Non-Scale Models of Economic Growth*

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Abstract

Growth models that incorporate non-rivalry and/or externalities imply that the size (scale) of an economy may influence its long-run growth rate. Such implied *scale effects* run counter to empirical evidence. This paper develops a general growth model to examine conditions under which balanced growth is void of scale effects. The model is general enough to replicate well known exogenous, as well as endogenous, (non-) scale models. We derive a series of propositions that show that these conditions for non-scale balanced growth can be grouped into three categories that pertain to (i) functional forms, (ii) the production structure, and (iii) returns to scale.

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1. Introduction

The recent resurgence of interest in the theory of economic growth can be attributed to its ambitious objective: to show how long-run growth can emerge as an equilibrium phenomenon that reflects the structural characteristics of the economy, such as the productivity of capital, the endogenous accumulation of knowledge, tastes, and economic policy.[1] This revival of attention to long-run growth issues is important, given the fundamental role that economic growth has historically played in determining the long-run welfare of societies. The emphasis on the endogenous factors as being key determinants of long-run growth rates contrasts sharply with the focus of the traditional neoclassical growth model, in which the long-run growth rate is determined by the (fixed) growth rate of population, possibly augmented by the exogenous rate of technological change.[2]

The new growth models can be categorized into two classes, according to the sources of growth. The term *R&D based-growth* refers to models in which the growth arises from technological innovation, as in Romer (1990). Models in which growth originates with private investment, either in physical or human capital, shall be referred to as *investment-based growth* models. The AK model, popularized by Barro (1990), is a one-sector version of the latter. Two key features of these new models have drawn their own set of criticisms, one based on their empirical implications, the other on the theoretical restrictions they impose on the underlying technologies. The primary motivation of this paper is to construct a model that still features endogenous growth, but addresses these shortcomings.

The empirical criticism pertains to the fact that both R&D and investment-based growth models that incorporate non-rivalry and/or externalities may exhibit *scale effects*, meaning that variations in the size or scale of the economy permanently alter the long-run equilibrium *growth rate*. For example, R&D-based growth models that follow Romer (1990) imply that a doubling of the population and resources devoted to R&D will increase the growth rate proportionately. Investment-based growth models in the tradition of Romer (1986) highlight how the character of the relationship

[1] See, for example, Romer (1986, 1990) and Barro (1990).
[2] See Solow (1956) and Swan (1956).

between scale and growth depends upon the existence and nature of production externalities. The basic investment-based model without any externalities is a non-scale model, as are models in which production externalities depend upon the average size of the economy-wide per capita capital stock.[3] In contrast, the Barro (1990) AK model introduces government expenditures as a pure non-rival public good that generates externalities that grow with the size of the economy; it therefore has a scale effect.

But existing empirical evidence does not support the presence of scale effects. For example, Jones (1995a) shows that variations in the level of research employment have exerted no influence on the long-run growth rates of the OECD economies, thus contradicting the predictions of the Romer (1990) model. In addition, the systematic empirical analysis of Backus, Kehoe, and Kehoe (1992) finds no conclusive evidence of a relation between per capita GDP growth and measures of scale.

In response to the empirical evidence, Jones (1995b), Segerstrom (1995), and Young (1998) have introduced models in which factor endowments and non-rival technology are still endogenous, but where long-run growth is not subject to scale effects. All three approaches focus on particular examples, with little suggestion of a comprehensive framework, or of the general properties that characterize growth without scale effects. In light of the empirical evidence, however, non-scale growth models constitute a new class of models whose general properties warrant careful examination and thorough understanding. This provides the motivation for the first goal of our paper: to establish the general characteristics of two-sector R&D and investment based growth without scale effects, that are consistent with the empirical evidence.

The second (theoretical) limitation of recent endogenous growth models is the requirement that to generate an equilibrium of ongoing growth, all production functions must exhibit constant returns to scale in the accumulated factors of production. This strong requirement imposes a strict knife edge restriction on the production structure and has been extensively criticized; see Solow

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[3]See e.g. Lucas (1988), Mulligan and Sala-i-Martin (1973), Bond, Wang, and Yip (1996) and Ladron-de-Guevara, Ortigueira and Santos (1997)

(1994).[4] For all other returns to scale, the balanced growth equilibrium is characterized by the absence of scale effects and for this reason non-scale growth equilibria should be viewed as being the norm, rather than the exception. Consequently, our second objective is to characterize clearly the nature of non-scale models and their relation to sectoral returns to scale.

Since the issue of non-scale equilibria pertains to the long run, we shall focus our attention on characterizing the equilibrium balanced growth path of the economy. This serves as a useful benchmark in that it provides insight into the nature of the equilibrium, while avoiding the analytical details involved in computing the entire transitional dynamic path.[5] Indeed, most of the recent two-sector endogenous growth literature focuses on balanced growth paths, and in this respect there are interesting parallels with the early work on multi-sector input-output models; see e.g. Solow and Samuelson (1953). An important characteristic of that early literature was that the equilibrium balanced growth rate was determined by production characteristics alone; demand conditions were irrelevant. Precisely the same holds true for non-scale growth models. We show that within a two-sector non-scale structure, the equilibrium growth rate is completely determined by the technologies of both sectors, *independent of demand*. This is in contrast to the AK model, for example, in which the equilibrium growth rate depends upon both demand determinants, e.g. the rate of time preference, as well as the technology; see e.g. Barro (1990).

Our results are summarized in a series of 5 propositions that characterize various aspects of the balanced growth equilibrium. Proposition 1 provides necessary and sufficient conditions for positive equilibrium growth. These conditions are analogous to those ensuring positive output in the early input-out models, thus reflecting the parallels noted above. Proposition 2 provides three alternative sets of conditions that suffice to ensure a balanced growth path. First, if both production functions have constant returns to scale in all endogenous and exogenous factors, then along the balanced growth path all variables grow at the exogenously given rate of population growth. This case includes the traditional neoclassical model. Second, if the production functions assume general

[4]Mulligan and Sala-i-Martin (1993) establish a slightly weaker condition for endogenous growth in the two-sector model.

[5]We investigate the transitional dynamics in a subsequent paper, Eicher and Turnovsky (1997).

Cobb-Douglas forms, without restrictions on the returns to scale, then the two sectors may grow at differential constant rates determined by production characteristics.[6] The third category is the novel case where the two production functions are separably homogeneous in the exogenously growing scale factor, labour, on the one hand, and the two endogenously accumulating factors, capital and knowledge, on the other. In that case the growth rate is determined by the ratio of the elasticity of the exogenous scale factor in either sector, relative to the deviation from constant returns to scale in that sector.

Balanced growth and sectoral returns to scale are intimately related, and various aspects of this relationship are highlighted in Propositions 3 and 4. First, we show how the relative magnitudes of the growth rates of the technology and output sectors depend upon the returns to scale of the endogenously growing factors in comparison to the returns to scale of the exogenously growing scale factor. Next, we find that increasing and decreasing returns to scale in the endogenously accumulating factors *in both sectors* may be consistent with positive balanced growth, though positive *per capita* growth requires increasing returns in at least one sector. In Jones' (1995b) non-scale model, the equilibrium growth rates in both sectors are determined by the production parameters in the knowledge sector alone. Proposition 5 provides a characterization of the conditions under which the equilibrium growth rates of both sectors are determined by the structural elasticities of only one sector.

Despite the technical nature of some of our propositions, we nevertheless regard the issues being addressed as being of general significance. We have already observed that issues of balanced growth in multisector models have a long history, dating back to Solow and Samuelson (1953). Furthermore, knife-edge stability issues, such as those associated with endogenous growth models, go back even further to Harrod (1939) and Domar (1946). The tradeoffs that we emphasize between returns to scale, the generality of the production functions, and the equilibrium growth rates have important implications for empirical analysis. Furthermore, the fact that the equilibrium growth rate is

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[6]We do not restrict the Cobb-Douglas production function to constant returns to scale. Indeed, the general Cobb-Douglas function is very convenient for parameterizing variable returns to scale.

determined by production characteristics alone (independent of demand) has consequences for macroeconomic policy. Finally, Section 4 shows how our model can replicate the essential features of exogenous, endogenous, and (non-) scale endogenous growth models, thereby providing a unifying framework for considering a wide variety of growth models.

2. A General Two-Sector Growth Model

We begin by outlining the structure of a general two-sector model of economic growth with exogenous population growth and with accumulating capital and technology. We follow previous non-scale models and parameterize its scale by the population size. The model is general in the sense that we restrict neither the parameters nor the forms of the production function. It is also general in that we can replicate the features of a wide variety of growth models that encompass both two-sector investment-based growth, as well as R&D-based growth.

This level of generality comes at the expense of having to abstract from issues related to specific microfoundations. We make these abstractions, not because we feel that such issues are unimportant, but to facilitate the identification of characteristics common to alternative approaches. All the models presented below can be given microfoundations, enabling the equilibrium we derive to be attained in a decentralized economy. However, investment-based and R&D-based models differ substantially in their respective decentralized structures, while the corresponding planner's problems can be analyzed within one unified framework. Decentralized versions of the investment-based model have been examined by Mulligan and Sala-i-Martin (1993) and Ladron-de-Guevara, Ortigueira and Santos (1997). These models differ profoundly from the decentralized R&D-based non-scale models of, for example, Jones (1995b), Segerstrom (1995), and Jones and Williams (1996). This is because the latter feature intermediate and final good sectors, and distinctly richer non-competitive market structures, which determine how the output of the R&D sector is allocated through the economy.

The objective of the planner is to maximize the intertemporal utility of the representative agent:

$$\frac{1}{1-g}\int_0^\infty c^{1-g}e^{-rt}dt \qquad r>0;\ \ g>0 \qquad (1a)$$

where c denotes per capita consumption and the utility function is of the constant elasticity form, so that $1/g>0$ is the intertemporal elasticity of substitution. The economy produces two goods, final output and technological change (new knowledge). The final good is produced using three productive factors: the social stocks of technology, labour, and capital, in accordance with the production function

$$Y = F[A, qN, fK] \qquad 0 \le q \le 1;\ 0 \le f \le 1 \qquad (1b)$$

where: Y denotes output of the final good, A denotes the stock of technology, K denotes the stock of physical capital, N denotes the population (labour force), which we assume grows at the steady rate $\dot{N}/N \equiv n$, all at time *t*. The fractions of labour and capital devoted to the production of the final good are q and f, respectively. The stocks refer to the amalgam of private stocks and those representing possible social spillovers, and may therefore be termed *social stocks* . Hence, the elasticities that will play a crucial role in our discussion below refer to the sum of private and social elasticities and the measures of returns to scale are characterized in terms of the social production function $F(.)$. Physical capital does not depreciate, and accumulates as a residual after aggregate consumption needs, *cN*, have been met:

$$\dot{K} = Y - cN \qquad (1c)$$

In contrast to physical capital, technology is not only a public good, but it is also produced in an alternative sector in accordance with the production function:

$$\dot{A} = J[A, (1-q)N, (1-f)K] \qquad (1d)$$

using the same three factors of production, namely the common stock of existing technology and the remaining fractions of labour and capital, $(1-q)$ and $(1\text{-}f)$, respectively. Equation (1d) encompasses the entire range of ways that the accumulation of knowledge has been specified in the previous literature. The earliest growth models, starting with Arrow (1962), basically assume $\dot{A} = J[.]A^{h_A}$ with $h_A < 1$, implying that growth of knowledge eventually ceases. By contrast, $h_A > 1$ implies exploding growth, and hence Romer (1990) introduced the knife edge case $h_A = 1$, which, however, exhibits scale effects. We start with the most general R&D function that allows for labour and capital to contribute to technological change, and derive the general conditions on the structure of both the output and R&D sectors to generate growth without scale effects.

The planner's problem is to maximize the intertemporal utility function, (1a), subject to the production and accumulation constraints, (1b) - (1d), and the usual initial conditions. His decision variables are: (i) the rate of per capita consumption; (ii) the fractions of labour and capital to devote to each activity; (iii) the rate of accumulation of physical capital and technology. The optimality conditions to this central planning problem can be summarized as follows:

$$c^{-g} = nN \tag{2a}$$

$$nF_N = mJ_N \tag{2b}$$

$$nF_K = mJ_K \tag{2c}$$

$$F_K f + \frac{m}{n} J_K (1-f) = r - \frac{\dot{n}}{n} \tag{2d}$$

$$\frac{n}{m} F_A + J_A = r - \frac{\dot{m}}{m} \tag{2e}$$

where n, m are the shadow values of aggregate physical capital and knowledge, respectively.[7]

Equation (2a) equates the marginal utility of consumption to that of physical capital. Equations (2b) and (2c) determine the sectoral allocations of labour and capital such that their respective shadow values are equated across sectors. Equation (2d) equates the marginal return to physical capital to the return on consumption, both measured in terms of units of final output. Analogously, (2e) equates the marginal return to knowledge to the return on consumption, both expressed in units of knowledge.[8]

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[7]To the extent that the social production function incorporate externalities that yield increasing returns to scale, issues of indeterminacy of equilibrium may arise; see Benhabib and Farmer (1994). These issues do not pose a particular problem in two-sector models with fixed labor supplies and we do not explicitly address them here; see also Mulligan and Sala-i-Martin (1993).

[8]In addition, the following transversality conditions must hold: $\lim_{t\to\infty} nKe^{-rt} = \lim_{t\to\infty} mAe^{-rt} = 0$.

3. Balanced Growth Equilibrium

We define a balanced growth path as being one along which all real quantities grow at constant, though not necessarily identical, rates. With capital being accumulated from new final output, we require the output/capital ratio, Y/K, along the balanced growth to remain constant. Together with the product market equilibrium, (1c), this implies that the consumption/output ratio C/Y must remain constant as well, so that $\hat{Y} = \hat{K} = \hat{C} \equiv \hat{c} + n$. where $\hat{}$ denotes the percentage growth rate; i.e. $\hat{x} \equiv \dot{x}/x$.

The balanced growth rates of these real quantities can be obtained by taking the differentials of the production functions (1b) and (1c). This leads to the following pair of linear equations in $\hat{A}, \hat{K}$

$$(1 - s_K)\hat{K} - s_A\hat{A} = s_N n \tag{3a}$$

$$-h_K\hat{K} + (1 - h_A)\hat{A} = h_N n \tag{3b}$$

where: $s_x \equiv F_x x/F$ and $h_x \equiv J_x x/J; x = A, N, K$ denote the structural elasticities in the production and knowledge sectors, respectively. For the present we assume that all three factors of production are necessary for the production of both goods, so that all elasticities are strictly positive. In general, these elasticities are functions of all variables in the production functions for technology and output, except in the Cobb-Douglas case when they are exogenously given constants.[9]

For $n > 0$, equations (3a) and (3b) jointly determine the rates of growth of physical capital, output, and consumption, on the one hand, and knowledge, on the other, as functions of the population growth rate and the various production elasticities in the two sectors:

$$\hat{A} = \frac{\{h_N(1 - s_K) + s_N h_K\}n}{\Delta} \equiv b_A n \tag{4a}$$

$$\hat{K} = \frac{\{s_N(1 - h_A) + h_N s_A\}n}{\Delta} \equiv b_K n \tag{4b}$$

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[9] For example, in the case of the CES production function these elasticities are $a_x(x/F[x])^{-J}$, where a_x represents the various distributional parameters and J is the substitution parameter.

where $\Delta \equiv (1-h_A)(1-s_K)-h_K s_A$. These expressions are discussed further in Section 3.1 below.[10]

Further restrictions along a balanced growth path can be obtained by considering the optimality conditions (2). Differentiating (2a), with respect to *t* yields:

$$\hat{v} = -g\hat{c} - n = -g\hat{K} - (1-g)n \tag{5a}$$

Thus, the constant growth rate of capital (output) obtained from (4b) determines a corresponding constant growth rate of the shadow value of capital. Moreover, combining the optimality conditions (2c) and (2d) we obtain

$$F_K[A, qN, fK] = r - \hat{v} \tag{5b}$$

The constancy of $\hat{n}$, together with that of r, thus implies that along the balanced growth path the marginal physical product of capital in the output sector, F_K, must be constant.

In addition, the balanced growth path is characterized by constant growth in the shadow value of knowledge, m, and constant sectoral allocation parameters q, f.[11] To establish this we need to flesh out the full dynamic evolution of the economy, by combining the equilibrium conditions (2) with the system dynamics in (1b) - (1d). This has been pursued in Eicher and Turnovsky (1997), where we focus explicitly on the transitional dynamics. The key steps in analyzing the dynamic evolution of the economy, relevant to its balanced growth characteristics are the following.

First, we must express the dynamics of the system in terms of stationary variables. Given the differential growth rates of the two sectors, we define the following variables having the requisite stationarity properties: $a \equiv A/N^{b_A}$; $k \equiv K/N^{b_K}$; $x \equiv C/N^{b_K}$; $q \equiv (nN^{b_K})/(mN^{b_A})$, [where b_A, b_K are the balanced growth rates defined in equations (4a), (4b)]. The short-run sectoral labour and capital allocation decisions, (2b) and (2c), can be solved for $q = q(k,a,q)$, $f = f(k,a,q)$. The

[10]At first sight (4a) and (4b) seem to imply that for n = 0, the growth rate of capital and technology are necessarily zero. This is not so, Equations (4a) and (4b) are derived under the assumptions of $s_N, h_N, n > 0$. Formulations that rely on $s_N, h_N, n = 0$ are identical to the system that has been thoroughly studied by Mulligan and Sala-i-Martin (1993). They show that for this system to generate positive growth rates requires the denominator in (4a) and (4b) be equal zero. This case is briefly discussed in Section 4.2 below.

[11]Being positive fractions, q, f must also lie between zero and one.

macrodynamic equilibrium can thus be represented by a fourth order system in the stationary variables (k, a, x, q).

Along the stationarity growth path, $\dot{q} = 0$, which combined with (4a) and (4b) implies

$$\hat{m} - \hat{v} = b_K n - b_A n = \hat{K} - \hat{A} \tag{5c}$$

so that the shadow value of knowledge grows at a constant rate, which deviates from that of the shadow value of capital so as to compensate for the differential growth rates of the quantities themselves. Thus we conclude that along the balanced growth path, the ratio of the value of the two capital stocks, nK/mA, remains constant. Finally, the stationarity of k, a, q, along the balanced growth path implies the required constancy of the sectoral allocations, q, f.

As discussed by Eicher and Turnovsky (1997), the dynamic non-scale system is always one dimension higher than is the corresponding endogenous growth model, exhibiting constant returns to scale in the accumulating factors, and this complicates its formal stability analysis. For example, while two-sector Lucas-type models reduce to third order dynamic systems, the present model leads to a fourth order system, as noted above. We are able to establish analytically that $\Delta > 0$ is a necessary condition for this system to be saddlepath stable. Beyond that, extensive simulations leave no doubt that the system is in fact saddlepath stable for all sets of plausible parameter values, thereby justifying our focus on the balanced growth path.[12]

3.1 General Characterization of Balanced Growth Equilibrium

We now return to the equilibrium growth rates, (4a) and (4b), two features of which are noteworthy. First, in deriving these expressions we have not introduced *any* restrictions on the production functions. Second, while the solutions are linear in the population growth rate, they also depend critically upon the elasticities s_x, h_x. Since these are functions of *all* factors that contribute to output and technology (except in one case, the Cobb-Douglas), it becomes obvious that population

[12]These simulations in some respect parallel those conducted by Mulligan and Sala-i-Martin (1993), for their third order system. Ladron-de-Guevara, Ortigueira and Santos (1997) provide a full analytical discussion of the global behaviour of Lucas (1988) type growth models.

growth is but one of the possible forces that drives economic growth in the generalized growth model.

First, we consider the conditions under which for $s_N > 0, h_N > 0, n > 0$ the equilibrium growth rates (4a) and (4b) are positive.[13] Necessary and sufficient conditions for this to be so are provided by the Hawkins-Simon conditions, namely $s_K < 1, \Delta > 0$; see Hawkins and Simon (1949). As we have noted, stability of the underlying dynamic system implies $\Delta > 0$, while these two conditions together also imply $h_A < 1$.

We may summarize these results with the following proposition:

Proposition 1: (*Necessary and Sufficient Conditions for Positive Growth*). $\Delta > 0$ and $s_K < 1$, (and $h_A < 1$) are necessary and sufficient for positive growth of output, capital, consumption, and technology in the two-sector non-scale growth model.

Corollary 1 to Proposition 1: (*Condition for Positive Per Capita Growth)*. If, further, $s_K > 1 - s_N$ the *per capita* growth rate of output and capital will be positive.

The sufficiency of the additional condition in the corollary follows immediately from equation (4b) The interesting point of the corollary is that in a growing economy, increasing returns to scale in the two private factors, capital and labour, in the production of final output, suffices to ensure *per capita* growth of output, independent of further production conditions in the R&D sector.

In Section 4 below we will examine certain special cases, including allowing some elasticities to be zero. One case of particular importance is the conventional two-sector investment based endogenous growth model, in which the production functions are constant returns to scale in the endogenously accumulating assets. In this case, the underlying dynamics are reduced to a third order system; see Mulligan and Sala-i-Martin (1993), Bond, Wang, and Yip (1996), and Ladron-de-

[13]It will be observed that the structure of (3) is formally identical to that of traditional input-output models; see e.g. Dorfman, Samuelson, and Solow (1958). The latter are concerned with determining when a system is able to sustain positive output levels, given inter-industry production needs and positive quantities of the primary factor, labour. Equations (3) determine similar conditions in terms of growth rates.

Guevara, Ortigueira, and Santos (1997). This case leads to the restriction $\Delta = 0$, and although this is no longer a necessary stability condition for this lower order system, it is nevertheless a crucial condition for determining the conditions for positive growth; see Mulligan and Sala-i-Martin (1993).

While Proposition 1 and its corollary establish conditions for positive growth, *balanced* growth requires *all* variables to grow at *constant*, although not necessarily equal, rates. Without further restrictions on the production functions, the elasticities s_x, h_x in (4a) - (4b) cannot be assumed to be constant. Hence, technology, output, and capital may not grow at constant rates unless we restrict either the structure of the model or the functional forms of the production function. To examine the general conditions for balanced growth, we note that the solutions (4a) and (4b) imply the following linear relationship among the equilibrium growth rates of output, capital, knowledge, and labour:

$$\hat{Y} = \hat{K} = b_K n = b_A l\, n = l \hat{A}, \text{ where } l \equiv b_K / b_A \tag{6}$$

It is evident from (4a) and (4b) that the relative sectoral growth rate, l, depends upon the production characteristics of the model. Along a balanced growth path it must be true that b_A and b_K are both positive constants, $\overline{b}_A$, $\overline{b}_K$.

There are three possible conditions under which (4a) and (4b) will be constants. The first is if production in both the output and R&D sectors exhibit constant returns to scale in all three factors; $s_A + s_K + s_N = 1$; $h_A + h_K + h_N = 1$. Substituting these conditions into (5a) and (5b), we find that with constant returns to scale in both sectors the non-scale growth rate can be expressed as:

$$\hat{Y} = \hat{K} = \hat{A} = n \tag{7}$$

implying $\overline{b}_A = \overline{b}_K = l = 1$. Interestingly enough, constant returns replicate a growth rate identical to that of the Solow model, which thus represents the simplest example of a non-scale growth model. But in contrast to the Solow model, technology here is also endogenous, just as in Romer (1990) type scale models. The model differs from previous scale models, however, in that we allow all factors of production and non-rival technology to grow over time, as in Jones (1995b) type non-scale models.

But in contrast to previous non-scale models, technology in the general model is produced with labour, technology, and *physical* capital. It is important to note the ease and generality with which one can introduce the absence of scale effects, without imposing any restrictions on the functional forms.

Once we leave the world of constant returns to scale in both sectors, stronger restrictions on the elasticities themselves must be imposed to ensure constant growth rates. The only functional form that renders b_A and b_K constant, independent of returns to scale, is the Cobb-Douglas in which all production elasticities are constant. In this case, output and knowledge may grow at differential rates that are determined by the production elasticities as reflected in l.

The third case that ensures a balanced growth equilibrium consists of production functions that are homogeneously separable in the exogenous and endogenously growing factors:

$$F = (qN)^{s_N} f[A, fK] \tag{8a}$$

$$J = ((1-q)N)^{h_N} j[A, (1-f)K] \tag{8b}$$

where (i) f and j are homogeneous of degree $s \equiv s_K + s_A < 1$ and $r \equiv h_A + h_K < 1$, respectively, in the two endogenous factors, private capital and knowledge[14], and (ii) the respective degrees of homogeneity, s and r, of the endogenously growing factors are related to the constant shares of the exogenously growing scale factors, s_N, h_N by

$$\frac{1-s}{1-r} = \frac{s_N}{h_N} \tag{8c}$$

Introducing these restrictions into (4a) and (4b), we see that the two sectors grow at the common rate

$$\hat{Y} = \hat{K} = \hat{A} = \left(\frac{s_N}{1-s}\right)n = \left(\frac{h_N}{1-r}\right)n \tag{9}$$

[14] The condition that the two production functions must have decreasing returns to scale in the endogenously accumulated factors, knowledge and capital, comes from combining $\Delta > 0$ with (8c). The former condition is equivalent to: $(1-k)(1-r) + k(1-s) > 0$. Given positive labour elasticities, s_N, h_N this is consistent with (8c) if and only if $r < 1, s < 1$ Since the functions f and j are general, the individual elasticities, s_A, s_K, h_A, h_N, although constrained by conditions (i) and (ii) are not constant, in contrast to s_N, h_N.

implying that $\overline{b}_A = \overline{b}_K = \left(s_N/(1-s)\right)$ and $l = 1$. This case is a hybrid of the first two, in that the endogenously accumulated factors are unrestricted with respect to functional form, but restricted with respect to returns to scale (as in case (i)), while the exogenously growing factor must assume a specific functional form, but is unconstrained with respect to returns to scale (as in case (ii)). Note that if $s_N + s = 1,\ h_N + r = 1$ so that both sectors exhibit constant returns to scale in all three factors, the balanced growth rate (9) reduces to (7). We summarize our present discussion with:

Proposition 2: (*Sufficient Conditions for Balanced Growth*). Balanced growth in non-scale growth models will occur if the production functions in both sectors are either:

(i) Subject to constant returns to scale, in which case both sectors grow at a common rate equal to the rate of population growth $\left(\hat{Y} = \hat{K} = \hat{A} = n\right)$.

(ii) Cobb-Douglas (with arbitrary degrees of returns to scale that satisfy Proposition 1). In this case, the two sectors may grow at differential rates, related by $\hat{Y} = \hat{K} = b_K n = b_A l n = l\hat{A}$, where $l \equiv b_K / b_A$, where l depends upon production characteristics.

(iii) Of the homogeneously separable forms (8a), (8b), in which case both sectors grow at a common rate equal to $\hat{Y} = \hat{K} = \hat{A} = \left(s_N/(1-s)\right)n = \left(h_N/(1-r)\right)n$.

This proposition stresses the importance of economies of scale in determining the characteristics of non-scale growth models. It emphasizes the tradeoff between the generality of the assumed functional form of the production function, on the one hand, and the restrictions to returns to scale, on the other. Some of the sufficient conditions noted in the proposition can be found in the literature so that one of the contributions of the proposition is that it enables us to view existing results from a more general perspective. Condition (i), which we have already noted includes the one-sector Solow model, is characteristic of any technology that exhibits constant returns to scale in all factors but diminishing returns to scale in the accumulated factors, such as the traditional two-sector Uzawa (1961) model of production. The fact that the unconstrained Cobb-Douglas production function permits differential long-run sectoral growth rates, as in Condition (ii), was also characteristic of Jones (1995b). This result contrasts with two-sector endogenous growth models, having constant returns to scale in the accumulating factors, when both sectors grow at the same (endogenous) rate along the balanced growth path; see, for example, Lucas (1988), Mulligan and Sala-i-Martin (1993).

Proposition 2 carries important implications for the empirical examination of returns to scale. If, because of externalities, one wants the returns to scale to be completely unconstrained, then the two production functions must be restricted to being Cobb-Douglas to assure a balanced growth path. Then the model is perfectly consistent with positive per capita growth in one, or both, sectors. If one is prepared to accept the restriction of decreasing returns to scale in the accumulating factors and of common long-run growth rates across sectors, then the less restrictive homogeneously separable functional form may be acceptable. This case, for example, is consistent with positive per capita growth if there are increasing returns to all three factors. However, the unrestricted homogeneous production function is consistent with balanced growth if and only if it has constant returns to scale.

Proposition 2 also has important consequences for policy effectiveness. Both the neoclassical and the standard non-scale models imply that balanced growth rates are invariant with respect to macroeconomic policy. Segerstrom (1995) and Young (1998) have shown that policy is still effective if it can influence the scale of the economy directly (e.g., by being able to influence the rate of population growth). But if we maintain the assumption of an exogenous scale, the only way for public policy to influence growth is through its impact on the production capabilities of the economy.

To see this, we follow Barro (1990) and modify the production functions F and J to incorporate productive public expenditure in the form:

$$Y = F[A, qN, fK, cG] \tag{1b'}$$

$$\dot{A} = J[A, (1-q)N, (1-f)K, (1-c)G] \tag{1d'}$$

where c represents the share of public expenditures allocated to the final goods sector. Public services are financed contemporaneously by allocating a constant share, t, of output, $G = tY$. The growth rate of capital and technology are readily derived and are simple variants of (4a) and (4b), with s_K, h_K being replaced by $s_K + s_G$, $h_K + h_G$, respectively. From Proposition 2, it follows immediately that, in order for public expenditure to influence the long-term growth rate in non-scale models, the production functions must be non-constant returns to scale, and public expenditures must be able to influence the elasticities of output, such that $s_x = s_x(t),\ h_x = h_x(t)$. Empirical evidence provides some support for this avenue of influence. Aschauer (1989) shows that the inclusion of public capital into production function regressions changes the structural elasticities of capital and labour. Devarajan, Swaroop, and Zou (1996) show that the shares of current and capital public expenditures are not constant, and changes in their shares alter the growth rate of the economy.

3.2 Returns to Scale and Balanced Growth

To analyze further the effect of economies of scale on the growth rates we shall allow maximum flexibility by imposing Cobb-Douglas functional forms in both sectors, and by assuming that the two production functions *F* and *J* are homogeneous of degrees *k* and *a* in the three factors *A*, *N*, and *K*, separately, so that $s_A + s_K + s_N = s + s_N \equiv k$ and $h_A + h_K + h_N = r + h_N \equiv a$. We can then express the solutions for the equilibrium growth rates in the form

$$\hat{A} = \frac{n\{(a - h_A)(1 - s_K) - h_K(1 - k + s_A)\}}{\Delta} \qquad \text{(10a)}$$

$$\hat{K} = \frac{n\{(1 - h_A)(k - s_K) - (1 - a + h_K)s_A\}}{\Delta} \qquad \text{(10b)}$$

where it will be recalled that $\Delta \equiv (1 - h_A)(1 - s_K) - h_K s_A > 0$. From (10a) and (10b), together with the definitions of returns to scale, we find a general condition that relates growth rates to elasticities and returns to scale to be:

$$\hat{A} \gtrless \hat{K} \text{ according as } \frac{1-s}{s_N} \gtrless \frac{1-r}{h_N} \equiv \frac{1-k}{s_N} \gtrless \frac{1-a}{h_N} \qquad \text{(11)}$$

To interpret (11) we should note that the quantity $(1-s)/s_N$ describes the ratio of the deviation from constant returns to scale of the *endogenously* growing factors, *K* and *A*, to the returns to scale of the *exogenously* growing factor, *N*, in the final output sector. This quantity may be either positive (if there are decreasing returns in *K* and *A*) or negative (increasing returns). The quantity $(1-r)/h_N$ has an analogous interpretation in the knowledge producing sector. Condition (11) asserts that *ceteris paribus*, output (capital) will grow faster than technology if this ratio of returns to scale in endogenous factors versus the exogenous scale factor is larger in the technology sector than it is in the output sector. This will be so, for example, if the economies of scale of the endogenous factors in the output sector are sufficiently greater than they are in the technology sector. An alternative, novel interpretation of (11) is that even if the two sectors have identical returns to scale, they will in general grow at different rates. Specifically, if the production functions have diminishing (increasing) returns to scale in all factors, the sector in which labour is relatively more important will grow at the faster (slower) rate. On the other hand, output will grow faster than knowledge, irrespective of the labour elasticities if it is subject to increasing returns to scale in all three factors $(k>1)$, while knowledge is subject to corresponding decreasing returns to scale $(a<1)$. The converse applies if these returns to scale are reversed.

We summarize the general effect of relative returns to scale on relative growth rates in:

Proposition 3: (*Returns to Scale and Relative Sectoral Growth Rates*). Define y_i to be the deviation of the *endogenous* factors from constant returns to scale, relative to the returns to scale of the *exogenous* scale factor, in sector *i*, $i = Y, A$. If $y_Y > y_A$ then the growth rate of technology exceeds the growth rate of output and capital, and vice versa.

Equations (10a) and (10b) impose restrictions on the sectoral returns to scale in the two sectors for positive balanced growth to prevail. In general, given $\Delta > 0$, positive growth rates of technology, capital, and output require that *k* and *a* satisfy:

$$a(1-s_K)+kh_K > h_A(1-s_K)+h_K(1+s_A). \tag{12a}$$

$$as_A+k(1-h_A) > s_K(1-h_A)+s_A(1+h_K) \tag{12b}$$

Both conditions are automatically ensured by the condition $\Delta > 0$ if there are constant returns to scale in both sectors, a = k = 1; see Proposition 2. But constant returns to scale in one sector do *not* require constant returns in the other. Somewhat surprisingly, decreasing returns to scale in both sectors may be consistent with positive balanced growth. Manipulating equations (12a) and (12b), one can show that both sectors will experience positive equilibrium growth if returns to scale in the two sectors lie in the decreasing returns to scale region

$$1 > a > 1-\frac{\Delta}{1-s_K};\;\; 1 > k > 1-\frac{\Delta}{1-h_A} \tag{13}$$

where it is important to note that the condition for well-behaved saddlepath behaviour, $\Delta > 0$, imposes additional restrictions on the degree of increasing returns to scale. In addition, Equation (13) shows that if returns to scale decrease too strongly a balanced growth path will not exist.

Mulligan and Sala-i-Martin (1993) perform a similar analysis under more restrictive conditions, which we shall discuss in Section 4.2, below. However, their focus is on specific functional forms and *per capita* growth rates. For comparative purposes it is important to obtain corresponding results here for the general model. From (10a) and (10b) we easily see that per capita growth in both sectors obtains, i.e. $\hat{Y}-n=\hat{K}-n>0$, and $\hat{A}-n>0$, if and only if returns to scale in the two sectors satisfy:

$$(1-h_A)(k-1)+s_A(a-1)>0 \tag{14a}$$

$$(1-s_K)(a-1)+h_K(k-1)>0 \tag{14b}$$

From (14a) and (14b) we see that a necessary condition for positive per capita growth in either sector is that there be increasing returns to scale (in all three factors) in at least one sector. Equations (14a) and (14b) also imply that as long as $s_K < 1, h_A < 1$ (as required by Proposition 1), increasing returns to scale in both sectors is consistent with positive per capita balanced growth in which the growth rates of output and technology exceed *n*., i.e. positive per capita growth in both sectors.[15] If there are decreasing returns to scale in one sector, they must be more than offset by increasing returns to scale in the other sector, while decreasing returns to scale in both sectors are inconsistent with positive per capita growth of output. These results may be summarized in the following proposition:

[15] These conditions can be shown as follows. From the restrictions (12a), (12b) to sustain positive growth in both sectors, we find that decreasing returns in the technology sector, $a < 1$, requires that *k* satisfy both $k > 1 - \Delta / h_K$ and $k > s_K + h_K s_A / (1 - h_A) = 1 - \Delta / (1 - h_A)$. Similarly, decreasing returns in production, $k < 1$, requires $a > h_A + h_K s_A / (1 - s_K) = 1 - \Delta / (1 - s_K)$. and $a > 1 - \Delta / s_A$. Given that both production functions have decreasing returns to scale in *A*, *K*, we know that $1 - s_K > s_A$, $1 - h_A > h_K$, implying that $\left(1 - \Delta / (1 - h_A)\right) > \left(1 - \Delta / h_K\right)$ and that $\left(1 - \Delta / (1 - s_K)\right) > \left(1 - \Delta / s_A\right)$. From these inequalities, we immediately obtain (13) of the text.

Proposition 4: (*Returns to Scale and Positive Balanced Growth*).

(i) Positive sustained balanced growth rates in both sectors are consistent with increasing, constant, or decreasing returns to scale in one, or both, sectors.

(ii) Positive *per capita* growth requires that there be increasing returns to scale in at least one sector. Increasing returns to scale in both sectors suffice to ensure positive per capita growth.

4. Benchmark Non-Scale Models of Endogenous Growth

The general model we have developed encompasses several well known special cases with, and without, scale effects. It is instructive to show how easily the general model replicates the traditional R&D and investment based endogenous growth models, and to show how their structures and growth rates fit into our framework of propositions. We begin with R&D based models, which are most closely related to the general model presented above. Romer's (1990) R&D model and its extension to a non-scale version by Jones (1995b) lay the foundation for our introduction of a hybrid non-scale growth model that shares features of both the investment based and the R&D based models. Finally, we turn to pure investment based models, and show how the Mulligan and Sala-i-Martin (1993) two sector model and the traditional one sector AK models are simply special cases of our general non-scale model.

4.1 Two-sector R&D-based Endogenous Growth Models with(out) Scale Effects

The initial specification of endogenous technological change by Romer (1990) assumes that final output is generated by a production function that exhibits constant returns to scale in knowledge-augmented labour (labour efficiency units), *AN*, and capital, physical *K*. The quantity of new output is specified as a linear function of the fraction of quality-adjusted labour employed in the technology sector. In terms of our general framework such a model can be represented by:

$$Y = (qAN)^{s} K^{1-s}; \text{ i.e. } s_N = s_A = 1 - s_K = s \qquad (15a)$$

$$\dot{A} = (1-q)AN;\ \text{ i.e. } h_N = h_A = 1;\ h_K = 0 \tag{15b}$$

This representation of the research sector necessitates that the population (or, in Romer the stock of skilled labour) is stationary, i.e. $n = 0$, otherwise no balanced growth path would exist and the growth rate would explode. The fact that the elasticity of technology in the R&D sector equals unity does not violate Proposition 2, since several structural elasticities are assumed to be zero. Equation (15b) immediately implies that $\hat{A} = (1-q)N$. Taking percentage changes of (15a) and noting that in the absence of population growth $\hat{K} = \hat{A}$, we obtain

$$\hat{c} = \hat{Y} = \hat{K} = \hat{A} = (1-q)N \tag{16}$$

This is the Romer result in which the equilibrium growth rate of the economy is tied to the share of population engaged in research, $(1-q)N$. This share increases proportionately with the size of the population, *N*, which pinpoints the source of the scale effect.

Jones' (1995a) model can be viewed as an extension of the Romer type R&D based models to allow for population growth in the absence of scale effects. As he pointed out, the assumption made by Romer, $h_A = 1$, is arbitrary, and he showed that if $h_A < 1$, the scale of the economy (i.e. the size of the population) will not influence the equilibrium growth rate.[16] Jones assumes a Cobb-Douglas world and retains the production function (15a) for final output, but modifies the production function to reflect the assumption $h_A < 1$. In terms of our notation, Jones specifies:

$$Y = (AqN)^s K^{1-s};\ \text{ i.e. } s_N = s_A = 1 - s_K = s \tag{15a}$$

$$\dot{A} = A^{h_A}((1-q)N)^{h_N};\quad h_K = 0 \tag{15b'}$$

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[16]The same point is made by Young (1998) in a much more intricate manner. Young essentially provides rich microfoundations to Jones' insights by introducing endogenous product variety into a quality ladder model. In addition to the profit destruction (due to competitors' quality improvement), firms' profits now also dissipate because of the increasing product varieties. Since Young assumes that only quality, but not variety improvements, generate spillovers, it is possible that scale (increase in size or population) creates such an increase in variety that all profits from quality improvements dissipate.

As in Romer, physical capital is not required for the production of knowledge. Substituting the elasticities in (15a) and (15b') into (4a) and (4b), we obtain the following expressions:

$$\hat{K} - n = \hat{A} = \frac{h_N}{(1-h_A)} n \tag{17}$$

which precisely replicates the equilibrium growth rate in the Jones model, where per capita consumption, per capita output and capital, and technology all grow at a common rate determined by: (i) the growth rate of labour, and (ii) the elasticities of labour and knowledge in the R&D sector.

Balanced growth in both Romer's and Jones' models is determined solely by the characteristics of the research sector. Both authors, as do others, make the simplifying assumption that physical capital does not enter in the production of technology. But long-run growth continues to be determined in this way in the more general model which (i) preserves constant returns to scale in the production of output, but (ii) introduces physical capital into the production of knowledge, i.e. $h_K > 0$. We can refer to this as a hybrid model in the sense that *both* capital and technology enter in *both* production functions. This modification leads to

$$Y = (AqN)^s K^{1-s}; \text{ i.e. } s_N = s_A = 1 - s_K = s \tag{15a}$$

$$\dot{A} = A^{h_A}((1-q)N)^{h_N}((1-f)K)^{h_K} \tag{15b''}$$

The condition $\Delta > 0$ now becomes $1 > h_A + h_K$ and the corresponding equilibrium growth rates for the two sectors are:

$$\hat{K} - n = \frac{n(h_N + h_K)}{1 - h_A - h_K} = \hat{A} \tag{18}$$

The economy thus has constant returns to scale in the two endogenous factors, technology and capital, in the final goods sector, and decreasing returns to scale in these two factors in the production of technology. As a consequence, the equilibrium growth rate in the output sector exceeds that of the technology sector (assuming $n > 0$), and both are determined by the production elasticities in the technology sector alone. The parallels between (18) and (17) are clear; we can also see that as long as capital is productive in the technology sector, the equilibrium growth rate in both sectors will be raised.

It is striking that despite the fact that physical capital accumulation is an intrinsic part of the Romer, Jones, and the hybrid models, the balanced growth rate is nevertheless determined by the production characteristics of the R&D sector alone; the elasticities of the final goods sector are irrelevant. But from the solutions (4a) and (4b) we know that, in general, the balanced growth rate depends upon the production characteristics of *both* sectors of the economy. The present result thus raises the issue of the robustness of the specification adopted in previous R&D based growth models. To highlight how special the previous results have been, it is instructive to examine the alternative conditions under which the characteristics of only the *final output* sector are the crucial determinants of the equilibrium growth rate in the overall economy.

The key to this question is provided by the general solutions (4a) and (4b). From these equations we find that everything is reversed, and growth rates determined by the elasticities characterizing the final output sector alone, if the properties of the production functions in (15a) and (15b") are reversed to:

$$Y = A^{s_A}(qN)^{s_N}\left(fK\right)^{s_K} \tag{19a}$$

$$\dot{A} = A^{1-h}\left((1-q)N(1-f)K\right)^{h}; \text{ i.e. } h_K = h_N = 1 - h_A = h \tag{19b}$$

Now the production function for final output is unrestricted, while knowledge is produced by a constant returns to scale production function in capital-augmented labour, *AK*, and knowledge. Imposing the restrictions in (19b), the condition $\Delta > 0$ now becomes $1 > s_A + s_K$ and the corresponding equilibrium growth rates for the two sectors are:

$$\hat{A} - n = \frac{n(s_N + s_A)}{1 - s_A - s_K} = \hat{K} \tag{20}$$

The economy now features constant returns to scale in the reproducible factors in the production of technology and decreasing returns to scale in these two factors in the production of final output. As a consequence, the equilibrium growth rate in the knowledge sector exceeds that of the final goods sector, though both are determined by the production elasticities in the final goods sector alone. Further, output per capita will grow if and only if $s_A + s_N + s_K > 1 - s_A$, that is if and only if the returns to scale in the output sector are sufficiently large.

We may summarize these results with the following proposition:

Proposition 5 (*Single-Sector Determinants of Growth Rates*).

Equilibrium growth rates in *both* sectors (*i* and *j*) are determined *exclusively* by the structural elasticities of *one* sector, *j* if and only if the production function of sector *i* exhibits constant returns to scale in the endogenous factor produced in sector *i*, and in labour-efficiency units, as augmented by the endogenous factor produced in sector *j*.

4.2 Two-Sector Investment-based Non-Scale Models

Finally, we turn our attention to non-scale investment-based growth models. For this purpose it is convenient to consider the Mulligan and Sala-i-Martin (1993) model, who examine transitional dynamics and necessary conditions for balanced growth in such a model. While their model permits externalities, these are assumed to depend upon average per capita stocks and hence do not introduce scale effects. Since R&D is nonexistent in investment-based models, technology is termed human capital in the Mulligan and Sala-i-Martin model and has no public good characteristic.

The social production functions in the Mulligan-Sala-i-Martin model are all expressed in per capita terms. In terms of our notation, they are of the form

$$Y = N(q A/N)^{s_A} (f K/N)^{s_K} \equiv (qA)^{s_A} (fK)^{s_K} N^{1-s_A-s_K}$$

$$\dot{A} = N((1-q) A/N)^{h_A} ((1-f) K/N)^{m_K} \equiv ((1-q) A)^{h_A} ((1-f) K)^{h_K} N^{1-h_A-h_K}$$

which are both constant returns to scale in all *three* factors of production, *A*,*K*, and *N*. Taking percentage changes, (3a) and (3b) now are of the form

$$(1-s_K)\left(\hat{K}-n\right)-s_A\left(\hat{A}-n\right)=0 \tag{3a'}$$

$$-h_K\left(\hat{K}-n\right)+(1-h_A)\left(\hat{A}-n\right)=0 \tag{3b'}$$

This is identical to our case (i) of Proposition 2. Provided $\Delta > 0$, the solution to (3a') and (3b') is $\hat{K}-n=\hat{A}-n=0$, so that per capita growth is zero.

Mulligan and Sala-i-Martin (1993) seek a solution in which there is endogenously determined per capita growth, (i.e. $\hat{K}-n>0, \hat{A}-n>0$) and for this it is necessary that $\Delta \equiv (1-s_K)(1-h_A) - s_A h_K = 0$. This implies that equations (3a') and (3b') are linearly dependent and therefore they do not jointly determine the equilibrium growth rates, as in (4a) and (4b). Instead, the equilibrium growth rates are now determined by one of these independent equations, together with a condition involving demand; see (21) below.

For their model, Mulligan and Sala-i-Martin show that the necessary condition for positive per capita equilibrium growth, $\Delta = 0$, imposes strong conditions on the relationships between the two sectors' returns to scale in the *endogenously accumulating factors*. Specifically they show that the necessary condition $\Delta = 0$ will be met if: (i) $s_K = 1$ and either $h_K = 0$ or $s_A = 0$. The latter is the Rebelo (1991) AK model, in which output does not depend upon knowledge. Or: (ii) $h_A = 1$ and either $s_A = 0$ or $h_K = 0$, the latter being the Lucas (1988) models. Or: (iii) both sectors are subject to constant returns to scale in *A* and *K*. Or finally: (iv) if there are decreasing returns to scale in one sector that are exactly offset by decreasing returns to scale in the other sector. Decreasing or increasing returns to the accumulating factors in both sectors are thus inconsistent with balanced growth.

These results are clearly more stringent than the comparable conditions for per capita growth for our model reported in (14). Allowing for more general returns to scale (in all factors) adds considerable flexibility. Thus, in contrast to Mulligan and Sala-i-Martin, decreasing returns to scale in the accumulating factors is consistent with positive per capita growth, as long as the elasticity of the exogenous scale factor is sufficiently large, so as to ensure increasing returns to scale in all three factors. Likewise, increasing returns to scale in both accumulating factors is consistent with per capita positive growth, again in contrast to Mulligan and Sala-i-Martin. Conditions (14) can be expressed in terms of returns to scale in the accumulating factors, *A*, *K*, as follows:

$$(1-h_A)(s-1)+s_A(r-1) > -s_N(1-h_A)-h_N s_A \tag{14a'}$$

$$h_K(s-1)+(1-s_K)(r-1) > -s_N h_K - h_N(1-s_K) \tag{14b'}$$

from which it is evident that decreasing returns to scale in the accumulating factors ($s<1, r<1$) may be consistent with these equations, provided the labour elasticities are sufficiently large.

To see how the tastes help determine equilibrium in such an endogenous growth model it is convenient to go to the generic one sector AK model, parameterized by setting $h_x = 0, x = A, N, K$ (no technology sector), $s_A = s_N = 0; s_K = 1$, $n = 0$. Note that in this case both (3a') and (3b') degenerate, providing no information about the equilibrium growth rate. To determine the equilibrium growth rate in this case, we must return to the optimality conditions (5a) and (5b). Combining these two equations (assuming $n = 0$) yields the equilibrium rate of growth of consumption

$$\hat{c} = \frac{1}{g}(F_K - r) \tag{21}$$

where the marginal physical product of capital is constant by assumption. Since this is an equilibrium in which the ratios of consumption to capital and output to capital are constant, these two quantities grow at the rate indicated in (21). This equation indicates that the equilibrium growth rate in the simple AK model is determined by a combination of taste and technology parameters. The equilibrium consumption-capital ratio adjusts so as to equate the growth rates of physical capital and consumption. In the two-sector investment-based model, the growth rates would be obtained by combining (21) with one of the linearly independent equations (3a'), (3b').

5. Conclusion

Recent endogenous growth models have been characterized by scale effects, in the sense that the long-run growth rate is responsive to the size of the economy. This implication runs counter to empirical evidence suggesting that scale effects are absent in OECD economies. The scale property is also a knife-edge one. Unless the underlying production functions are constant returns to scale in the factors being endogenously accumulated, the balanced growth equilibrium will be one of non-scale rather than scale effects. For these two reasons the comprehensive study of non-scale growth equilibria is important and has provided the motivation for this paper.

We find that non-scale balanced growth obtain under three conditions that involve tradeoffs between the generality of the production functions and restrictions on returns to scale. The first condition places no restrictions on the form of the production function, but requires constant returns to scale in all factors. In this case all variables grow at the exogenously given rate of population growth, as in the Solow model. The second arises if the production functions are of Cobb-Douglas form (with arbitrary returns to scale), when the two sectors may grow at differential constant rates determined by production characteristics. Third is the intermediate case, where the two production functions are separably homogeneous in the exogenously growing scale factor, on the one hand, and the two endogenously accumulating factors, on the other. In this case both sectors must grow at a common rate, though not necessarily equal to that of labour.

The fact that the balanced growth rates are determined essentially by production conditions has interesting implications for policy effectiveness. Retaining the traditional assumption of an exogenous growth rate of labour, it implies that policy will affect long-run growth rates only through its impact on the underlying production structure and specifically the production elasticities. Evidence presented by Aschauer (1989) and others suggests that this avenue for public policy may in fact be empirically relevant. But since this channel operates only indirectly it is likely to vary qualitatively across economies and stages of development. This might explain the weak empirical evidence on the responsiveness of growth rates to variations in tax policy, obtained for example, by Easterly and Rebelo (1993) among OECD economies and by Stokey and Rebelo (1995) using US data. This evidence had previously been interpreted as evidence against those AK models that assign a powerful and direct role to fiscal policy as a determinant of growth.

We conclude our discussion with some comments on transitional dynamics. As we noted at the outset, the balanced growth steady-state equilibrium serves as an important benchmark. But one must, of course, be careful in drawing conclusions about any real world economy from these abstractions. For example, while long-run balanced growth rates may prove to be essentially independent of fiscal policy, most economies may be extremely slow to adjust. This implies that the effects of policy changes, though only temporary, may in fact endure for long periods of time, thereby rendering the nature of the transitional adjustment to be extremely important. Indeed, our preliminary simulations in Eicher and Turnovsky (1997) strongly suggest such slow adjustments, with the asymptotic speed of convergence being found to be around 2% at annual rates.

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