They introduced an IP formulation that minimized the total external border of land parcels to be acquired for development. The total external border was calculated using two sets of binary indicator variables. These variables determined which border segments of each parcel formed the external boundary of the parcel network. Önal and Briers [19] followed a similar modeling approach in the reserve selection context and accounted for the externality or internality of borders using only one set of variables. McDonell et al. [14] introduced a non-linear formulation of the minimum perimeter criterion that can be applied to irregularly shaped patches. They account for compactness through a weighted objective function. However, the non-linear nature of the formulation severely restricts the potential of this model to exactly optimize large-scale problems. Fischer and Church [2] employed a similar approach that linearizes the perimeter term used by McDonell et al. [14]. This is a significant improvement because the resulting linear structure allows powerful IP solvers to tackle larger problems.

Minimizing the perimeter of mature forest patches in harvest scheduling models requires a slightly different approach due to the dynamic nature of the problem. The added temporal dimension makes harvest scheduling models more complicated. This research builds on the works of Rebain and McDill [22, 23] and introduces an IP formulation that identifies management alternatives that would result in the development of large mature forest patches with minimum boundaries. The computational difficulty of the proposed formulation, as well as its impact on the compactness of the mature forest patches, is discussed. The cost of perimeter minimization is evaluated based on how much net present value must be forgone in order to achieve minimum boundaries. In addition, the impact of the formulation on the fragmentation of mature forest habitat is examined by comparing the total number of patches that evolve over the planning horizon with and without perimeter minimization. The models presented in the second part of this paper provide quantitative information on the tradeoffs between timber production and the provision of mature forest habitat.

2. Model Formulation

This section describes the boundary minimization component of the proposed IP model. The full model, called TOTALMIN (see Appendix), is a harvest scheduling model that can identify harvest layouts on a landscape and over time given various commodity production and sustainability objectives. The model includes harvest flow smoothing constraints, a minimum average ending age constraint, and maximum harvest opening size constraints. In addition, mature forest patch constraints are included to ensure that the spatial and temporal allocation of harvests would, in each planning period, guarantee the existence of patches of forests that are older than a minimum age and greater than a minimum size. An important feature of this approach is that these patches may be created over time by allowing currently immature forest patches to mature; i.e., the patches are not created solely by reserving existing old forest patches. One objective function of the model minimizes the total boundary of these patches. This construct is unique in that it allows dynamic but controlled habitat formation in a managed ecosystem.

The formulation of the mature forest patch criterion is a tighter version of the one presented in [22]. This was achieved by adding constraint sets (A8), (A10), and (A12) (see Appendix) to force variables O_{mt}, B_{ct} and BO_{mt} to turn on whenever constraints (A7), (A9) and (A11) allow them to turn on. The formulation of the maximum harvest area constraints is a generalization of the formulation presented in [13].

The key spatial units utilized by the model are management units, mature forest patches, clusters, and paths. Mature forest patches are contiguous sets of management units, each of which is older than the minimum age and has a combined area greater than the minimum patch size. Clusters and paths are both groups of contiguous management units (with no age limitations) whose combined areas just exceed the minimum mature patch size requirement (clusters) or the maximum harvest opening size requirement (paths), respectively.

2.1. The boundary minimization component

$$Min \sum_{t \in T} \mu_t \qquad (1)$$

subject to:

$$\sum_{m \in M} P_m BO_{mt} - 2 \sum_{pq \in N} CB_{pq} \Omega^t{}_{pq} = \mu_t \qquad \text{for } t = 1,2,\ldots T \qquad (2)$$

$$BO_{pt} + BO_{qt} - 2\Omega^t{}_{pq} \geq 0 \qquad \text{for } t = 1,2,\ldots T,\ pq \in N \qquad (3)$$

$$BO_{pt} + BO_{qt} - \Omega^{t}{}_{pq} \le 1 \qquad \text{for } t = 1,2,\ldots T,\ pq \in N \qquad (4)$$

$$BO_{mt} \in \{0,1\} \qquad \text{for } m \in M, \text{ and } t = 0,\ 1,\ldots,T \qquad (5)$$

$$\Omega_{pq} \in \{0,1\} \qquad \text{for } pq \in N \qquad (6)$$

where μ_t is the total perimeter of the mature forest habitat patches in period *t*; BO_{mt} is a binary variable whose value is 1 if management unit *m* is part of a patch that is big enough and old enough to constitute a large, mature patch (for the definition of this variable and its relation to harvesting decisions, see the full model in the Appendix); and $\Omega^{t}{}_{pq}$ is a binary variable that takes the value of 1 if adjacent management units *p* and *q* are both part of a patch that meets the minimum age requirement for large mature patches in period *t*. The parameters are *M*, which represents the set of management units in the forest; *T* – the number of periods in the planning horizon; P_m – the perimeter of management unit *m* in meters; CB_{pq} – the length of the common boundary between the two adjacent stands *p, q;* and N – the set of pairs of adjacent management units in the forest.

Equation (1) minimizes the sum of the perimeters that border all of the large, mature forest patches that evolve over the entire planning horizon. Constraint sets (2)-(4) work together. Constraints in (2) calculate the total perimeter of all groups of stands that fulfill the minimum age and area requirements for large mature forest patches in period *t*, and assign this value to the accounting variables μ_t. Constraints (3)-(4) control the values of the binary variables $\Omega^t{}_{pq}$ which replace the otherwise non-linear cross-product terms ($\Omega^t_{pq} = BO_{pt}BO_{qt}$) in (2). Notice that constraint set (4) is not necessary if objective function (1) is minimization. On the other hand, if maximizing the edge habitat is the objective, then constraint set (4) would be necessary and (3) could be dropped. Constraint sets (5) and (6) identify the mature patch size and the cross-product linearization ($\Omega^t{}_{PQ}$) variables as binary, respectively.

3. Methods

3.1. Measuring the spatial impact of perimeter minimization

The impact of the minimum perimeter model (TOTALMIN) on the patch shape, temporal patch overlap, and fragmentation of mature forest habitat was compared with that of a CONTROL Model. The CONTROL Model had the same constraint set as the TOTALMIN Model, but the perimeter minimizing objective function was replaced by the dual objectives of NPV and mature forest habitat maximization. The CONTROL Model formulation is as follows:

$$Max\, Z = \sum_{m \in M} A_m [c_{m0} X_{m0} + \sum_{t=h_m}^{T} c_{mt} X_{mt}] \tag{7}$$

$$Max\ \lambda \tag{8}$$

subject to: constraints (A2)-(A21) from the TOTALMIN Model (Appendix),

where Z is the discounted net revenue from the forest during the planning horizon, plus the discounted residual value of the forest; h_m is the first period in which management unit m is old enough to be harvested; A_m is the area of management unit m in hectares; and c_{mt} is the discounted net revenue per hectare if management unit m is harvested in period t plus the discounted residual forest value based on the projected state of the stand at the end of the planning horizon.

Equation (7) specifies the first objective function of the problem, namely to maximize the discounted net revenue from the forest during the planning horizon plus the forest value of each stand at the end of the planning horizon. Equation (8) specifies the second objective function of the CONTROL problem, to maximize the minimum amount of mature forest habitat in large patches over the time periods in the planning horizon. Equations (A13) and (8) work together to capture this value over all the time periods (the value of the variable λ) and maximize it. Although constraints (A14)–(A16) are not necessary in this model, they were retained for bookkeeping purposes, i.e., to tally the perimeter of the patches.

The solution to the CONTROL Model is a finite set of harvest schedules that are efficient (Pareto-optimal) with respect to the dual objectives of NPV and mature forest habitat maximization. Pareto-optimality, or efficiency, means that neither of the two objective function values – i.e., neither the NPV nor the mature patch habitat area – corresponding to any of these harvest schedules can be improved without compromising the other. Since the CONTROL Model tallies but does not optimize the perimeter of the patches, its solutions can serve as a baseline for the TOTALMIN Model to improve upon.

The spatial impacts of the TOTALMIN Model were analyzed on a 50-stand hypothetical forest planning problem (Figure 2). The forest could be considered slightly over-mature, since approximately 40% of the area is between 60-100 years old with an optimal financial rotation of 80 years. The average stand size was 18 ha, and the total forest area was 900 ha. As noted above, a 60-year planning horizon was considered, composed of three 20-yr periods. The four possible prescriptions for a given stand were: cut the management unit in period 1, period 2, or period 3, or do not cut it at all. Since the minimum harvest age was 60 years, some stands had fewer prescriptions if they were too young to harvest in the early periods of the model. A maximum harvest opening size of 40 ha was imposed, and groups of contiguous stands were allowed to be harvested concurrently as long as their combined areas did not exceed this limit. All individual management units were smaller than 40 ha. The wildlife species under consideration is assumed to need forest patches that are at least 50 ha in size and that are at least 60 years old. Since the minimum mature forest patch size is greater than the maximum management unit size, these patches must be composed of more than one management unit.

As mentioned earlier, the set of efficient solutions to the CONTROL Model provided a convenient series of feasible minimum habitat levels at which the impacts of the TOTALMIN Model could be examined. A multiple-objective optimization method, “Alpha-Delta” [31], was used to identify the set of efficient solutions for the CONTROL Model. This algorithm was specifically developed to solve multiple-objective integer programs like the models described in this paper. Using a weighted objective function with fixed weights, the algorithm finds solutions by sequentially constraining the feasible objective space. Giving a slight slope (Alpha) to the weighted objective function ensures that (1) the solutions are found progressively from one end of the efficient frontier to the other, and (2) these solutions are efficient. Each time a new solution is found, the feasible objective space is confined using the achievement values that correspond to the new solution. The two parameters of the algorithm, Alpha and Delta, were set to 1 degree and

0.01 ha, respectively. A detailed description of this algorithm and its parameters is provided in [31]. The example forest planning problem yielded 36 Pareto-optimal solutions (Figure 1). Corresponding to each solution is a harvest schedule indicating a set of management units to be harvested in each period. The line that connects these points in Figure 1 is the efficient frontier. This frontier separates the region where additional Pareto-optimal harvest schedules are known not to exist from the region where non-Pareto-optimal alternatives may exist [31].

For each Pareto-optimal solution to the CONTROL Model, the following information was collected: (1) NPV, (2) area of mature forest patch habitat in each planning period, (3) the total perimeter of the mature forest patches in each period, (4) the patch overlap between periods 1 and 2, and between periods 2 and 3, and (5) the total number of patches that evolved over the planning horizon. These data were used to describe the shape, the temporal connectivity and the fragmentation of the mature forest habitat that would evolve on the landscape under the solutions to the CONTROL Model. The attribute of shape was measured by the PAR of the patches. One PAR value was calculated for each planning period by dividing the total perimeter of patches that form in that period by the total area of these patches. The average of these three PARs was then calculated for each

Pareto-optimal solution. The temporal overlaps of the patches between planning periods 1-2 and 2-3 were expressed as the percentage of the total patch area in period 1 and 2, respectively. The fragmentation of mature forest habitat was accounted for by the number of patches that evolved during the entire planning horizon. The assumption behind describing fragmentation this way was that a mature forest habitat that consists of fewer patches with the same or greater total area is less fragmented. Again, the PARs and the number of patches corresponding to the CONTROL Model solutions served as a baseline to improve upon using the TOTALMIN Model.

Thirty-five of the 36 attainment values on the objective that maximized the minimum amount of mature forest habitat in the CONTROL Model served as minimum habitat levels for testing the perimeter minimizing formulation. The attainment value of zero, corresponding to the harvest schedule that yielded no patches (represented by Point A on Figure 1), was not used for obvious reasons. Thus, the TOTALMIN Model was solved 35 times by sequentially assigning the 35 attainment values (call these values a_i) from the CONTROL Model to variable λ, the right-hand-side of constraint (13).

3.2. Measuring the opportunity costs of perimeter minimization

Although the solutions to the above programs can be used to assess the potential reduction in perimeter, a second step is necessary to estimate how much NPV would have to be forgone to achieve these reductions. Thus, in Step 2, the following series of integer programs were solved.

For $i = 1,\ldots,35$*:*

$$Max\, Z = \sum_{m \in M} A_m [c_{m0} X_{m0} + \sum_{t=h_m}^{T} c_{mt} X_{mt}] \tag{9}$$

subject to:

$$\sum_{m \in M} A_m BO_{mt} \geq a_i \qquad \text{for } t = 1,2,\ldots T \tag{10}$$

$$\sum_{t=1}^{T} \mu_t \leq b_i \tag{11}$$

where a_i is the attainment value on objective function (7) of solution i, obtained by solving the CONTROL Model; and b_i is the objective function value, i.e., the minimal perimeter, of solution i obtained by solving the integer programs described in Step 1. Parameter a_i provides a lower bound on the minimum area of mature forest patches while b_i provides an upper bound on the total perimeter of mature forest patches. The rest of the constraints were the same as in the TOTALMIN Model. This 2-step procedure is referred to as the

PERMIN Procedure. Step 2 is necessary to avoid minimum perimeter solutions with NPVs that are lower than the potential maximum (weak Pareto-optima).

The objective function values (NPV) of the solutions to the IPs in Step 2 were compared to those of the CONTROL Model. The cost of perimeter minimization was expressed as the percentage difference between the pairs of NPVs for each minimum habitat level. In addition, the solution times of perimeter minimization were compared with those of the CONTROL Model.

3.3. Measuring the tradeoffs between the NPV and perimeter objectives

Minimizing the perimeter of the patches while requiring the minimum habitat area to be greater than or equal to a predefined level does not guarantee that the minimum PAR solution would be found; it only guarantees a minimum perimeter solution. Solutions might exist that have longer perimeters but larger habitat areas, resulting in a lower PAR than the minimum perimeter solution. Minimizing the PAR directly, on the other hand, was not pursued here because the obvious formulation of this concern would have required a non-linear objective function. This non-linearity can be avoided by minimizing the perimeter and maximizing the area of the patches simultaneously in a multiple-objective programming framework. A constraint that requires a small minimum area of mature habitat is also needed to avoid the zero perimeter / zero habitat scenario. From a harvest scheduling perspective, however, a third objective, such as maximizing the net revenues of the forest, is also needed in the model to ensure that the solutions are economically efficient. In order to avoid the complexities of dealing with three objectives simultaneously, we approached the problem by finding sets of harvest schedules that are efficient with respect to the NPV and perimeter objectives and constrained to provide various minimum areas of large, mature patch habitat. In this context, an efficient harvest schedule is one for which no other harvest schedule exists that would produce at least the minimum area of large, mature patch habitat in each period and yield more net revenues or less perimeter without compromising one of these latter two objectives.

An additional argument against minimizing the perimeter subject to minimum habitat area is that the average cost (NPV forgone per unit PAR improvement) of the minimum perimeter solution might be higher than some of the longer perimeter solutions. The decision maker (DM) might want to select a harvest schedule that yields a certain amount of mature forest habitat in large and relatively compact patches at a lower average cost. Exploring the tradeoffs between the NPV and the perimeter at various levels of minimum habitat area provides insight on the quality of the minimum perimeter solution and while potentially identifying Pareto-optimal alternatives with lower average costs.

The tradeoffs between the NPV and the perimeter of the mature forest patches were illustrated by generating the set of Pareto-optimal solutions for these two objectives at three different levels of minimum habitat area. This was done using the Alpha Delta algorithm [31] to solve a bi-criteria model that maximizes the NPV (Equation (9)) and minimizes the total perimeter of the patches (Equation (1)) subject to constraints (A2)-(A21). The right-hand-side of Constraint (A13), λ, which represents the required minimum habitat level, was set at three different values: 64.38, 119.06, and 153.99 ha. These values were chosen to be evenly distributed along the efficient frontier of the CONTROL Model. The parameters of the Alpha Delta algorithm, Alpha and Delta, were set to 1 degree and 1 meter, respectively. Again, this setting ensured that a large number of Pareto-optimal solutions would be found.

CPLEX 9.0 [7] was used to generate the Pareto-optimal solutions to these models and to solve the IPs in the perimeter minimization and NPV maximization phases. A program that automates the Alpha Delta algorithm was written in Microsoft Visual Basic 6 using the ILOG CPLEX Callable Libraries. The relative MIP gap tolerance parameter (optimality gap) was set to 0.00001 (0.001%), the MIP variable selection strategy parameter was set to ‘3’ (i.e., strong branching), and the strong branching parallel thread limit was set to 2. The setting was used because in preliminary runs parallel strong branching proved to be the most efficient variable selection strategy. All the IPs were solved on a Dual-AMD Athlon ™ MP 2400+ (2.00 GHz) computer with 2.0 GB RAM.

4. Results and discussion

4.1. The spatial impacts of perimeter minimization

On average, the TOTALMIN Model reduced the PAR by 27% and the number of patches by one third (Table 1). The proportion of the patch that is part of the mature forest habitat in both periods 1 and 2 and in both periods 2 and 3 increased by 72% and 145%, respectively. The average cost, as measured by the reduction in the NPV of the forest, of minimizing the perimeter while ensuring that a certain area of mature forest habitat is maintained in each period is 7.6% and ranges from about 5 to 10%. In addition, the average time to solve the perimeter minimization problem was 17.49 minutes per problem compared to the 0.81 minutes to solve the CONTROL Model.

4.1.1. Impact on compactness

Figure 2 shows the spatial impact of the CONTROL and TOTALMIN Models on an example problem. The light-shaded polygons represent the management units that are scheduled to be cut in the given period, while the dark-shaded ones identify those that constitute large, mature forest patches. In this example, the required minimum amount of mature forest habitat was 154 ha. The three numbers in each polygon indicate the unit ID, the initial age-class, and the period when the unit is scheduled for harvest (a period of "0" indicates that the management unit is not scheduled to be cut during the planning horizon). The planning periods in this example are 20 years long. The ages-classes of units that are not scheduled to be cut in the respective periods are also shown.

The improvement in compactness is striking in the second and third periods (Figure 2, left). The TOTALMIN Model reduced the PAR from 68.77 m/ha to 40.86 m/ha in the second period, and from 61.87 m/ha to 40.86 m/ha in the third period; no improvement was made in the first period. Similar degrees of improvement occurred in the majority of the 35 example runs (Table 1). The smallest improvements were typically made in the first period because the initial spatial structure of the forest limits the number of potential harvest schedules that can yield more compact patches.

4.1.2. Impact on temporal connectivity

Observe in Figure 2 that the mature forest patch that evolves in the second period in the solution to the TOTALMIN Model remains intact in the third period. Does minimizing the total perimeter result in static habitat patches? The data from 35 runs indicate that, on average, more than 90% of the area that is part of mature forest patches in period 2 is also part of the patches in period 3 (Table 1). This means that the patches do not change much from period 2 to 3 when the perimeter is minimized under the TOTALMIN Model. This is good news for mature forest specialists that cannot move freely and rapidly to avoid habitat loss. Similar models with longer planning horizons are needed to determine whether these patches remain intact after the third planning period. Unfortunately, the computational difficulty of solving these IP problems with four or more planning periods is still too great to allow us to conduct such experiments. As a note, Tahvonen [29] provides evidence that under certain economic conditions long-term stationary equilibrium exists between lands allocated for timber production versus old-growth preservation where the area allocated for old growth is permanently fixed rather than rotating to different parts of the landscape. One could speculate that such a long-term equilibrium is likely to occur sooner with spatially more constrained patch attributes (e.g., with minimum boundary requirements).

4.1.3. Impact on habitat fragmentation

The CONTROL Model produced an average of 4.51 patches over the planning horizon. This was reduced to exactly three with the TOTALMIN formulation (Table 1), which means that in all cases there was exactly one patch in each period. This result is not surprising. Combining two or more disjoint patches into one, while keeping the total area of the patches above a predefined limit, generally yields a lower total perimeter. These results suggest that minimizing the perimeter tends to lead to fewer and larger patches.

Further tests would be needed to conclude whether the proposed perimeter minimizing formulation could serve as an effective and computationally less expensive alternative to models that directly enforce temporal or spatial connectivity (e.g., [19, 20]). In sum, the TOTALMIN Model appears to be effective in reducing the PAR and the number of the patches and in increasing the temporal overlap.

4.2. Tradeoffs between the NPV and perimeter minimization

The perimeter minimizing formulation finds the maximum reduction possible in the length of the perimeter while maintaining at least a specific minimum area of mature patches in all periods. However, solutions may exist that result in nearly as much perimeter reduction at substantially lower cost. To address this issue, the rest of the discussion focuses on analyzing the tradeoffs between the NPV and the perimeter of the patches at three different levels of minimum habitat area: 64.38, 119.06, and 153.99 ha. Figure 3 illustrates these tradeoffs. Each point on the graphs represents an efficient (or, equivalently, a Pareto-optimal) harvest schedule. These sequences of solutions were generated by the Alpha-Delta method using the TOTALMIN formulation with the addition of the NPV objective (Equation 22). The lines that connect these points form the efficient frontiers between the NPV and the perimeter of the patches at each of the three levels of minimum habitat area. The rightmost points (Point 1, 10, 24) on these curves represent harvest schedules that were obtained by maximizing the NPV of the forest without regard to the perimeter of the patches while maintaining the predefined minimum habitat area requirements. The leftmost points (Point 9, 23, T) are harvest schedules obtained by the PERMIN Procedure described above. The points between these two extremes represent efficient solutions that offer compromises in the attainment of the NPV and the perimeter objectives.

These graphs show how much NPV would have to be forgone in order to reduce the perimeter of the patches to a desired level while maintaining a minimum total area of these mature patches. They can help the DM eliminate solutions where a disproportionately high amount of one objective would have to be given up to gain minimal improvement on the other. For example, Point B on the lowermost graph would be preferred over Point T by most people since the harvest schedule it represents results in only slightly more total perimeter as Point T at a much lower (approx. $50,000 less) cost. The maps in Figure 4 show the mature forest patches in grey that would evolve in period 2 and 3 if the harvest schedules corresponding to Point T and B in Figure 3 were chosen. The 'T' scenario would result in slightly more elongated patches than the 'B' scenario (Figure 4).

4.2.1. The average costs of perimeter minimization

Another way of analyzing these alternative harvest schedules is to look at the average cost (i.e., average NPV forgone) of achieving one unit of PAR improvement over the solutions to the CONTROL Model. The DM could easily eliminate the least cost-efficient alternatives from further consideration based on this statistic. Table 2 shows the

NPVs, PARs and average costs that correspond to each of the efficient harvest schedules represented by the points in Figure 3.

Note that harvest schedule B in the rightmost table yields patches with a lower average PAR than harvest schedule T, the minimum perimeter solution. Further, its average cost per unit of PAR improvement is also lower. This confirms that perimeter minimization (i.e., the PERMIN Procedure) does not necessarily generate patches with minimum PARs. Second, the large fluctuation of the average costs along the efficient frontiers also suggests that good alternative solutions might be overlooked if only the minimum perimeter solution is identified.

Clearly, finding the best compromise forest management regime for a given landscape is not trivial if several conflicting objectives are present. Balancing the inherent tradeoffs between these objectives requires the forest planners and decision makers to examine the problem from different perspectives.

Conclusions

The models presented in this article enable forest planners to control the size and shape of the mature forest habitat patches that evolve on a landscape as a result of harvest schedules. At the same time, they assure that the areas of contiguous harvest openings never exceed a predefined limit. This is not accomplished without significant computational costs. The integer programming model proposed here is much more complex and harder to solve to optimality than the models without the patch size and perimeter components. Yet, the steep increase in computational cost may be worthwhile if the models identify spatial benefits that can be attained at relatively low opportunity costs (NPV forgone) with perimeter minimization. This study has shown that minimizing the perimeter of the patches tends to make their shape more compact and to increase their temporal connectivity while yielding fewer and larger patches. Furthermore, as computers and software packages are constantly getting faster and more powerful, there is reason to be optimistic in regard to the future use of these complex models.

The multiobjective approach discussed in the second part of this study can be used to generate efficient harvest schedules with respect to the three goals of maximizing the NPV of the forest, maximizing the minimum amount of mature forest habitat in large patches, and minimizing the perimeter of these patches. This approach can help DMs select their best compromise solution after having seen several alternatives. In many cases it would not be possible to set up restrictions or goals on the size, shape and other spatial attributes of the patches prior to the planning process because of the difficulty in quantifying these parameters up front. The approach proposed here enables the DM to see what is possible before setting specific targets for these attributes. Finding the set of efficient harvest schedules with respect to the three objectives, however, is not trivial. The approach followed by this study only identifies a small subset of the possible efficient harvest schedules. This would be true even if the NPV-perimeter tradeoffs were generated at more than three levels of minimum habitat area. Although generating the complete set of tradeoffs between the three objectives could provide the DM with more information, this benefit might be offset by the increased computational expense.

Analyzing the impacts of perimeter minimization on the spatial and temporal connectivity of mature forest patches within landscapes of various spatial structures is a subject of future research. Further research is also needed to identify improved integer programming formulations of these problems that reduce solution times.

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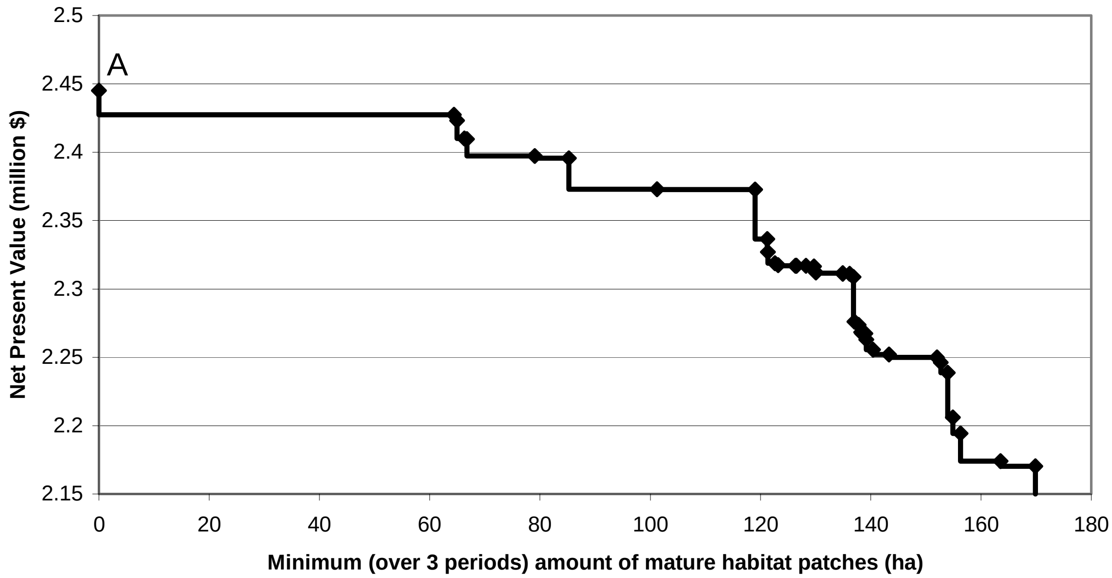

Figure 1. Pareto-optimal solutions to the CONTROL Model.

Table 1. The impact of perimeter minimization

	Average (per problem)								
	NPV ($)	Cost (NPV forgone relative to that of the CONTROL Model)			Temporal overlap between periods		P/A ratio (m/ha)	Number of patches	Solution time (minutes)
		mean	max	Min	1-2	2-3			
CONTROL	2,303,532	-	-	-	34.35%	37.21%	69.26	4.51	0.81
TOTALMIN	2,129,501	7.56%	10.57%	4.75%	59.04%	91.06%	50.31	3.00	17.49

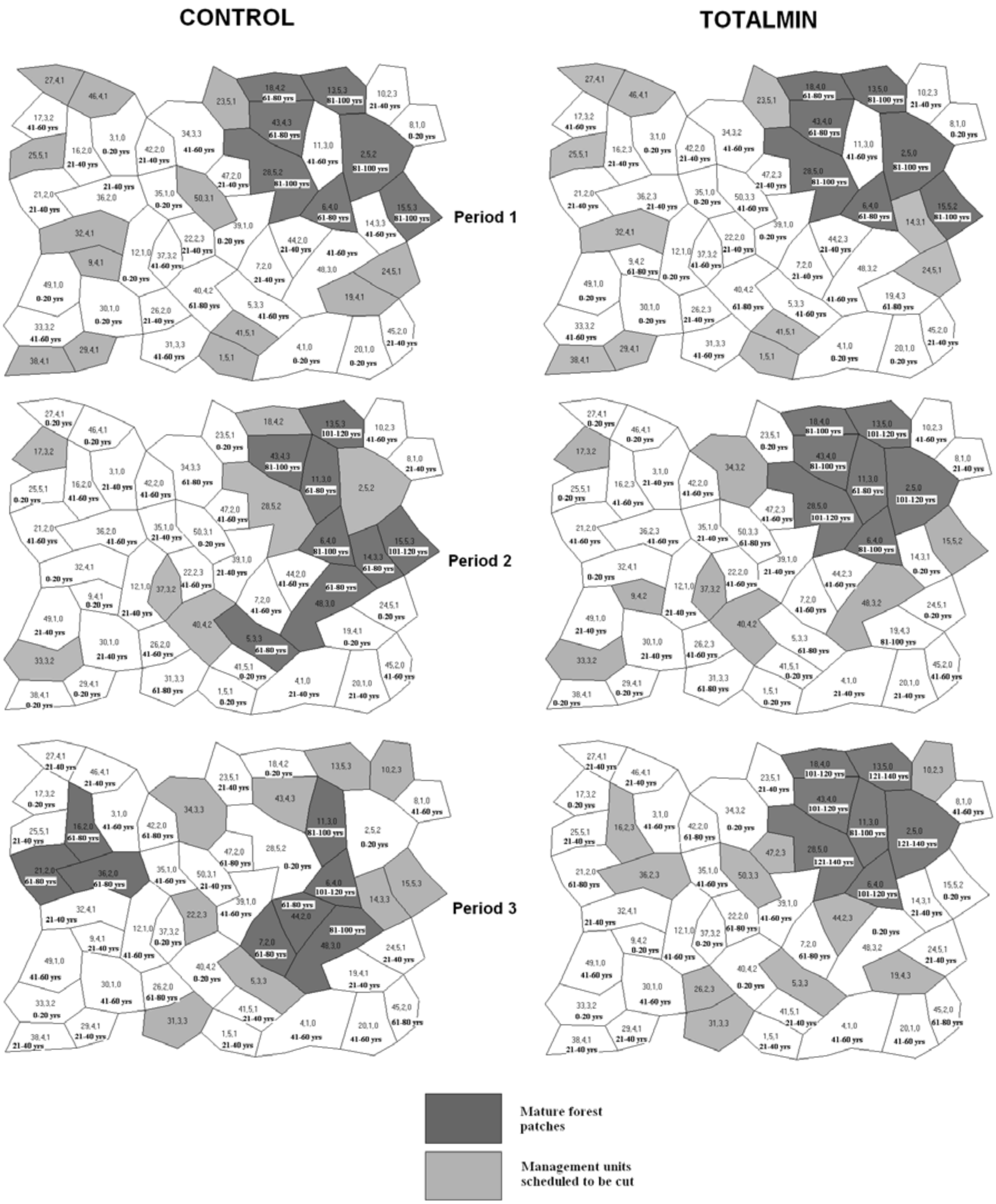

Figure 2. Mature forest habitat patches generated by the TOTALMIN vs. CONTROL Models. (Labels in the polygons indicate the management unit ID, the initial age class of the management unit, and the period in which the management unit is scheduled to be harvested. The ages of management units that are not cut in the given period are also indicated.)

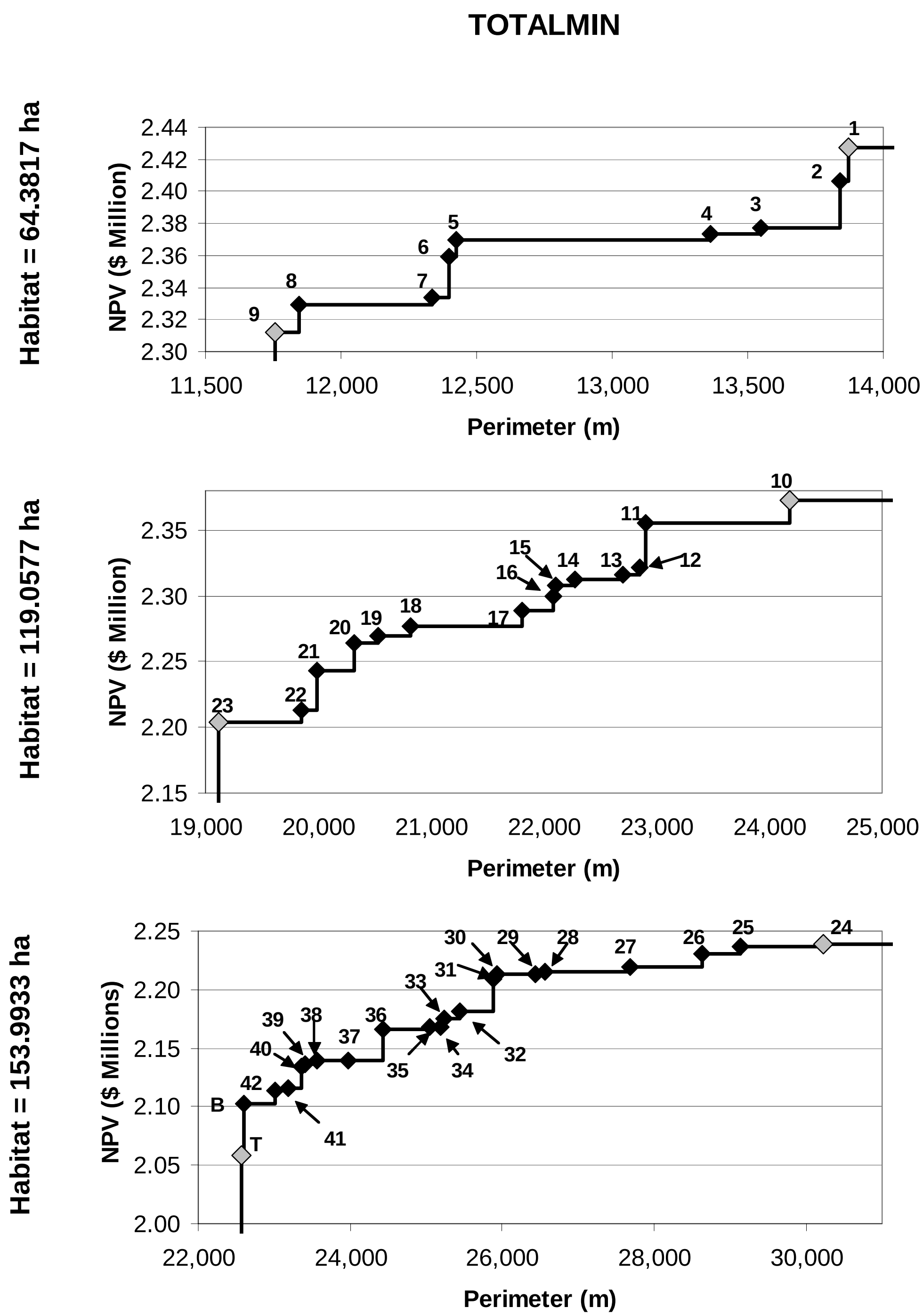

Figure 3. Tradeoffs between the NPV and the total perimeter of mature forest patches at three levels of minimum habitat area (TOTALMIN).

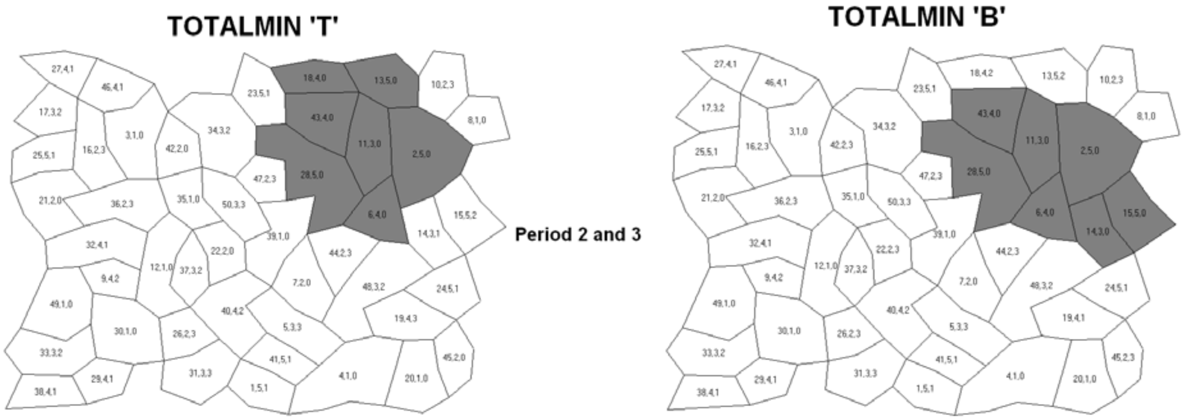

Figure 4. Mature forest patches generated by the TOTALMIN "T" and "B" Models. (Labels in the polygons indicate the management unit ID, the initial age class of the management unit, and the period in which the management unit is scheduled to be harvested. Grey areas identify mature forest patches.)

Table 2. The cost of perimeter minimization with the TOTALMIN Model

	Habitat >= 64.3817 ha				Habitat >= 119.0577 ha				Habitat >= 153.9933 ha		
	NPV ($)	Average P/A Ratio (m/ha)	Average Cost per unit PAR imp.		NPV ($)	Average P/A Ratio (m/ha)	Average Cost per unit PAR imp.		NPV ($)	Average P/A Ratio (m/ha)	Average Cost per unit PAR imp.
1	**2,427,424**	**70.71**	**N/A**	**10**	**2,372,633**	**67.50**	**N/A**	**24**	**2,238,791**	**64.68**	**N/A**
2	2,406,217	70.16	$38,028	**11**	2,355,799	63.76	$4,501	**25**	2,236,222	62.61	$1,242
3	2,376,803	64.50	$8,142	**12**	2,321,840	55.30	$4,163	**26**	2,230,087	60.32	$1,998
4	2,373,246	66.88	$14,145	**13**	2,315,861	61.14	$8,937	**27**	2,219,769	59.56	$3,718
5	2,369,735	61.75	$6,434	**14**	2,312,124	61.98	$10,960	**28**	2,215,169	58.44	$3,785
6	2,359,213	61.26	$7,211	**15**	2,308,129	55.47	$5,362	**29**	2,213,536	57.27	$3,410
7	2,333,438	58.99	$8,019	**16**	2,299,392	59.87	$9,606	**30**	2,212,977	56.51	$3,158
8	2,329,005	60.50	$9,632	**17**	2,288,670	60.58	$12,144	**31**	2,209,136	55.92	$3,386
9	**2,312,173**	**60.05**	**$10,805**	**18**	2,276,818	57.99	$10,080	**32**	2,181,014	54.48	$5,662
				19	2,269,390	56.66	$9,528	**33**	2,175,380	52.60	$5,250
				20	2,263,789	56.66	$10,047	**34**	2,168,379	53.19	$6,131
				21	2,243,177	53.96	$9,564	**35**	2,168,253	51.76	$5,458
				22	2,212,710	54.53	$12,336	**36**	2,165,597	50.41	$5,128
				23	**2,204,132**	**51.21**	**$10,344**	**37**	2,139,321	51.50	$7,550
								38	2,139,242	50.46	$6,999
								39	2,136,270	50.39	$7,176
								40	2,133,828	50.40	$7,353
								41	2,115,789	49.24	$7,969
								42	2,113,340	49.18	$8,095
								B	2,102,264	48.15	$8,259
								T	**2,058,601**	**48.38**	**$11,056**

Appendix: The Full Model (TOTALMIN)

$$Min \sum_{t \in T} \mu_t \tag{A1}$$

subject to:

$$X_{m0} + \sum_{t=h_m}^{T} X_{mt} \leq 1 \qquad \text{for } m \in M \tag{A2}$$

$$\sum_{m \in M_{ht}} v_{mt} \cdot A_m \cdot X_{mt} - H_t = 0 \qquad \text{for } t = 1, 2, \ldots T \tag{A3}$$

$$b_{lt} H_t - H_{t+1} \leq 0 \qquad \text{for } t = 1, 2, \ldots T\text{-}1 \tag{A4}$$

$$-b_{ht} H_t + H_{t+1} \leq 0 \qquad \text{for } t = 1, 2, \ldots T\text{-}1 \tag{A5}$$

$$\sum_{m \in M_p} X_{mt} \leq |M_p| - 1 \qquad \text{for all } p \in P \text{ and } t = k_p, \ldots, T \tag{A6}$$

$$\sum_{j \in J_{mt}} X_{mj} - O_{mt} \geq 0 \qquad \text{for } m \in M, \text{ and all } t \text{ such that } J_{mt} \neq \emptyset \tag{A7}$$

$$\sum_{j \in J_{mt}} X_{mj} - |J_{mt}| O_{mt} \leq 0 \qquad \text{for } m \in M, \text{ and all } t \text{ such that } J_{mt} \neq \emptyset \tag{A8}$$

$$\sum_{m \in MC_c} O_{mt} - |MC_c| B_{ct} \geq 0 \qquad \text{for } c \in C, \text{ and } t = 1, 2, \ldots, T \tag{A9}$$

$$\sum_{m \in MC_c} O_{mt} - B_{ct} \leq |MC_c| - 1 \qquad \text{for } c \in C, \text{ and } t = 1, 2, \ldots, T \tag{A10}$$

$$\sum_{c \in C_m} B_{ct} - BO_{mt} \geq 0 \qquad \text{for } m \in M, \text{ and } t = 1, 2, \ldots, T \tag{A11}$$

$$\sum_{c \in C_m} B_{ct} - |C_m| BO_{mt} \leq 0 \qquad \text{for } m \in M, \text{ and } t = 1, 2, \ldots, T \tag{A12}$$

$$\sum_{m \in M} A_m BO_{mt} \geq \lambda \qquad \text{for } t = 1,2,\ldots T \tag{A13}$$

$$\sum_{m\in M} P_m BO_{mt} - 2\sum_{pq\in N} CB_{pq}\Omega^t{}_{pq} = \mu_t \qquad \text{for } t = 1,2,\ldots T \tag{A14}$$

$$BO_{pt} + BO_{qt} - 2\Omega^t{}_{pq} \geq 0 \qquad \text{for } t = 1,2,\ldots T,\ pq \in N \tag{A15}$$

$$BO_{pt} + BO_{qt} - \Omega^t{}_{pq} \leq 1 \qquad \text{for } t = 1,2,\ldots T,\ pq \in N \tag{A16}$$

$$\sum_{m\in M} A_m[(Age_{0t}^T - \overline{Age}^T)X_{0t} + \sum_{t=h_m}^{T}(Age_{mt}^T - \overline{Age}^T)X_{mt}] \geq 0 \tag{A17}$$

$$X_{mt} \in \{0,1\} \qquad \text{for } m \in M, \text{ and } t = 0, h_m, h_m+1,\ldots,T \tag{A18}$$

$$B_{ct} \in \{0,1\} \qquad \text{for } c \in C,\ t = 1, 2, \ldots, T \tag{A19}$$

$$O_{mt},\ BO_{mt} \in \{0,1\} \qquad \text{for } m \in M, \text{ and } t = 0, 1,\ldots,T \tag{A20}$$

$$\Omega_{pq} \in \{0,1\} \qquad \text{for } pq \in N \tag{A21}$$

where the variables are:

X_{mt} = A binary decision variable whose value is 1 if management unit *m* is to be harvested in period *t* for $t = h_m, h_{m+1},\ldots,T$. In other words, X_{mt} represents a harvesting prescription for management unit *m*. When $t = 0$, the value of the binary variable is 1 if management unit *m* is not harvested at all during the planning horizon (i.e., X_{m0} represents the "do-nothing" alternative for management unit *m*). Note: in constraint sets (A7-8), index *j* is used, in addition to *t*, to denote harvest periods. The new identifier is needed in these constraints because t is already used to define the period for which O_{mt} applies.

λ = the minimum area of mature forest habitat patches over all periods;

μ_t = the total perimeter of mature forest habitat patches in period t;

H_t = a continuous variable indicating the total volume of sawtimber in m^3 harvested in period t;

O_{mt} = a binary variable whose value may equal 1 if management unit m meets the minimum age requirement for mature patches in period t, i.e., the management unit is old enough to be part of a mature patch;

B_{ct} = a binary variable whose value is 1 if all of the stands in cluster c meet the minimum age requirement for mature patches in period t, i.e., the cluster is part of a mature patch;

BO_{mt} = a binary variable whose value is 1 if management unit m is part of a cluster that meets the minimum age requirement for large mature patches, i.e., the management unit is part of a patch that is big enough and old enough to constitute a large, mature patch;

Ω^{t}_{pq} = a binary variable whose value is 1 if adjacent management units p and q are both part of a cluster that meets the minimum age requirement for large mature patches in period t;

and the parameters are:

M = the set of management units in the forest;

A_m = the area of management unit m in hectares;

T = the number of periods in the planning horizon;

h_m = the first period in which management unit m is old enough to be harvested;

M_{ht} = the set of management units that are old enough to be harvested in period t;

v_{mt} = the volume of sawtimber in m^3/hectare harvested from management unit m if it is harvested in period t;

b_{lt} = a lower bound on decreases in the harvest level between periods t and $t+1$ (where, for example, $b_{lt} = 1$ requires non-declining harvest; $b_{lt} = 0.9$ would allow a decrease of up to 10%);

b_{ht} = an upper bound on increases in the harvest level between periods t and $t+1$ (where, for example, $b_{ht} = 1$ allows no increase in the harvest level; $b_{ht} = 1.1$ would allow an increase of up to 10%);

M_p = the set of management units in path p;

P = the set of all paths, or groups of contiguous management units, whose combined area is just above the maximum harvest opening size (the term "path," as used in this paper, is defined in the following discussion);

k_p = the first period in which the youngest management unit in path p is old enough to be harvested;

J_{mt} = the set of all prescriptions under which management unit m meets the minimum age requirement for mature patches in period t;

MC_c = the set of management units that compose cluster c;

C = the set of all clusters, or groups of contiguous management units whose combined area is just above the minimum large, mature patch size (the term "cluster," as used in this paper, is defined in the following discussion);

C_m = the set of all clusters that contain management unit m;

P_m = the perimeter of management unit m in meters;

N = the number of pairs of management units in the forest that are adjacent;

CB_{pq} = the length of the common boundary between the two adjacent stands p, q in meters;

Age_{mt}^{T} = the age of management unit m at the end of the planning horizon if it is harvested in period t; and

$\overline{Age}^{T}$ = the target average age of the forest at the end of the planning horizon.

Equation (A1) minimizes the sum of the perimeters that border all of the large, mature forest patches that evolve over the entire planning horizon. Forest planning models generally consider management actions and the consequent state of the forest over a finite time period known as the planning horizon. The planning horizon is then subdivided into discrete planning periods, and it is assumed that all of the activities that occur within a given planning period happen at one point, typically the midpoint, of the period. In the example problems discussed in the paper, the planning horizon is 60 years, with three 20-year planning periods.

Constraint set (A2) consists of logical constraints that allow only one prescription to be assigned to a management unit, including a do-nothing prescription. Harvest variables

(X_{mt}) are only created for periods where the stand is old enough to be harvested. Constraint set (A3) consists of harvest accounting constraints that assign the harvest volume for each period to the harvest variables (H_t). Constraint sets (A4) and (A5) are flow constraints that restrict the amount by which the harvest level is allowed to change between periods. In the example problems in this paper, harvests were allowed to increase by up to 15% from one period to the next or to decrease by up to 3%.

Constraint set (A6) consists of adjacency constraints generated with the Path Algorithm [13]. They limit the maximum size of a harvest opening, a restriction often imposed for legal or policy reasons, by prohibiting the concurrent harvest of any contiguous set of management units whose combined area just exceeds the maximum harvest opening size. The exclusion period imposed by these constraints equals one planning period, but the constraints can be modified easily to impose longer exclusion periods in integer multiples of the planning period. A "path" is defined for the purposes of the algorithm as a group of contiguous management units whose combined area just exceeds the maximum harvest opening size. These paths are enumerated with a recursive algorithm described in [13]. A constraint is written for each path and period in which all of the management units in the path are old enough to be harvested. (In the initial periods of the planning horizon, some of the management units in a path may not be mature enough to be harvested.) The constraints prevent the concurrent harvest of all of the management units in that path, since this would violate the maximum harvest opening size.

Constraint sets (A7-13) are the mature patch size constraints. Constraint sets (A7-8) determine whether or not management units meet the minimum age requirement for mature patches. These constraints sum over all of the prescription variables (variables X_{mj}) for a management unit under which the unit would meet the age requirement for mature patches in a given period. O_{mt} equals 1 if and only if one of these prescriptions has a value of 1, indicating that the management unit will be old enough in that period to be part of a large mature patch. As an example, if the initial age class of management unit *m* is 3 (41-60 years) and the minimum age requirement for mature patches is age class 4 (61-80 years), then only under prescriptions $X_{m0} = 1$ or $X_{m3} = 1$ ($J_{m2} = \{0, 3\}$) can this unit become old enough by period 2 to be part of a mature patch. One pair of these constraints is written for each management unit in each period in which it is possible for the management unit to meet the age requirement for a mature patch (i.e., when $J_{mt} \neq \varnothing$). For example, if the initial age class of management unit *m* is 1 or 2 (0-40 years) in the above example, then under no prescription will this unit become old enough by period 2. O_{m2} will never turn on in this case.

Constraint sets (A9-10) determine whether or not a cluster of management units meets the minimum age requirement for mature patches. All possible clusters are enumerated using a recursive algorithm described in [23]. A cluster meets the age requirement for mature patches in period *t* if all of the management units that compose that cluster meet the age requirement, as indicated by the set of O_{mt} variables for the management units in that cluster. B_{ct} takes a value of 1 if and only if cluster *c* meets the age requirement in period *t*. These pairs of constraints are written for each cluster in each period including those in which for one or more stands in the cluster, the set J_{mt} is empty. It is clear, however that in these periods, the cluster in question cannot meet the age requirement under any harvest scheduling scenario. We relied on the IP solver's preprocessor to identify and eliminate these constraints.

Constraint sets (A11-12) determine whether or not individual management units are part of a cluster that meets the minimum age requirement for mature patches, i.e., whether a management unit is part of patch that is big enough and old enough. Since the clusters overlap, this constraint set is necessary to properly account for the total area of large, mature patch habitat. These constraints set the values of the variables that indicate whether a management unit is part of a patch that meets the minimum age and size requirement for large, mature patches in period *t*. $BO_{mt} = 1$ if and only if at least one of the clusters it belongs to meets the age requirement in that period. Constraint set (A13) specifies that the total mature patch area for each period must be larger than λ in all periods.

Constraint sets (A14-16) work together. Constraints in (A14) calculate the total perimeter of all groups of stands that fulfill the minimum age and area requirements for large mature forest patches in period *t*, and assign this value to the accounting variables μ_t. The total perimeter ($\sum_{t \in T} \mu_t$) is minimized by objective function (1). Constraints (A15-16) control the values of the binary variables $\Omega^t{}_{pq}$ which replace the otherwise non-linear cross-product terms ($\Omega^t_{pq} = BO_{pt} BO_{qt}$) in (A14). Notice that constraint set (A16) is not necessary if objective function (A1) is minimization. On the other hand, if maximizing the edge habitat is the objective, then constraint set (A16) would be necessary and (A15) could be dropped.

Constraint (A17) is an ending age constraint. It requires the average age of the forest at the end of the planning horizon to be at least $\overline{Age}^T$ years, preventing the model from over-harvesting the forest. In the example problems in this paper, the minimum average ending age was set at 40 years, or ½ the optimal economic rotation.

Constraint sets (A18-21) identify the stand prescription, mature patch size, and the cross-product linearization ($\Omega^{t}{}_{PQ}$) variables as binary.