REVIEW OF INTERNATIONAL ECONOMICS
Manuscript No: #8493, Acceptance Date: October 2, 1998

International Capital Markets and Non-Scale Growth*

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Abstract

New growth models exhibit "scale effects," meaning that variations in the levels of key variables exert permanent influences on growth rates. Such predictions run counter to recent empirical evidence. In this paper we extend a general non-scale model to the open economy. With complete capital markets we find that only output and capital, but not consumption, retain their non-scale structure. Introducing capital market imperfections, the model is again fully non-scale. Debt subsidies are analyzed and shown to provide capital flow reversals consistent with the recent experience in East Asia.

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JEL Classification: F34, F41
Abbreviations: None
Number of Figures: 3
Date: October 9, 1998

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1. Introduction

An important implication of "new growth" models is that they exhibit "scale effects", meaning that variations in the *levels* of key variables such as the investment rate, research effort, and policy variables (tax rates) exert *permanent* influences on national growth rates. But these theoretical predictions run counter to recent empirical evidence obtained from studies based on the US and other OECD countries.[1] This has led to the development of a new class of so-called "non-scale" growth models, in which technology and capital accumulation are still endogenous, but long-run growth rates are now independent of changes in policy and other scale variables.[2] Instead, long-run growth rates are determined by the exogenous growth rate of labor in conjunction with production elasticities. In this respect these new models are closer in spirit to the traditional Solow-Swan neoclassical growth model, which in fact emerges as a special case.

One further attractive feature of non-scale models is that a balanced growth equilibrium obtains with few restrictions on returns to scale. This is in contrast to previous endogenous growth models, which require constant returns to scale in the accumulated factors of production for balanced growth to prevail.[3] It also contrasts with the traditional neoclassical growth model, for which constant returns to scale in capital and labor must hold. Thus the non-scale growth model has the important advantage of flexibility, in that the relevance of the production elasticities to growth provides a possible explanation for the observed diversity of long-run growth rates.

The discussion of this new strand of the literature has taken place exclusively within closed economies. The literature thus abstracts from factor mobility and its influences on growth, just at a time when international capital flows have become an integral part of the economic performance of industrialized economies. This paper extends the discussion of non-scale growth to the open

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[1] For example, OECD data support neither the claim of R&D based growth models that a doubling of the resources devoted to R&D efforts should increase the rate of growth proportionately, nor the proposition of AK models that an increase in investment rates results in higher growth [Jones (1995b)]. Tax rates have been shown to be in ineffective in influencing long-run growth in the US; see Backus, Kehoe, and Kehoe (1992) and Stokey and Rebelo (1995).

[2] The term non-scale reflects the characteristic that a country's growth rate is independent of the scale of the economy as measured, for example, by the size of population; see Jones (1995a), Segerstrom (1995), Young (1998). Eicher and Turnovsky (1997) provide a general characterization of non-scale growth using a two-sector production technology.

[3] Solow (1994) refers to this as a knife-edge condition. See Jones (1997) for discussion of this issue.

economy, doing so with a twofold objective in mind. First, we wish to characterize the dynamic structure of a simple open economy non-scale model and examine the extent to which the non-scale nature of the model will be preserved once international capital flows are introduced. Second, we seek to contrast the results with the standard open economy AK model, on the one hand, and the neoclassical growth model, on the other.

To keep the analysis tractable we restrict the production technology to only a single sector. Initially we consider a conventional pure small open economy facing a perfect world capital market, and assume that capital accumulation is subject to convex adjustment costs, a feature common to small open economy models (Turnovsky 1997b). Subsequently, we modify the model by introducing a capital market imperfection, in the form of an upward sloping supply schedule of funds.

Opening the non-scale economy to international trade in goods and assets leads to several important new results. First, the presence of a perfect world capital market generates a sharp dichotomy between the behavior and determinants of the equilibrium growth rates of consumption and output. As in the open economy AK model, consumption is always on its balanced growth path and its equilibrium growth rate is determined by domestic preferences and the net of tax foreign interest rate. On the other hand, the long-run growth rate of output (and capital) is now determined by the production and population growth parameters alone (and is independent of tax rates). They thus retain the non-scale structure and are subject to transitional dynamics.[4] Thus in the presence of perfect world capital markets, the non-scale property of the closed economy is only partially observed; the characteristics of this economy may therefore be termed *semi-non-scale*.

The introduction of the capital market imperfection fundamentally changes both the dynamic structure and long-run policy effectiveness. The evolution of all variables becomes interrelated and subject to transitional dynamics. Borrowing constraints thus yield an equilibrium in which the equilibrium growth rates of *both* output and consumption are determined by the production and population growth parameters alone, precisely as in a non-scale closed economy. A *full* non-scale equilibrium is thus obtained.

[4]This contrasts sharply with the open economy AK model, in which the production side is also always on its steady balanced growth path (Turnovsky 1996).

The examination of the transitional dynamics in the presence of international capital market imperfections reveals interesting insights. Recent events in East Asia and Mexico highlight that simple one-time policy changes may lead to an initial period of excessive capital inflows, followed by substantial capital outflows at some later stage.[5] Conventional dynamic growth models of open economies are usually characterized by one-dimensional transitional adjustment paths, and therefore require at least two time-separate and offsetting policy prescriptions in order to account for such reversals in capital flows (Bond et. al. 1996, Turnovsky 1997b, Chapter 5). By contrast, our model is able to generate such capital flow reversals as part of the transitional dynamics following a one-time policy event.

Specifically, we examine the economic effects of a debt subsidy, which is often seen as a culprit of recent events in Thailand and Korea. We show that while the equilibrium level of capital and output remain unchanged, a subsidy to debt increases the debt-capital ratio permanently, exclusively through an increase in the level of debt. The higher debt and higher cost of borrowing raise the costs of debt service so that long-run consumption per capita must decline, since output remains unchanged. The transitional adjustment followed by capital and debt is especially interesting, since it involves a loop consisting of three distinct phases. Initially, a reduction in borrowing costs raises the incentives to accumulate both debt and capital. During the intermediate phase, the increased debt raises debt service costs, leaving less output for investment, so that capital accumulation slows and eventually declines. Finally, during the third stage the reduction in capital and the higher associated debt costs, eventually more than offset the benefits of the initial subsidy, causing debt to decline, along with capital, toward the new long-run equilibrium.

We also examine an increase in a distortionary income tax and find that in contrast to a debt subsidy it leads to proportionate long-run declines in the stocks of capital and debt. While a variety of transitional time paths are possible, monotonic adjustment in both variables is the most plausible.

[5]For example, Thailand's recent financial liberalization involved increased subsidies on foreign borrowing, leading to (perceived) excessive capital inflows that eventually led to a balance of payments crisis with associated net capital outflows; see Guitan (1998).

2. Small Open Economy

We consider a small open economy that consumes and produces a single traded commodity. Each individual is endowed with a fixed quantity of labor, L_i. Labor is fully employed so that total labor supply, equal to population, N, grows exponentially at the steady rate $\dot{N} = nN$. Individual domestic output, Y_i, of the traded commodity is determined by the individual's private capital stock, K_i, his labor supply, L_i, and the aggregate capital stock $K = NK_i$. Assuming a Cobb-Douglas production function, individual output is determined by:

$$Y_i = \alpha' L_i^{1-\sigma} K_i^{\sigma} K^{\eta} \equiv \alpha K_i^{\sigma} K^{\eta} \qquad 0 < \sigma < 1,\ \eta \gtrless 0 \tag{1a}$$

This formulation is akin to the earliest endogenous growth model of Romer (1986). The spillover received by an individual from the aggregate stock of capital can be motivated in various ways. One is to interpret K as knowledge capital, as Romer suggested. Another, is to assume N specific inputs (subscripted by i) with aggregate K representing an intra-industry spillover of knowledge.[6]

Each private factor of production has positive but diminishing marginal physical productivity. To assure the existence of a competitive equilibrium the production function must exhibit constant returns to scale in the two private factors [Romer (1986)]. In contrast to the standard neoclassical growth model, we do not insist that the production function exhibits constant returns to scale, and indeed total returns to scale are $1+\eta$, and are increasing or decreasing, according to whether the spillover from aggregate capital is positive or negative.[7]

Aggregate consumption in the economy is denoted by C, so that the per capita consumption of the individual agent at time t is $C/N = C_i$, yielding the agent utility over an infinite time horizon represented by the intertemporal isoelastic utility function:

$$\Omega \equiv \int_o^{\infty} (1/\gamma) C_i^{\gamma} e^{-\rho t} dt; \quad -\infty < \gamma < 1 \tag{1b}$$

with $1/(1-\gamma)$ equal to the intertemporal elasticity of substitution.

[6] A negative exponent can be interpreted as reflecting congestion, along the lines of Barro and Sala-i-Martin (1992).
[7] When production functions exhibit non-constant returns to scale, the existence of a balanced growth equilibrium requires the production function to be Cobb-Douglas, as assumed in (1a); see Eicher and Turnovsky (1997).

Agents accumulate physical capital, with expenditure on a given change in the capital stock, I_i, involving adjustment (installation) costs that we incorporate in the quadratic (convex) function

$$\Phi(I_i, K_i) \equiv I_i + hI_i^2/2K_i \quad = I_i\left(1 + hI_i/2K_i\right)$$

This equation is an application of the familiar Hayashi (1982) cost of adjustment framework, where we assume that the adjustment costs are proportional to the *rate* of investment per unit of installed capital (rather than its level). The linear homogeneity of this function is necessary if a steady-state equilibrium having ongoing growth is to be sustained. For simplicity we assume that the capital stock does not depreciate, so that the net rate of capital accumulation is given by:

$$\dot{K}_i = I_i - nK_i \tag{1c}$$

In addition, the agents also accumulate foreign bonds, B_i, which pay a fixed rate of return, *r*, determined exogenously in the world bond market. We shall assume that income from current production is taxed at the rate τ_y, income from bonds is taxed at the rate τ_b, while in addition, consumption is taxed at the rate τ_c. We shall focus on the distortionary aspects of taxation and assume that revenues from all taxes are rebated to the agent as lump sum transfers T_i. Thus the individual agent's instantaneous budget constraint is described by:

$$\dot{B}_i = (1-\tau_y)Y_i + \left[r(1-\tau_b) - n\right]B_i - (1+\tau_c)C_i - I_i\left[1 + (h/2)(I_i/K_i)\right] + T_i \tag{1d}$$

Aside from the fact that we have allowed for non-constant returns to scale and population growth, the above model is a standard endogenous growth model of a small open economy. But it is precisely these two factors that render the model within the class of so-called non-scale growth models. We will show below, that the model is consistent with long-run stable growth, provided returns to scale are appropriately constrained. This contrasts with earlier models of endogenous growth and externalities in which exogenous population growth could be shown to lead to explosive growth rates; see Romer (1990). We should also point out that the standard AK model emerges as the special case where $\sigma + \eta = 1$, $n = 0$, and the neoclassical if $\eta = 0$.

The agent's decisions are to choose his rates of consumption, C_i, investment, I_i, and asset accumulation, B_i, K_i, to maximize the intertemporal utility function, (1a), subject to the accumulation equations, (1c) and (1d). The optimality conditions with respect to C_i and I_i are respectively

$$C_i^{\gamma-1} = \lambda(1+\tau_c) \tag{2a}$$

$$1 + h\left(I_i/K_i\right) = q \tag{2b}$$

where λ is the shadow value (marginal utility) of wealth in the form of internationally traded bonds and q is the value of capital in terms of the (unitary) price of foreign bonds.[8] Equation (2a) equates the marginal utility of consumption to the tax-adjusted shadow value of wealth, while the latter equates the marginal cost of an additional unit of investment, which is inclusive of the marginal installation cost hI_i/K_i, to the market value of capital. Equation (2b) may be solved to yield the following expression for the rate of capital accumulation:

$$\dot{K}_i/K_i = I_i/K_i - n = (q-1)/h - n \equiv \varphi_i \tag{3}$$

With all agents being identical, equation (3) implies that the growth rate of the aggregate capital stock, is $\varphi = \varphi_i + n$, so that

$$\dot{K}/K = I/K - n = (q-1)/h \equiv \varphi \tag{3'}$$

Optimizing with respect to B_i and K_i implies the arbitrage relationships

$$\rho - \left(\dot{\lambda}/\lambda\right) = r(1-\tau_b) \tag{4a}$$

$$\left((1-\tau_y)\sigma Y_i/qK_i\right) + \left(\dot{q}/q\right) + (q-1)^2/2hq = r(1-\tau_b) \tag{4b}$$

[8]Since the shadow values λ, q pertain to individuals they should have subscripts i appended to them. But as agents are identical, their respective shadow values are the same and for notational simplicity the subscripts are suppressed.

Equation (4a) is the standard Keynes-Ramsey consumption rule, equating the marginal return on consumption to the after-tax rate of return on holding a foreign bond.[9] With ρ, r, τ_b all being constants, it implies a constant growth rate of the marginal utility λ. Likewise (4b) equates the after-tax rate of return on domestic capital to the after-tax rate of return on the traded bond. The former comprises three components. The first is the marginal after-tax output per unit of installed capital, (valued at the price q), while the second is the rate of capital gain. The third element equals $(qI_i - \Phi)/qK_i$ and reflects the fact that an additional benefit of a higher capital stock is to reduce the installation costs (which depend upon I_i/K_i) associated with new investment.

Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality conditions must be imposed:[10]

$$\lim_{t\to\infty} \lambda B_i e^{-\rho t} = 0; \quad \lim_{t\to\infty} \lambda q K_i e^{-\rho t} = 0 \tag{4c}$$

3. Aggregate Dynamics

Our objective is to analyze the dynamics of the aggregate economy about a stationary growth path. Along such an equilibrium path, aggregate output and the aggregate capital stock are assumed to grow at the same constant rate, so that the aggregate output-capital ratio remains unchanged. Summing the individual production functions (1a) over the N agents, the aggregate production function is:

$$Y = \alpha K^{\eta+\sigma} N^{1-\sigma} \equiv \alpha K^{\sigma_K} N^{\sigma_N} \tag{5}$$

where: $\sigma_N \equiv 1-\sigma$ = share of labor in aggregate output, $\sigma_K \equiv \eta + \sigma$ = share of capital in aggregate output. Thus $\sigma_K + \sigma_N = 1 + \eta$ measures total returns to scale of the social aggregate production function. Taking percentage changes of (6) and imposing the long-run condition of a constant Y/K ratio, the long-run equilibrium growth of capital and output, g, is given by

[9] Our notation suggests that $B_i > 0$, so that the agent is a net lender abroad, being taxed on his foreign income earnings. However, we do not rule out the possibility that $B_i < 0$, in which case he is a net borrower and τ_b is a subsidy. The case of a debtor economy is addressed in Section 4.

[10] The transversality condition on debt is equivalent to the national intertemporal budget constraint.

$$g \equiv \left(\sigma_N/(1-\sigma_K)\right)n > 0 \tag{6}$$

We shall show below that as long as the dynamics of the system are stable, $\sigma_K < 1$, in which case the long-run equilibrium growth is $g > 0$, as indicated. Under constant returns to scale, $g = n$, the rate of population growth. Otherwise *g* exceeds *n* or is less than *n*, that is there is positive or negative per capita growth, according to whether returns to scale are increasing or decreasing, $\eta \gtrless 0$.

To analyze the transitional dynamics of the economy about the long-run stationary growth path, it is convenient to express the system in terms of the relative price of installed capital, *q*, and the following stationary variables:

$$c \equiv C/N^{(\sigma_N/(1-\sigma_K))};\quad k \equiv K/N^{(\sigma_N/(1-\sigma_K))};\quad b \equiv B/N^{(\sigma_N/(1-\sigma_K))} \tag{7}$$

Under standard conditions of constant social returns to scale $[\sigma_N + \sigma_K = 1]$ and the quantities in (7) reduce to standard per capita quantities; i.e. $c = C/N = C_i$, etc. Otherwise they represent "scale-adjusted" per capita quantities.

3.1 Consumption Dynamics

To determine the growth rate of consumption we take the time derivative of (2a) and combine with (4a) to find that the individual's consumption grows at the constant rate:

$$\dot{C}_i/C_i = \left((r(1-\tau_b)-\rho-n)/(1-\gamma)\right) \equiv \psi_i \tag{8}$$

With all individuals being identical, the growth rate of aggregate consumption is $\psi = \psi_i + n$, so that

$$\dot{C}/C = \left((r(1-\tau_b)-\rho-\gamma n)/(1-\gamma)\right) \equiv \psi \tag{8'}$$

Differentiating *c* in (7) and using (8'), the growth rate of the scale-adjusted per capita consumption is:

$$\dot{c}/c = \left((r(1-\tau_b)-\rho-\gamma n)/(1-\gamma)\right) - \left(\sigma_N/(1-\sigma_K)\right)n \equiv \psi - g \tag{9a}$$

Equations (8), (8') and (9a) all share the property that with a perfect world capital market, the corresponding consumption growth rates are constant and independent of the production characteristics of the domestic economy. In addition, these equilibrium growth rates vary inversely with the tax on foreign bond income, but are independent of all other tax rates. These aspects of the dynamics of consumption remain unchanged from the basic AK model discussed by Turnovsky (1996).[11]

3.2 Capital and the Price of Capital

The dynamics of capital accumulation are, however, distinctly different from those of the standard open economy AK model, where like consumption, capital always lies on its balanced growth path. In the present model we find that the scale-adjusted capital-labor ratio, k, and the relative price of capital, q, converge to a long-run steady growth path along a transitional locus. To derive this path we differentiate k in (7) with respect to time and combine with (3'), to obtain:

$$\dot{k}/k = \left[\left((q-1)/h\right) - \left(\sigma_N/(1-\sigma_K)\right)n\right] = \varphi - g \tag{9b}$$

To derive the law of motion for the relative price of the capital good, we substitute the production function, (1a), the aggregation condition, $K = NK_i$, and (7) into the arbitrage condition (4b). The latter can then be expressed as

$$\dot{q} = r(1-\tau_b)q - (q-1)^2/2h - (1-\tau_y)\alpha\sigma k^{\sigma_K - 1} \tag{9c}$$

Thus (9b) and (9c) comprise a pair of equations in q and k, that evolve independently of consumption.

In order for the domestic capital stock ultimately to follow a path of steady growth (or decline), the stationary solution to (9b), (9c), attained when $\dot{q} = \dot{k} = 0$, must have (at least) one *real* solution. Setting $\dot{q} = \dot{k} = 0$ we see that the steady-state values of q and k, $\tilde{q}$ and $\tilde{k}$, are determined recursively as follows. First, the steady-state price of installed capital is:

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[11] It is important to note that this dependence of the consumption path on exogenous factors would not vanish if we introduced another sector to the non-scale model, as in Jones (1995a) or Eicher and Turnovsky (1997). The exogeneity of the consumption path is a function of the constant return to foreign bonds and is unrelated to the accumulation of domestic variables. In fact, the rate of return to any domestic factors must adjust to match the foreign return.

$$\tilde{q} = 1 + h\big(\sigma_N/(1-\sigma_K)\big)n = 1 + hg \tag{10a}$$

Having determined $\tilde{q}$ from this equation, the equilibrium scale-adjusted capital-labor ratio, $\tilde{k}$, is then determined from the steady-state arbitrage condition:

$$(1-\tau_y)\alpha\sigma\tilde{k}^{\sigma_K-1} + (\tilde{q}-1)^2/2h = r(1-\tau_b)\tilde{q} \tag{10b}$$

In order to be viable, the long-run equilibrium must satisfy the transversality conditions. Substituting (4a), (3') into (4c) and evaluating this requires that:

$$r(1-\tau_b) > g > 0, \tag{11}$$

That is, the after-tax interest rate on foreign bonds must exceed the growth rate of domestic aggregate output. Observe that the condition (11) ensures that (10a) and (10b) imply a unique steady state equilibrium having: (i) a positive equilibrium growth rate of capital (output), $\tilde{\varphi} = g$, and, (ii) a positive scale-adjusted capital-labor ratio, $\tilde{k}$.[12]

The equilibrium of the production side is thus fundamentally different in the non-scale economy from that of the simple AK technology. First, it is characterized by transitional dynamics, (9b), (9c), the nature of which will be discussed below. Second, in contrast to the AK technology where the existence of a balanced equilibrium growth rate depends upon the size of the adjustment costs, the transversality condition (11) *always* ensures the existence of a unique equilibrium growth rate of capital.[13] Moreover, as is evident from (6), the steady-state growth rate of aggregate capital (and output) shares the standard characteristic of non-scale growth models, namely that it is: (i) strictly positive and (ii) depends upon the returns to scale in the production function. Specifically, the growth rate is greater than or less than that of labor, according to whether there are increasing or decreasing returns to scale in aggregate production. Furthermore, it is independent of the taxes levied upon

[12] This may be shown as follows:

$$(1-\tau_y)\alpha\sigma\tilde{k}^{\sigma_K-1} = r(1-\tau_b)\tilde{q} - \left(\tilde{q}-1\right)^2/2h$$

Using (10a) and (11) the right hand of this equation exceeds $g(1+hg/2) > 0$, thus implying $\tilde{k}^{\sigma_K-1} > 0$ and hence $\tilde{k} > 0$.

[13] The AK model corresponds to $\sigma + \eta = \sigma_K = 1$ in (10b). This yields a quadratic equation in *q*, which may or may not have a real solution. In the case that it does, the smaller root yields the equilibrium growth rate of output $\varphi = (q-1)/h$.

interest or capital income. This implication contrasts with the AK model of the small open economy in which the growth of output increases with the former tax rate and decreases with the latter.

One further contrast is the response of the relative price of capital, q, to the cost of adjustment h. Setting $\sigma_K = 1$ in (10b), we see that in the AK model an increase in h lowers the return to capital arising from its favorable impact on installation costs; see (4b). For the return to capital to remain equal to the fixed return on foreign bonds, a q must *decline*. In the non-scale economy, with the equilibrium growth rate determined by production elasticities, an increase in h requires a *higher q*, in order for the growth rate of capital to equal the equilibrium growth rate of output; see (10a).

Linearizing (9b) and (9c) around (10a) and (10b), the local transitional dynamics of capital and its shadow price can be represented by the system:

$$\begin{pmatrix} \dot{k} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & \tilde{k}/h \\ -(1-\tau_y)\alpha\sigma(\sigma_K - 1)\tilde{k}^{\sigma_K - 1} & r(1-\tau_b) - g \end{pmatrix} \begin{pmatrix} k - \tilde{k} \\ q - \tilde{q} \end{pmatrix} \tag{12}$$

From (10a) we see that a necessary and sufficient condition for the equilibrium growth rate of output, g, to be positive is that $\sigma_K < 1$. This condition implies $\eta < \sigma_N = 1 - \sigma$, so that the share of external spillover generated by private capital accumulation, and hence the overall social increasing returns to scale, cannot exceed the exogenously growing factor's share (labor) in production. Assuming that this condition is met, the determinant of the matrix appearing in (12) is negative, implying that the dynamics are a saddlepoint. As usual, we assume that the capital stock accumulates slowly, so that k evolves gradually from its initial value, k_0, while the shadow value of capital may adjust instantaneously to new information.

Fig. 1 illustrates the phase diagram for the linearized dynamic subsystem (12). The value $\tilde{q} = 1 + hg$ corresponds to the $\dot{k} = 0$ locus. The value $q^* = 1 + hr(1-\tau_b)$ denotes the value of q at which the slope of the $\dot{q} = 0$ locus becomes vertical. Steady-state equilibrium is at the intersection of the $\dot{k} = 0$ and the $\dot{q} = 0$ loci, with XX being the negatively sloped saddlepath through that point.

3.3 Accumulation of Foreign Debt

As in the simple AK model, an important aspect of this equilibrium is that differential growth rates of consumption and domestic output can be sustained. This is a consequence of the economy being small in the world bond market and we now consider the implications for its net asset position.[14]

The domestic government is assumed to maintain a continuously balanced budget. Thus we assume that all tax revenues are rebated back to the private sector, in accordance with

$$T = NT_i = \tau_y \alpha K^{\sigma_K} N^{\sigma_N} + \tau_b rB + \tau_c C \qquad (13)$$

Aggregating the individual consumer's flow budget constraint, (1d), and substituting for (13) implies that the aggregate net rate of accumulation of traded bonds by the private sector, the nation's current account balance, is described by

$$\dot{B} = rB + \alpha K^{\sigma_K} N^{\sigma_N} - C - I\big(1 + (h/2)(I/K)\big) \qquad (14)$$

In the Appendix we show that provided the transversality conditions (4c) hold, the linearized solution to the scale-adjusted per capita stock of bonds, starting from the initial stock of bonds, b_0, is given by:

$$b(t) = -\big(M/(r-g)\big) - \big(L/(r-g-\mu)\big)e^{\mu t} + \big(b_0 + M/(r-g) + L/(r-g-\mu)\big)e^{(\psi-g)t} \qquad (15)$$

where M and L are constants defined in the Appendix.

As we can observe from (15), traded bonds are subject to transitional dynamics, in the sense that their growth rate $\dot{b}/b$ varies through time. There are two cases. First: if $\psi < g$, $b \rightarrow -M/(r-g)$ so that asymptotically bonds grow at the same rate as capital, g. If $\psi > g$, the scale adjusted stock of traded bonds grows at the rate $\psi - g$, with the aggregate stock of traded bonds growing at the rate ψ. Which case is relevant depends critically upon the size of the consumer rate of time preference relative to the rate of return on investment opportunities, as in the AK model (Turnovsky 1996).

4. Upward Sloping Supply Curve of Debt

[14] We shall assume that the country is sufficiently small so that it can maintain a growth rate which is unrelated to that in the rest of the world. Ultimately, this requirement imposes a constraint on the growth rate of the economy. If it grows faster than the rest of the world, at some point it will cease to be small. While we do not pursue the issue here, we should note that the issue of convergence in international growth rates is an important one.

The assumption that the economy is free to borrow or lend as much as it wants at a fixed interest rate in a perfect world capital market is strong. While it may be a reasonable assumption for developed economies having access to highly integrated capital markets, it is less realistic for developing economies, which because of risk considerations have restricted access to world financial markets. The key institutional factor that we wish to take into account is that world capital markets assess the risk associated with lending to specific economies in terms of their credit worthiness and their ability to service the associated debt costs. As an indicator of this, we use the aggregate debt-capital (equity) ratio of the economy and assume that the interest rate charged by the world capital markets increases with this ratio. This leads to the upward sloping supply schedule for debt, expressed by assuming that the borrowing rate, $r(Z/K)$, charged on foreign debt, Z, is of the form:

$$r(Z/K) = r^* + \omega(Z/K); \quad \omega' > 0 \tag{16}$$

where r^* is the exogenously given world interest rate and $\omega(Z/K)$ is the country-specific borrowing premium that increases with the nation's debt-capital ratio.[15] The homogeneity of the relationship is required to sustain an equilibrium with ongoing growth.[16]

In specifying (16) we are viewing the imperfection of the bond market from the standpoint of a borrowing nation. This seems more natural in the sense that it is the debtor nation that in reality is the source of risk. But recognizing that $Z = -B$, the stock of net credit, one can formulate the analysis symmetrically in terms of a downward sloping supply of credit to the world credit market. However, since most developing economies are debtor nations, we shall assume $Z \geq 0$. With this formulation an

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[15] A rigorous derivation of (1d) presumes the existence of risk. Formal justifications along these lines are provided by Eaton and Gersovitz (1981) and Kletzer (1984). Since we do not wish to develop a full stochastic model, we should view (17) as representing a convenient reduced form, one supported by empirical evidence; see Edwards (1984).

[16] Various formulations can be found in the literature. The original formulation by Bardhan (1967) expressed the borrowing premium in terms of the absolute stock of debt; see also Obstfeld (1982). Other authors, such as Cooper and Sachs (1985) have argued how a country, by adopting growth-oriented policies, can shift the upward-sloping supply function outward, so that at each level of debt a lower borrowing premium is charged. This effect can be incorporated by assuming that the borrowing premium depends upon the level of debt *relative* to some measure of earning capacity, and therefore debt-servicing capacity, such as capital, or output that depends upon capital. This latter formulation is the one we have adopted; see also van der Ploeg (1996).

increase in r^* describes an increase in the world interest rate, while an exogenous shift in the function ω represents a change in the country-specific borrowing rate.

As we will see the dynamics change dramatically when the economy faces an upward sloping supply curve of debt. The increasing cost of borrowing ties the consumption and production decisions, in contrast to the case of a perfect world capital market discussed in Section 3 which permits a decoupling of these two sets of decisions.[17]

The representative agent's decision remains as the maximization of the utility function (1b), subject to the capital accumulation equation (1c) and the flow budget constraint now expressed as:

$$\dot{Z}_i = (1+\tau_c)C_i + I_i\left[1+(h/2)(I_i/K_i)\right] - (1-\tau_y)Y_i + \left[(1-\tau_z)r(Z/K) - n\right]Z_i - T_i \qquad (17)$$

The constraint (17), expressed from the standpoint of a debtor, asserts that to the extent that the agent's consumption, outstanding interest payments plus investment expenses exceed his net revenue, he will increase his stock of debt. This is the mirror image of (1d), the only difference being that $\tau_z > 0$ now refers to a subsidy to interest owed on outstanding debt, rather than a tax on interest income earned.

It is important to emphasize that in performing his optimization, the representative agent takes the interest rate as given. The interest rate facing the debtor nation, as reflected in its upward sloping supply curve of debt is a function of the economy's *aggregate* debt, which the individual agent in making his decisions assumes he is unable to influence.

Performing the optimization, we find that the optimality conditions with respect to C_i and I_i remain given by (2a), (2b), with the latter implying (3), as before. The optimality conditions with respect to debt and capital are now modified to incorporate the endogenous interest rate as a function of the nation's debt-capital ratio, Z/K:

$$\rho - (\dot{\lambda}/\lambda) = (1-\tau_z)r(Z/K) \qquad (4a')$$

$$\left((1-\tau_y)\sigma Y_i/qK_i\right) + (\dot{q}/q) + (q-1)^2/2hq = (1-\tau_z)r(Z/K) \qquad (4b')$$

This changes the dynamics fundamentally.

[17] The same contrast occurs in the AK model with an upward sloping supply curve of debt; see Turnovsky (1997a).

As in Section 3.3, the government rebates all revenues in accordance with (13), or writing the equation in terms of aggregate debt,

$$T + r\tau_z Z = \tau_y \alpha K^{\sigma_K} N^{\sigma_N} + \tau_c C \qquad (13')$$

The only difference is that with τ_z being applied to debt, as long as $Z > 0$, it is a subsidy and an expense, rather than a source of revenue. Combining (17) with (13') implies that the economy's net rate of accumulation of debt, its current account deficit, is described by:

$$\dot{Z} = C + I\left[1 + (h/2)(I/K)\right] - \alpha K^{\sigma_K} N^{\sigma_N} + r(Z/K)Z \qquad (17')$$

4.1 Macrodynamic Equilibrium

Transforming the system in terms of the stationary "scale-adjusted" per capita variables defined in (7), together with the price of capital, *q*, the equilibrium dynamics are now expressed by:

$$\dot{c}/c = \left(r(z/k)(1-\tau_z) - \rho - \gamma n\right)/(1-\gamma) - \left(\sigma_N/(1-\sigma_K)\right)n \equiv \psi - g \qquad (9a')$$

$$\dot{k}/k = \left[((q-1)/h) - \left(\sigma_N/(1-\sigma_K)\right)n\right] = \varphi - g \qquad (9b)$$

$$\dot{q} = r(z/k)(1-\tau_z)q - (q-1)^2/2h - (1-\tau_y)\alpha\sigma k^{\sigma_K - 1} \qquad (9c')$$

$$\dot{z} = \left(r(z/k) - g\right)z - \alpha k^{\sigma_K} + c + \left((q^2 - 1)/2h\right)k \qquad (14')$$

Since the cost of borrowing depends upon the nation's debt-capital ratio, all four dynamic equations are linked in an indecomposable system, the properties of which are briefly discussed below. In particular, all variables, including consumption, are now subject to transitional dynamics.

The steady state growth path is obtained when $\dot{c} = \dot{k} = \dot{z} = \dot{q} = 0$, so that the corresponding steady state values of c, k, z, q, denoted by tildes, are determined by:

$$\left((1-\tau_z)r(\tilde{z}/\tilde{k}) - \rho - \gamma n\right)/(1-\gamma) = g \qquad (18a)$$

$$\tilde{q} = 1 + h\left(\sigma_N/(1-\sigma_K)\right)n = 1 + hg \qquad (18b)$$

$$(1-\tau_y)\alpha\sigma\tilde{k}^{\sigma_K-1}/\tilde{q}+(\tilde{q}-1)^2/2h\tilde{q}=(1-\tau_z)r\left(\tilde{z}/\tilde{k}\right) \qquad (18c)$$

$$\tilde{c}+\left((\tilde{q}^2-1)/2h\right)\tilde{k}-\alpha\tilde{k}^{\sigma_K}+\left(r(\tilde{z}/\tilde{k})-g\right)\tilde{z}=0 \qquad (18d)$$

This steady state has a simple recursive structure. As in the small economy facing a perfect world capital market, the steady-state price of installed capital is determined by (18b), so that the equilibrium growth rate equals *g*. Having only restricted access to the world financial market has no adverse impact on the country's long-run growth rate of output. But in contrast to the previous model, long-run domestic consumption now grows at the same rate as does domestic output. This is achieved through the adjustment in the country's debt to capital ratio, $\widetilde{(z/k)}$, such that the net cost of borrowing generates an equilibrium growth rate of consumption that equals the exogenously determined growth rate of output, *g*. Having determined both $\tilde{q}$ and $\widetilde{(z/k)}$, (18c) determines the scale adjusted capital-labor ratio, $\tilde{k}$, such that the after-tax rate of return on capital equals the after-tax cost of debt. Finally, given $\tilde{q}$, $\widetilde{(z/k)}$, and $\tilde{k}$, (18d) determines the equilibrium scale-adjusted per capita consumption, $\tilde{c}$.

The equilibrium growth rate must be consistent with the transversality condition. As before, this reduces to $\tilde{r}(1-\tau_z)>g$. Substituting from (18a), this can be expressed in terms of exogenous parameters as $\gamma<\rho(1-\sigma_K)n/(\sigma_N+\sigma_K-1)$ As long as this condition is met, we have a unique steady-state equilibrium.

We can now characterize and compare the steady state (18) to that of the exogenous interest rate case of Section 3. In contrast to the previous case, the endogenous interest rate ties both output and consumption growth to the exogenous production and population growth parameters embodied in *g*. A subsidy on debt, τ_z, which in the pure small open economy would raise the consumption growth rate permanently, will now have only a transitory effect. This is because it will encourage the economy to accumulate debt, raising the debt-capital ratio and the cost of borrowing, offsetting the effects of the subsidy, to the point where the net cost of borrowing is unchanged; see (18a). With $\tilde{q}$ being determined independently, and the long-run after tax cost of debt unchanged, (18c) implies that the equilibrium stock of capital, $\tilde{k}$, remains unchanged. Thus the increase in the debt-capital ratio is accomplished entirely by an increase in debt, $\tilde{z}$. The higher debt and the higher cost of borrowing raises the costs of debt service, so that with output remaining unchanged, long-run consumption per capita must decline.

The long run borrowing costs, determined by the debt-capital ratio, $\widetilde{(z/k)}$, are independent of the domestic income tax, τ_y. Thus an increase in τ_y must lead to a proportional adjustment in $\tilde{k}$ and $\tilde{z}$. With $\tilde{q}$ fixed, the arbitrage condition (18c) implies that an increase in τ_y must lead to a reduction in $\tilde{k}$ and therefore a proportional reduction in $\tilde{z}$. The resulting effect on per capita consumption, $\tilde{c}$, is ambiguous. While the reduction in capital lowers output, the reduced debt costs leaves more resources available for consumption. On balance we find that a higher income tax rate will reduce long-run consumption if and only if $C/Y > (1-\sigma_K)$.

3.2 Transitional Dynamics

The linearized dynamics to this system are expressed by the fourth order system:

$$\begin{pmatrix} \dot{k} \\ \dot{z} \\ \dot{q} \\ \dot{c} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \tilde{k}/h & 0 \\ a_{21} & \tilde{r} + r'\left(\tilde{z}/\tilde{k}\right) - g & \tilde{q}\tilde{k}/h & 1 \\ a_{31} & (1-\tau_z)r'\left(\tilde{q}/\tilde{k}\right) & (1-\tau_z)\tilde{r} - g & 0 \\ -(1-\tau_z)r'\left(\tilde{z}\tilde{c}/\tilde{k}^2\right)\Big/(1-\gamma) & (1-\tau_z)r'\left(\tilde{c}/\tilde{k}\right)\Big/(1-\gamma) & 0 & 0 \end{pmatrix} \begin{pmatrix} k-\tilde{k} \\ z-\tilde{z} \\ q-\tilde{q} \\ c-\tilde{c} \end{pmatrix} \tag{19}$$

where $a_{21} \equiv \alpha(1-\sigma_K)\tilde{k}^{\sigma_K - 1} - \frac{\tilde{z}}{h}\left[\tilde{r} + r'\frac{\tilde{z}}{h} - g\right] - \frac{\tilde{c}}{\tilde{k}}$; $\quad a_{31} \equiv -(1-\tau_z)r'\frac{\tilde{z}\tilde{q}}{\tilde{k}^2} + \left(1-\tau_y\right)\alpha\sigma(1-\sigma_K)\tilde{k}^{\sigma_K - 2}$.

It is straightforward to show that both the determinant and the trace of the matrix in (19) are positive, implying that there are either two or four eigenvalues with positive real parts. Various sufficient conditions can be established to ensure that there are in fact just two positive roots, in which case with capital and debt, k, z evolving gradually, and c, q allowed to jump instantaneously, the dynamics are represented by a unique stable saddlepath. Being second order, it allows for nonlinear adjustment paths for both capital and debt along which both variables may overshoot their respective long-run equilibria. The simplest condition to ensure a unique stable saddlepath is $C/Y > (1-\sigma_K)$.

Henceforth we assume that the stability properties are ensured so that we can denote the two stable roots by μ_1, μ_2, with $\mu_2 < \mu_1 < 0$. The stable solution is of the form:

$$k(t) - \tilde{k} = B_1 e^{\mu_1 t} + B_2 e^{\mu_2 t} \tag{20a}$$

$$z(t) - \tilde{z} = B_1 \nu_{21} e^{\mu_1 t} + B_2 \nu_{22} e^{\mu_2 t} \tag{20b}$$

$$q(t) - \tilde{q} = B_1 \nu_{31} e^{\mu_1 t} + B_2 \nu_{32} e^{\mu_2 t} \tag{20c}$$

$$c(t) - \tilde{c} = B_1 \nu_{41} e^{\mu_1 t} + B_2 \nu_{42} e^{\mu_2 t} \tag{20d}$$

where B_1, B_2 are arbitrary constants and the vector $\left(1 \quad \nu_{2i} \quad \nu_{3i} \quad \nu_{4i}\right)'$ $i = 1,2$ (where the prime denotes vector transpose) is the normalized eigenvector associated with the stable eigenvalue, μ_i. That is, $\left(1 \quad \nu_{2i} \quad \nu_{3i} \quad \nu_{4i}\right)'$ satisfies:

$$\begin{pmatrix} -\mu_i & 0 & \left(\tilde{k}/h\right) & 0 \\ a_{21} & \tilde{r} + r'\left(\tilde{z}/\tilde{k}\right) - g - \mu_i & \left(\tilde{q}\tilde{k}/h\right) & 1 \\ a_{31} & (1-\tau_z)r'\left(\tilde{q}/\tilde{k}\right) & (1-\tau_z)\tilde{r} - g - \mu_i & 0 \\ -(1-\tau_z)r'\left(\tilde{z}\tilde{c}/\tilde{k}^2\right)/(1-\gamma) & (1-\tau_z)r'\left(\tilde{c}/\tilde{k}\right)/(1-\gamma) & 0 & -\mu_i \end{pmatrix} \begin{pmatrix} 1 \\ \nu_{2i} \\ \nu_{3i} \\ \nu_{4i} \end{pmatrix} = 0 \tag{21}$$

The arbitrary constants, B_1, B_2, appearing in the solution (20) are obtained from initial conditions, specifically that the economy starts out with given initial stocks of capital and debt, k_0, z_0. Setting t = 0 in (20a), (20b) and letting $d\tilde{k} \equiv \tilde{k} - k_0$, $d\tilde{z} \equiv \tilde{z} - z_0$, B_1, B_2 are given by:

$$B_1 = \left(d\tilde{z} - \nu_{22} d\tilde{k}\right)/\left(\nu_{22} - \nu_{21}\right); \;\; B_2 = \left(\nu_{21} d\tilde{k} - d\tilde{z}\right)/\left(\nu_{22} - \nu_{21}\right) \tag{22}$$

The constants B_1, B_2 thus depend upon the specific shocks, and once determined, the complete solution for the equilibrium evolution follows from (20).

We shall focus our attention on the dynamics of capital, debt, and the relative price of capital. These depend critically upon ν_{2i}, ν_{3i} $i = 1, 2$. From (27) we obtain:

$$\nu_{3i} = \left(h/\tilde{k}\right)\mu_i < 0; \quad i = 1,2 \tag{23a}$$

$$\nu_{2i} = -\left(a_{31} + \left[(1-\tau_z)\tilde{r} - g - \mu_i\right]\left(h/\tilde{k}\right)\mu_i\right)\Big/\left(\tilde{r}'(1-\tau_z)\left(\tilde{q}/k\right)\right); \quad i = 1,2 \qquad \text{(23b)}$$

In general, ν_{2i} can be positive or negative. A weak condition for $\nu_{22} > \nu_{21} > 0$ is that the interest elasticity of the debt supply function exceed $1-\sigma_K$.[18] This condition establishes a relationship between the elasticities of the marginal productivities of foreign debt and capital. In general, the slope along the transitional path in *z*-*k* space is given by:

$$dz/dk = \left(B_1\nu_{21}\mu_1 e^{\mu_1 t} + B_2\nu_{22}\mu_2 e^{\mu_2 t}\right)\Big/\left(B_1\mu_1 e^{\mu_1 t} + B_2\mu_2 e^{\mu_2 t}\right) \qquad \text{(24)}$$

where B_1, B_2 are given by (22). Note that since $0 > \mu_1 > \mu_2$ as $t \to \infty$ this converges to the new steady state along the direction $\left(dz/dk\right)_{t\to\infty} = \nu_{21} > 0$.

Increase in τ_z: In our analysis of the steady state we have seen that, in contrast to the case where the interest rate on bonds is determined exogenously in the world market, an endogenously determined interest rate links output and consumption growth to the parameters embodied in *g*. A subsidy on debt, τ_z, which raised the consumption growth rate permanently in the pure open economy, now has only a transitory effect. While the long-run stock of national debt increases, the long-run stock of capital remains unchanged. This is illustrated in Fig 2 by a long-run move from P to S. The transitional dynamics are along the path PQRS. With $d\tilde{k} = 0$ for this shock, $B_2 = -B_1$ and the slope along this transitional locus in (30) as:

$$dz/dk = \left(\nu_{21}\mu_1 e^{\mu_1 t} - \nu_{22}\mu_2 e^{\mu_2 t}\right)\Big/\left(\mu_1 e^{\mu_1 t} - \mu_2 e^{\mu_2 t}\right) \qquad \text{(25)}$$

Evaluating this expression at $t = 0, t \to \infty$, we see that the locus both begins its transition and converges to the new steady state, in a positive direction, as drawn. Since the long-run stock of capital is unchanged this must imply a transitional loop.

The intuition for this adjustment path can be broken into three distinct phases. First, the immediate effect of a higher debt subsidy is to lower the net costs of borrowing. In order for asset

[18] To establish this, substitute (13c) into the expression for a_{31}. This condition suffices to ensure $a_{31} < 0$.

market equilibrium to prevail the rate of return on capital must decline and given the instantaneous stock of capital, its shadow value, q, must immediately rise. Thus the reduction in borrowing costs and the rise in the shadow value of capital generates an incentive to accumulate both additional debt and capital. This is represented by the movement in the positive direction PQ in Fig 2. The second phase starts as the increased debt raises debt service costs, leaving less output for investment so that capital accumulation slows and eventually declines. This part of the transition is represented by the movement in the negative direction QR. The reduction in capital and accumulating debt raises debt costs even further, eventually more than offsetting the benefits of the initial subsidy. This causes debt to decline along with capital, as represented by the movement along the final segment RS toward the new steady-state equilibrium at S. This adjustment may be summarized in the following proposition:

> **Proposition 1:** An increase in the subsidy to foreign debt, τ_z. leads to capital flow reversals (an adjustment loop) as both foreign and domestic capital overshoot their respective new steady-state levels during the transition.

Increase in τ_y: To characterize the transitional dynamics in this case, we recall from our analysis of the steady state that a higher income tax reduces the long-run stock of capital and debt proportionately. We have also shown that irrespective of the shock that both debt and capital will converge asymptotically to the new equilibrium in a positive direction, which in this case means that they will both be reduced together in the direction $(dz/dk)_{t\to\infty} = \nu_{21} > 0$. But earlier stages of the transition are ambiguous. It is straightforward to show that:

$$\left(\frac{dz}{dk}\right)_{t=0} = \frac{(\nu_{21}\mu_1 - \nu_{22}\mu_2)d\tilde{z} + \nu_{21}\nu_{22}(\mu_2 - \mu_1)d\tilde{k}}{(\mu_1 - \mu_2)d\tilde{z} + (\nu_{21}\mu_2 - \nu_{22}\mu_1)d\tilde{k}}; \ \frac{d\tilde{z}}{\tilde{z}} = \frac{d\tilde{k}}{\tilde{k}} < 0$$

The reduction in long-term debt, $d\tilde{z} < 0$, will tend to generate an immediate reduction in capital and debt, for precisely analogous reasons to those just discussed above in conjunction with the subsidy to debt. At the same time, the reduced after-tax return on capital income must be compensated either by an initial reduction in q, or an initial increase in $\dot{q}$ order to maintain portfolio balance equilibrium, given the initial unchanging cost of debt; see (9c'). It seems most plausible that the long-run reduction in the capital stock, $d\tilde{k} < 0$, will induce a decline in q, thus adding to the incentives to reduce the stock of capital in the short run. However, we are unable to rule out the possibility that the initial response consists of an increase in q accompanied by a large increase in $\dot{q}$, thus generating an initial increase in capital. Figure 4 illustrates a variety of possible time paths whereby the long-run reduction in capital and debt are accomplished. Of these, we view the monotonic paths PSQ and PTQ as most likely.[19] We summarize the transitional dynamics in this case with the proposition:

> **Proposition 2:** An increase in the income tax, τ_y. leads to a monotonic reduction in debt during the transition. Domestic capital may, however, increase during early stages of the transition, before converging to a lower steady-state level.

5. Conclusions

In this paper we have extended a general non-scale growth model to an open economy. Initially, the economy is assumed to face a perfect world capital market, and subsequently we have examined the economy in the presence of international capital market imperfections. With the introduction of a perfect world capital market, the non-scale structure of the closed economy is no longer fully retained. Consumption growth is determined by a combination of tastes and borrowing costs, as in AK models. But in contrast to the AK model, the dynamics of capital and its relative price are subject to transitional dynamics that can be conveniently represented in terms of what we call "scale-adjusted" per capita quantities. The long-run growth rate of domestic output and capital is determined by a combination of the exogenously given growth rate of labor, together with the production elasticities of capital and labor. Most importantly, it is unaffected by either the taxation of foreign interest or domestic income, though both will have transitory effects. The former will generate a short-run increase in the growth rate of output, leading to the accumulation of capital; the latter will have the opposite effect. In the long run both these are reflected in the adjustment of the factor mix chosen by the economy.

[19] This is confirmed with some numerical simulations that we have conducted.

The second part of the paper has provided a condition that yields a full fledged non-scale growth model for the open economy. By introducing international capital market imperfections we not only provide a mechanism for the dynamics of the consumption, output, and capital growth rates to be linked in a non-scale fashion. More importantly, perhaps, we provide an explanation for capital flows reversals that are consistent with one-time policy changes. Effectively the model implies that financial liberalization may generate capital inflow reversals during the transition from one stationary state to another.

Appendix

Utilizing the normalizations in (7) and substituting the expressions for aggregate investment and capital, (3'), into (14) enables this equation to be expressed in the scale-adjusted per capita form:

$$\dot{b} = (r-g)b + \alpha k^{\sigma_K} - c - (q^2 - 1)/2h\,k \qquad \text{(A.1)}$$

Starting from given initial stock, b_0, and using the stable solution to (12) the linearized solution to this equation is:

$$b(t) = \left(b_0 + \frac{M}{r-g} + \frac{L}{r-g-\mu} - \frac{c(0)}{r-\psi}\right)e^{(r-g)t} - \frac{M}{r-g} - \frac{L}{r-g-\mu}e^{\mu t} + \frac{c(0)}{r-\psi}e^{(\psi-g)t} \qquad \text{(A.2)}$$

where: $M \equiv \alpha\tilde{k}^{\sigma_K} - \left((\tilde{q}^2-1)/2h\right)\tilde{k}$; $L \equiv \left(k_0 - \tilde{k}\right)\left[\alpha\sigma_K\tilde{k}^{\sigma_K-1} - \left((\tilde{q}^2-1)/2h\right) - \mu\tilde{q}\right]$

In order to ensure national intertemporal solvency, the transversality condition $\lim_{t\to\infty}\lambda B e^{-\rho t} = \lim_{t\to\infty}\lambda(0)N_0 b e^{(g-r(1-\tau_b)t)} = 0$ must be satisfied and this will hold if and only if

$$r(1-\tau_b) - g > 0 \qquad \text{(A.3a)}$$

$$r(1-\tau_b) - \psi > 0 \qquad \text{(A.3b)}$$

$$c(0) = (r-\psi)\left(b_0 + M/(r-g) + L/(r-g-\mu)\right) \qquad \text{(A.3c)}$$

Condition (A.3a) is ensured by (12), while (A.3b) imposes an upper bound on the rate of growth of consumption. This latter condition reduces to $\gamma < \rho/[r(1-\tau_b) - n]$, imposing an upper limit on the intertemporal elasticity of substitution. This is certainly met in the case of a logarithmic utility function and given the empirical evidence indicating small elasticities of substitution $(\gamma < 0)$, will hold under less restrictive conditions as well. The third condition determines the feasible initial level of consumption and imposing this condition, (A.2) reduces to (15) of the text.

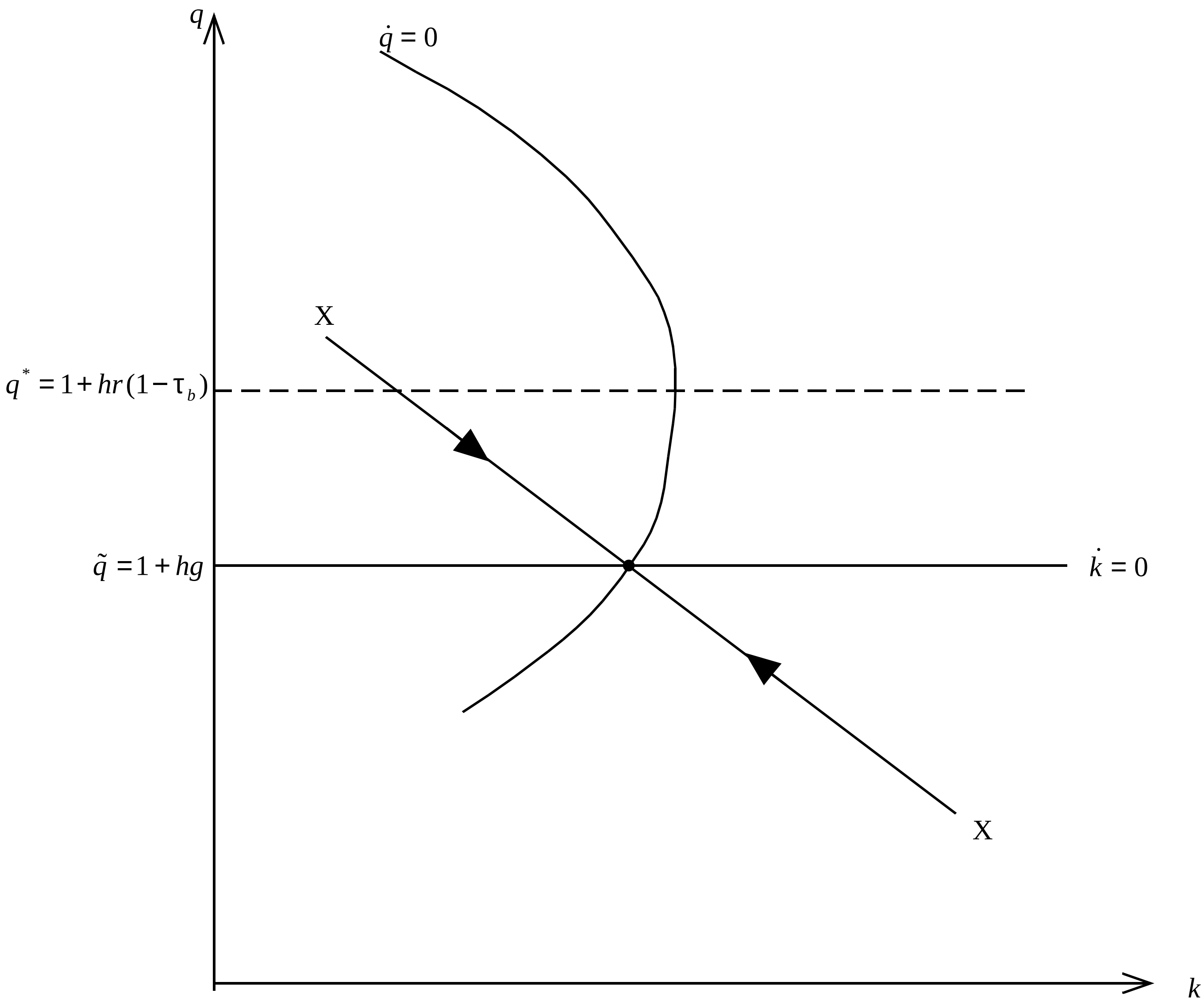

Figure 1. Phase Diagram

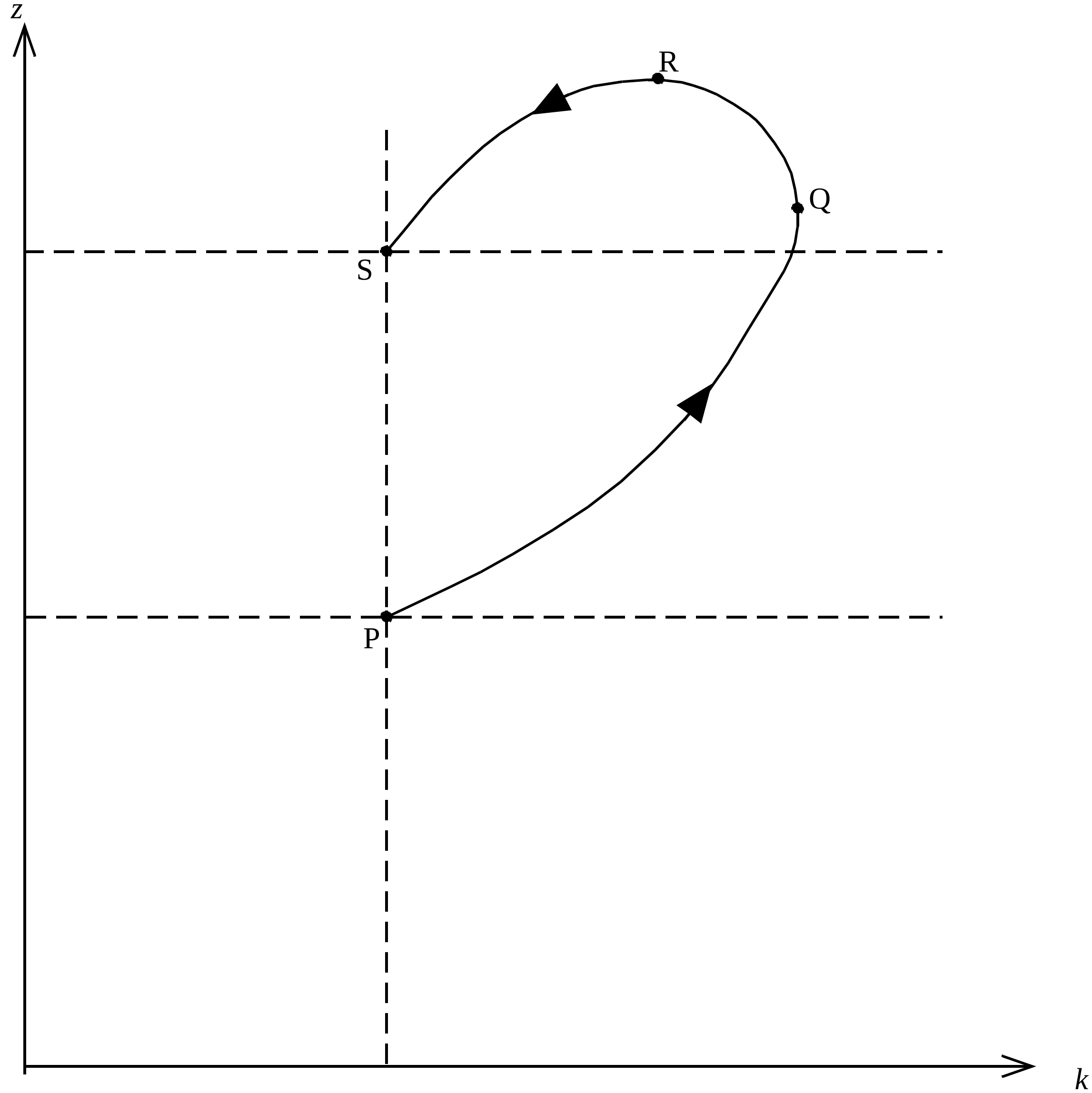

Figure 2. Transitional Dynamics: Increase in τ_z

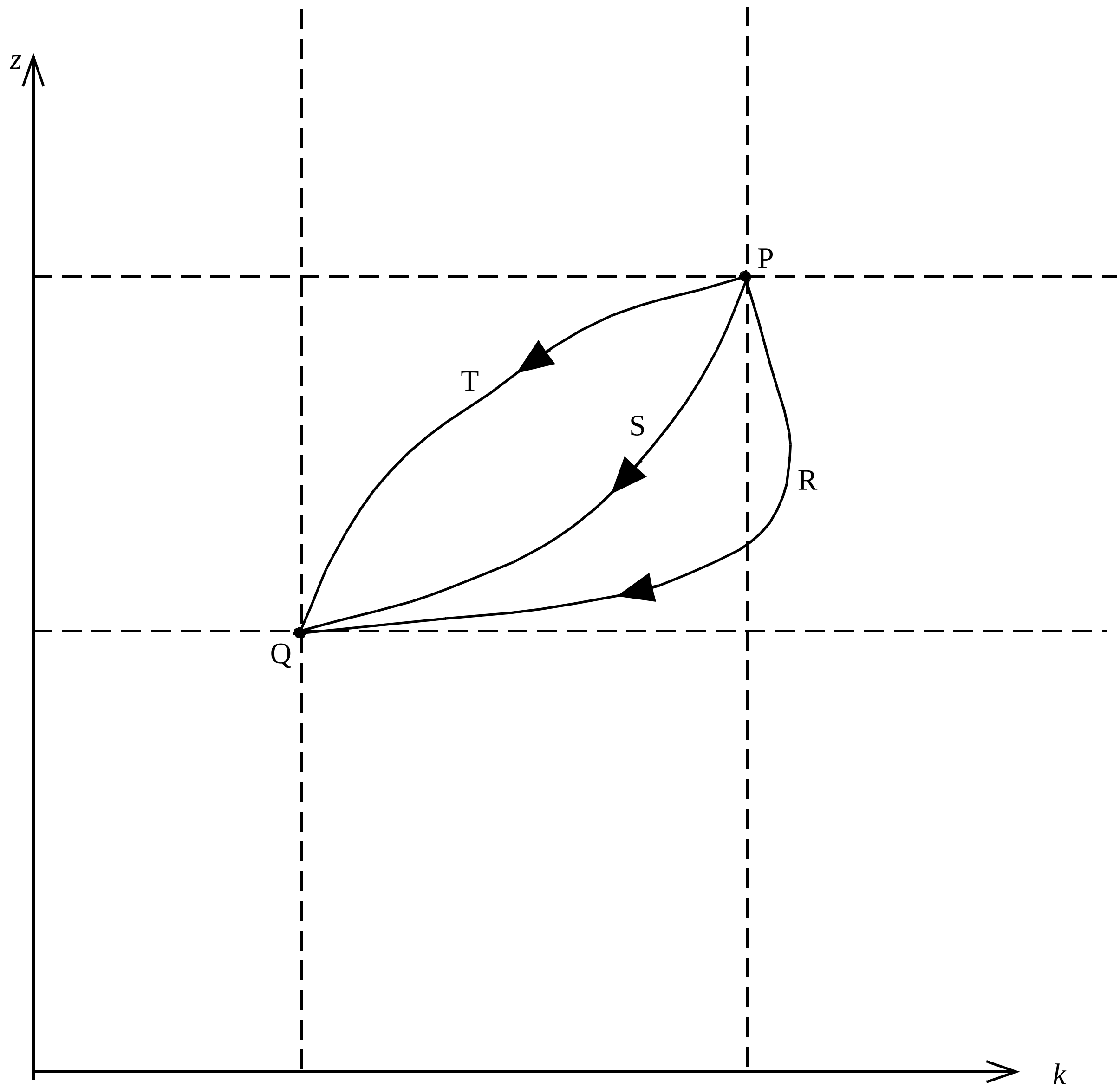

Figure 3. Transitional Dynamics: Increase in τ_y

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