CONVERGENCE IN A TWO-SECTOR NON-SCALE GROWTH MODEL*

Theo S. Eicher

Stephen J. Turnovsky

University of Washington

Revised Version
July 1999

*An earlier version of this paper was presented at the Third International Conference on Computing in Economics and Finance, held at Stanford University. Comments by Charles Jones, Jon Temple, an anonymous referee, and the Editor, Oded Galor on previous versions of this paper are greatly appreciated. Eicher thanks the von Humboldt Foundation for financial support and Nuffield College, Oxford, and the University Tübingen for their hospitality. Turnovsky's research was supported in part by the Castor endowment

Abstract

Much of the convergence debate has focused solely on output. Recent empirical evidence suggest that crucial inputs, such as technology and capital, may exhibit markedly distinct convergence patterns. We examine the convergence characteristics of a two-sector non-scale model of growth that features population growth and endogenous technology. The model replicates key economic ratios and speeds of convergence with relative ease. Most important, however, is that capital and technology differ strikingly in their convergence paths and speeds. The non-constancy of the convergence rates and the non-proportionality of the endogenous variables during transition suggests further refinements for the empirical tests of convergence.

Key Words: non-scale growth, convergence, capital, technology

JEL classification: O41

1. Introduction

Neoclassical and endogenous growth models offer strikingly different predictions with respect to both the determinants of long-run growth rates and speeds of convergence along transitional growth paths. The neoclassical model predicts that the per capita growth rates of output of parametrically distinct countries converge to their respective national rates of technological change, which may or may not be country-specific, but which remain unexplained. By contrast, endogenous growth models imply that long-run national growth rates are in general sensitive to national characteristics, such as tastes, technology, and tax structure. Controlling for technological change, Mankiw, Romer, and Weil (1992) and Barro and Sala-i-Martin (1995) present empirical evidence to suggest that countries do converge to identical growth rates, thus supporting the neoclassical growth model's explanation of long-run growth rates over that of the endogenous growth model.

However, the neoclassical approach to convergence overemphasizes capital accumulation at the expense of technological change. Bernard and Jones (1996a, 1996b) have shown that while growth rates of output among OECD countries converge, the growth rates of manufacturing technologies exhibit markedly different time profiles. These distinct growth and convergence patterns of technology and output also challenge the first generation of endogenous growth models. Because the dynamics in these models are typically described by a one-dimensional locus, the first generation of endogenous growth models constrain output and technology to accumulate in proportion, much like in the neoclassical model.[1]

In this paper we show how the empirical findings regarding the differential rates of convergence of output and technology can be reconciled by introducing a two-sector *non-scale* model of capital accumulation that incorporates endogenous technological change and population growth.[2] This model is in many respects a hybrid of neoclassical and endogenous growth models, as it can be shown that both the traditional neoclassical and endogenous growth models may emerge as special cases (see Eicher and Turnovsky, 1999).

Non-scale growth models, introduced initially by Jones (1995a), were motivated by the "scale effect" characteristic of some endogenous growth models. This was particularly troublesome in the seminal Romer (1990) model, with its implication that the rate of growth of the economy is proportional to the absolute size of the research sector. This implies that an increase in the size of the population, other things equal, must necessarily lead to faster growth rates, a prediction that contradicts the empirical evidence. To resolve scale effects, two variants of the non-scale growth model have evolved. The first approach leads to an equilibrium in which long-run per capita growth is proportional to the rate of population growth, with the proportionality depending upon the technological production characteristics of the final output and technology sectors.[3] The only way that policy can influence the long-run growth rate is by influencing either the population growth rate or the technological parameters; see Eicher and Turnovsky (1999). Empirical evidence suggesting that neither country size, nor policies (such as taxation) seem to have mattered for long run growth in OECD economies in the twentieth century provides some support for this position.[4]

The implication that long-run growth cannot be sustained in the absence of population growth has itself been the source of criticism. Accordingly, a second, alternative class of non-scale growth models, that eliminates the effect of country size, but still permits growth in the absence of population growth, has more recently emerged. These models permit at least a limited role for government policy to influence the long-run growth rate, through taxes and subsidies to research and development.[5] The model we present falls into the first category and we shall interpret what we shall view as its success in replicating the economy as providing some support for the original form of non-scale growth model.

No comprehensive analysis of the convergence properties of the first class of non-scale models currently exists. In a preliminary examination of this issue, Jones (1995a) obtained excessively slow rates of convergence, mainly because he assumed that the sectoral allocation of each factor remains constant during the transition. Dinopulous and Thompson (1998) analyze the transition dynamics of the alternative non-scale model numerically to find that the rate of convergence is approximately equal to the rate of population growth, but they do not highlight separate transition of output, capital and technology. The key contribution of the present paper is to show how the stable transition path in the two-sector non-scale growth model is characterized by a two-dimensional stable saddlepath, which permits the growth rates and the convergence speeds *to vary both across* *time and variables*. This is in sharp contrast with both the traditional one-sector neoclassical model and previous endogenous growth models, such as Bond, Wang, and Yip (1996) and Ortiguiera and Santos (1997), in which the stable dynamic adjustment path is a conventional one-dimensional manifold.[6] In these cases all variables converge to their respective steady states at *identical and constant rates*; that is the economy possesses a unique speed of convergence. The presence of a two-dimensional stable manifold introduces important flexibility to the convergence characteristics, by allowing capital, output, and technology to converge at different time-varying rates toward possibly different long-run equilibrium growth rates. These properties are consistent with Bernard and Jones (1996a, 1996b) who show that different sectors exhibit distinctly different convergence time profiles, suggesting that the process of convergence is more complex than indicated by changes in any single aggregate measure.

Our numerical simulations of the two-sector non-scale model are encouraging. The model replicates key economic ratios and reasonable speeds of convergence with relative ease. We show that reasonable asymptotic convergence speeds are achieved with a wide variety of parameter values. The numerical results support Jones (1995b) preliminary finding that larger shares of technology in the R&D sector increase the speed of convergence. In addition we highlight that the accumulation of knowledge slows the accumulation of capital, and hence reduces the speed of adjustment. Therefore the convergence speed is shown to be lower than in previous endogenous growth models. However, we point out that during the transition the speed of convergence might vary significantly. Finally we emphasize how distinct the convergence properties of the respective endogenous factors might be during the transition. A positive shock to the output sector is accompanied by increased capital accumulation, but initial *divergence* of technology. Essentially the non-linear transition allows for a greater adjustment of the capital intensive output sector at the expense of R&D. This result sharply contrasts with the implications based on growth models that feature linear and proportional adjustments. The non-constancy of the convergence rates and the non-proportionality of the endogenous variables in transition should provide impetus for further refinements in the empirical specifications of growth regressions.

2. A Two-Sector Non-Scale Economy

We focus on a centrally planned economy and use social production functions, in which externalities are internalized. The population, *N*, is assumed to grow at the steady rate $\dot{N}/N = n$. The economy produces final output, *Y*, and new technology, *A*, utilizing the social stocks of technology, labor, and physical capital, *K*, according to the Cobb-Douglas production functions:[7]

$$Y = a_F A^{s_A}[qN]^{s_N} K^{s_K} \qquad 0 < s_i < 1;\ \ i = A, N, K \tag{1a}$$

$$\dot{A} = a_J A^{h_A}[(1-q)N]^{h_N} - d_A A \quad 0 < h_i < 1;\ \ i = A, N \tag{1b}$$

where $a_F,\ a_J$ represent exogenous technological shift factors to the production functions and q is the fraction of labor employed in the final good sector, $s_i,\ h_i$. are the productive elasticities, and d_A represents the rate of depreciation of technology. According to equations (1a) and (1b), physical capital is specific to the production of final output, whereas new technology is a non-exclusive non-rival public good to both sectors. Labor is the only factor subject to intersectoral allocation Physical capital accumulates residually, after aggregate consumption, *C*, and depreciation needs, $d_K K$, have been met

$$\dot{K} = Y - C - d_K K \tag{1c}$$

While the introduction of physical capital depreciation is standard, the depreciation of technology is less obvious. One interpretation is of skill depreciation of workers (as in Eicher 1996), and another is as a measure of technology product obsolescence.[8]

The planner's problem is to maximize the intertemporal utility of the representative agent:

$$\frac{1}{1-g}\int_0^\infty (C/N)^{1-g} e^{-rt} dt \qquad r > 0;\ \ g > 0 \tag{1d}$$

where C/N denotes per capita consumption, subject to the production and accumulation constraints, (1a) - (1c), and the usual initial conditions. His decision variables are: (i) the rate of per capita consumption, C/N; (ii) the sectoral allocation of labor, q; (iii) the rates of accumulation of physical capital and technology. The optimality conditions are set out in the Appendix and have been discussed in Eicher and Turnovsky (1999) under more general conditions.

Before examining the dynamics, we characterize the balanced growth equilibrium, defined as a growth path along which all variables grow at constant, but possibly different, rates. In accordance with the stylized empirical facts (Romer 1989), we assume that the output/capital ratio, Y/K, is constant. A key feature of the non-scale model is that the equilibrium percentage growth rates of technology and capital, $\hat{A}$ and $\hat{K}$ respectively, are determined entirely by the production conditions. Taking the differentials of the production functions (1b) and (1c), and solving we find:

$$\hat{A} = \frac{h_N(1-s_K)n}{(1-h_A)(1-s_K)} = \frac{h_N}{(1-h_A)} n \equiv b_A n \tag{2a}$$

$$\hat{K} = \hat{Y} = \hat{C} = \frac{\left[(1-h_A)s_N + h_N s_A\right]}{(1-h_A)(1-s_K)} n \equiv b_K n \tag{2b}$$

and thus the per capita growth rate of output (capital) is:

$$\hat{Y} - n = n\{(1-h_A)[s_A + s_N + s_K - 1] + s_A[h_A + h_N - 1]\}/(1-h_A)(1-s_K) \tag{2c}$$

Hence countries converge to identical growth rates, if either their production technologies are identical, or their production functions are constant returns to scale. If production technologies differ across countries, growth rates exhibit conditional convergence.

Equations (2) illustrate the basic characteristic of the first type of non-scale models, namely that the equilibrium growth rates in general are proportional to the population growth rate. Jones (1995a) provides an extensive discussion of the relevance of this class of non-scale growth model, in light of the mixed evidence on the correlation between population growth and output growth (see e.g. Barro and Sala-i-Martin 1995). He emphasizes that this aspect of the model, which is a direct consequence of non-constant returns to scale, relies foremost on the creation of new technology, and on the growth of effective researchers. A zero effect of the population growth rate on the per capita output growth rate is consistent with either (i) increasing returns to scale in one sector, offset by appropriate decreasing returns to scale in the other; or (ii) constant returns to scale in both sectors. Our numerical results on convergence speeds are robust across this relationship between long-run per capita growth and population growth. For example, Table 3 below notes one case where the asymptotic speed of convergence is 2.3% when the two sectors are subject to constant returns to scale in all factors, and another when both sectors have decreasing returns to scale.

Finally, the differential equilibrium growth rates of physical capital and knowledge are reflected in the relative percentage growth rates of their respective shadow values, n, m, which from equations (A.3) and (A.5) in the Appendix satisfy:

$$\hat{n} - \hat{m} = (b_A - b_K)n \tag{2d}$$

3. Dynamics of a Two-Sector Non-Scale Model

To derive the dynamics about the balanced growth path we define the following stationary variables: $y \equiv Y/N^{b_K}$; $k \equiv K/N^{b_K}$; $c \equiv C/N^{b_K}$; $a \equiv A/N^{b_A}$; $j \equiv J/N^{b_K}$; $q \equiv n/mN^{(b_A - b_K)}$, where $J \equiv a_J A^{h_A}[(1-q)N]^{h_N}$. For convenience, we shall refer to *y*, *k*, *c*, and *a* as *scale-adjusted* quantities. This allows us to rewrite scale-adjusted output and technology as

$$y = a_F q^{s_N} a^{s_A} k^{s_K} \tag{3a}$$

$$j = a_J (1-q)^{h_N} a^{h_A} \tag{3b}$$

Given this normalization, we can use the first order condition (A.1b) to solve implicitly for the share of labor in production, $q = q(q, a, k)$; $\P q/\P q > 0$, $\P q/\P k > 0$, $sgn(\P q/\P a) = sgn(s_A - h_A)$[9] We show in the Appendix that the dynamic system, linearized around the equilibrium, $\tilde{k}, \tilde{a}, \tilde{q}, \tilde{c}$, may be approximated by the following fourth order system:

$$\begin{pmatrix} \dot{k} \\ \dot{a} \\ \dot{q} \\ \dot{c} \end{pmatrix} = \begin{pmatrix} s_K \frac{\tilde{y}}{\tilde{k}} + s_N \frac{\tilde{y}}{\tilde{q}} \frac{\P q}{\P k} - b_K n - d_K & s_A \frac{\tilde{y}}{\tilde{a}} + s_N \frac{\tilde{y}}{\tilde{q}} \frac{\P q}{\P a} & s_N \frac{\tilde{y}}{\tilde{q}} + \frac{\P q}{\P q} & -1 \\ \frac{-h_N \tilde{j}}{1-\tilde{q}} \frac{\P q}{\P k} & (h_A - 1)\frac{\tilde{j}}{\tilde{a}} - \frac{h_N \tilde{j}}{1-\tilde{q}} \frac{\P q}{\P a} - b_A n - d_A & -\frac{h_N \tilde{j}}{1-\tilde{q}} \frac{\P q}{\P q} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ \frac{\tilde{c} s_K}{g\tilde{k}}\left(a_{11} - \frac{\tilde{c}}{\tilde{k}}\right) & \frac{\tilde{c} s_K a_{12}}{g\tilde{k}} & \frac{\tilde{c} s_K a_{13}}{g\tilde{k}} & 0 \end{pmatrix} \begin{pmatrix} k - \tilde{k} \\ a - \tilde{a} \\ q - \tilde{q} \\ c - \tilde{c} \end{pmatrix} \tag{4}$$

where the matrix in (4) is $A \equiv (a_{ij})$, and we define:

$$a_{31} \equiv \left[\frac{d}{\tilde{a}} a_{21} + \frac{s_A h_N}{s_N} \frac{\tilde{j}}{a(1-\tilde{q})^2} \frac{¶q}{¶k} - \frac{g}{\tilde{c}} a_{41}\right]\tilde{q}; \quad a_{32} \equiv \left[\frac{d}{\tilde{a}} a_{22} + \frac{s_A h_N}{s_N} \frac{j}{\tilde{a}(1-\tilde{q})^2} \frac{¶q}{¶a} - \frac{g}{\tilde{c}} a_{42}\right]\tilde{q},$$

$$a_{33} \equiv \left[\frac{d}{\tilde{a}} a_{23} + \frac{s_A h_N}{s_N} \frac{j}{\tilde{a}(1-\tilde{q})^2} \frac{¶q}{¶k} - \frac{g}{\tilde{c}} a_{43}\right]\tilde{q}, \text{ and } d \equiv h_A + \frac{s_A h_N}{s_N}\left(\frac{\tilde{q}}{1-\tilde{q}}\right)$$

It is readily shown that the determinant of this matrix is proportional to $(h_A - 1)(s_K - 1)$. Imposing the condition $s_K < 1,\ h_A < 1$ implies that the determinant is positive which means that it has either 0, 2, or 4 positive roots. The dynamic efficiency condition, (i) $¶F/¶K \equiv s_K\left(\tilde{y}/\tilde{k}\right) > b_K n$, rules out the first case. Unfortunately, due to the complexity of the system, we cannot find as general a condition to rule out the case of 4 unstable roots. However, the conditions (ii) $g \geq 1$, (iii) $s_k \leq 1/(1+g)$ are two simple and plausible restrictions that suffice to eliminate the third case of explosive growth. These conditions are met by our simulations and indeed, in all of the many simulations carried out over a wide range of parameter sets, 2 positive roots were always obtained.[10] Thus since the system features two state variables, *k* and *a*, and two jump variables *c*, and *q*, we are confident that equilibrium is characterized by a unique stable saddlepath.[11]

In previous growth models, in which all variables moved in proportion to one another, the associated unique stable eigenvalue sufficed to characterize the transition. With two stable roots $0 > m_1 > m_2$, say, the speeds of adjustment change over time, although asymptotically all scale-adjusted variables converge to their respective equilibrium at the rate of the slower growing eigenvalue, $-m_1$.[12] In particular, since scale-adjusted output is determined by capital and technology and the proportion of labor allocated to manufacturing during the transition, its convergence speed is also not constant over time, but is a composite of the others.

In general, we define the speed of convergence at time *t*, of a variable *x* say, as

$$f_x(t) \equiv -\left(\frac{\dot{x}(t) - \dot{\tilde{x}}}{x(t) - \tilde{x}}\right) \tag{5}$$

where $\tilde{x}$ is the equilibrium balanced growth path, which may or may not be stationary, depending upon the specific variable. The expression (5) measures the rate of convergence at any instant of time in terms of the percentage rate of change in the distance $x(t)-\tilde{x}$. With the non-monotonic adjustment path, it is possible for the measure (5) to be negative, in which case the variable is diverging at that instant. In the case where $\tilde{x}$ is constant or follows a steady growth path and the stable manifold is one dimensional, this measure equals the magnitude of the unique stable eigenvalue (see Ortigueira and Santos 1997). But we are primarily interested in the convergence speeds of *per capita* quantities, in particular, per capita output and technology, Y/N and A/N, which in steady-state equilibrium grow at the rates $(b_K-1)n$, and $(b_A-1)n$. As usual in endogenous growth models this implies that per capita variables are *non-stationary* unless both sectors exhibit overall constant returns to scale. Applying the measure (5) to either per capita variable, we find:

$$f_{Y/N}(t)=-\frac{\dot{y}(t)}{y(t)-\tilde{y}}-(b_K-1)n=f_y(t)-(b_K-1)n \tag{6a}$$

$$f_{A/N}(t)=-\frac{\dot{a}(t)}{a(t)-\tilde{a}}-(b_A-1)n=f_a(t)-(b_A-1)n \tag{6b}$$

Thus, for example, per capita output has the same time profile as does scale-adjusted output, except that its speed of convergence has to be adjusted by the equilibrium growth rate of per capita output. Asymptotically, $f_{Y/N}(t)$ converges to its steady-state growth path at the rate $-m_1-(b_K-1)n$. Thus, per capita output, Y/N, will converge slower, or faster than does scale-adjusted output, *y*, depending upon whether the equilibrium balanced growth rate of per capita output is positive or negative. This in turn depends upon returns to scale. Moreover, although scale-adjusted capital and knowledge both converge asymptotically at the same rate, $-m_1$, their corresponding per-capita quantities, K/N, A/N, converge to their respective nonstationary growing equilibrium paths at the differential rates $m_1-(b_K-1)n$, $m_1-(b_A-1)n$, respectively.

4. Numerical Analysis of Transitional Paths

To obtain a clear idea about the implied speeds of convergence, we turn to numerical simulations. Table 1 shows that the values we employ for our fundamental parameters are essentially identical to those suggested by previous calibration exercises.[13] The final goods sector exhibits constant returns to scale in capital and labor, but increasing returns to scale with the inclusion of knowledge. The technology sector is subject to increasing returns to scale in labor and knowledge.[14] These assumptions are made so as to obtain a plausible equilibrium per capita output growth rate.

Table 1: Benchmark Parameters

Production parameters:	$a_F = 1,\ s_N = 0.6,\ s_K = 0.4,\ s_A = 0.3,\ a_J = 1,\ h_N = 0.5,\ h_A = 0.6$
Preference parameters:	$r = 0.04;\ g = 1$
Depreciation and population parameters:	$d_K = 0.05,\ d_A = 0.01,\ n = 0.015$

We group the resulting endogenous variables into three categories. The *balanced per capita growth rates* of capital (output), and technology; *key equilibrium ratios*, including the output-capital ratio, the share of consumption in output, and the share of labor employed in the output sector; the *convergence speeds* of per capita capital (output) and technology. All turn out to be remarkably plausible as shown in Table 2 and extensive sensitivity analysis carried out in Eicher and Turnovsky (1998) proves our results are robust across alternative parameter sets.

Table 2: Benchmark Equilibrium Values

$\widehat{(Y/N)}$	$\widehat{(A/N)}$	$\widetilde{(Y/K)}$	$\widetilde{(C/Y)}$	$\tilde{q}$	$\tilde{f}_{Y/N}$	$\tilde{f}_{A/N}$
0.009	0.003	0.285	0.740	0.941	0.0196	0.0256

Given the larger increasing returns in the final good sector, the growth rate of capital and output of exceeds that of technology, with per capita output growing at nearly 1%, and per capita knowledge at 0.3%.[15] The capital-output ratio is approximately 3.5, while around 74% of output is devoted to consumption. Approximately 94% of the work force is employed in the output sector, with the balance of 6% employed in producing knowledge.

The asymptotic speed of convergence is provided by the larger of the two stable eigenvalues, m_1. This value implies that per capita output converges asymptotically at an annual rate of nearly 2%. This convergence rate is achieved with plausible underlying parameter values that also replicate key ratios of the economy exceedingly well. The slower per capita growth of knowledge implies that per capita knowledge converges at a faster rate of nearly 2.6%. The rate of convergence of output is significantly lower than the approximately 7% found for the Solow model or the 10% in the Lucas model.[16] Neither of the two later models implies convergence rates for technology (for which there exists also no empirical guidance), although human capital converges at the same speed as output in the Lucas model.

Convergence has become a hotly debated empirical issue. The question seldom addressed is if the rates of convergence are in fact constant over time - a theoretical implication that is solely a function of the simplified modeling structure as we have shown above. In individual country studies, Barro and Sala-i-Martin (1995) find significant variations in the convergence speeds across countries and reject constancy in Japan, but not in the US and Europe. Even more controversial is the implied speed of convergence. The seminal contributions that have shaped this research agenda argued that countries converge to their steady state level at about 2-3 percent a year.[17] However, recent studies suggest that these estimates ignore a number of econometric issues, as a result of which they are downwardly biased. Once one controls for factors such as omitted variables (country specific effects), the endogeneity of the dependent variables, and measurement errors the estimates of the convergence rates both increase above 2% and are much more sensitive to the time period, the set of countries and their stages of development. The new consensus that seems to be emerging is that convergence rates potentially differ significantly across countries and over time, leading to a much wider range of estimates of the convergence rate.[18] This finding is also consistent with the time-varying convergence rates generated by our model. The fact that initial values of the convergence rate of per capita output exceed 10% (see Fig.1d) is consistent with the empirical evidence that less developed countries converge at faster rates than do OECD economies.

Table 3: Alternative Equilibria

	$\hat{(Y/N)}$	$\hat{(A/N)}$	$\tilde{(Y/K)}$	$\tilde{(C/Y)}$	$\tilde{q}$	$\tilde{f}_{Y/N}$	$\tilde{f}_{A/N}$
$a_F = 2$	0.009	0.003	0.285	0.740	0.941	0.0196	0.0256
CRS in Y and A $s_N = 0.6, s_K = 0.3,\ s_A = 0.1$ $h_N = 0.5, h_A = 0.5$	0	0	0.350	0.814	0.985	0.0228	0.0228
DRS in Y and A $s_N = 0.55, s_K = 0.25,\ s_A = 0.1$ $h_N = 0.45, h_A = 0.45$	-0.0024	-0.0027	0.411	0.847	0.986	0.0231	0.0234

As part of a sensitivity analysis, we report alternative simulations in Table 3. This table provides examples of how long-run growth rates of per capita capital, output, and technology are altered in response to changes in the returns to scale in either sector. Constant (decreasing) returns to scale in both sectors generates zero (negative) per capita long-run growth rates of output and technology. Table 3 also illustrates the invariance of long-run growth rates with respect to parameters (e.g. a_F) that are known to influence long-run growth rates in previous endogenous growth models. Instead, the table verifies how in non-scale models, long-run growth rates are determined exclusively by the magnitudes of the assumed production elasticities in technology and output, in conjunction with the population growth rate (i.e. the quantities b_A, b_K equations (2)).

The asymptotic convergence speed of per capita output in the benchmark economy is around 2%, consistent with much of the empirical evidence. Most importantly for our purposes, Table 3 provides evidence of the robustness of this result to significant changes in the underlying production parameters, particularly overall returns to scale. We have also conducted sensitivity analyses (not reported) of the asymptotic speed of convergence with respect to the elasticities in the knowledge production sector, about which we have relatively little information. For example, we find that constraining the production function of knowledge to constant returns to scale, the asymptotic rates of convergence of both output and knowledge range from around 1% (for $h_A = 0.75, h_N = 0.25$) to around 5% (for $h_A = 0.25, h_N = 0.75$). There is no consensus on the exact values of h_A, h_N, and s_A, however. Nevertheless the calibration results hinge crucially on all these parameters.[19] While the model is remarkably robust to significant changes in the parameters, the model highlights the feature first discussed by Jones (1995a). Essentially the convergence speed is increasing in the share of technology in the R&D function and in output. The result is not surprising, since technology is also the origin of the (positive) externality, but it underlines the necessity for future research into the exact properties of aggregate and industry level R&D functions.

To obtain a complete picture of the convergence speed, and to address the diversity of the empirical findings it is helpful to look beyond the asymptotic speed of convergence, and examine the transition closer. We have pointed out in the dynamic analysis that the two dimensional stable manifold generates time varying convergence speeds. In addition, the rates of convergence of capital and knowledge toward their respective steady-state growth rates may well diverge along substantial portions of the transitional path, so that the convergence speed of any one of these variables would be misleading as an economy-wide measure. There is no reason to expect identical rates of convergence across variables or time.

In order to provide clear intuition into the adjustment process, we conduct a simulation where only one parameter, the productivity of output is doubled (from $a_F = 1$ to $a_F = 2$). The phase diagram, Figure 1a, shows that initially, the increase in productivity of the output sector attracts resources to that sector, and away from the knowledge-producing sector. The figure shows that the scale-adjusted capital stock begins to *accumulate* accompanied by a *reduction* in the scale-adjusted knowledge, since the per capita rate accumulation of the latter is less than the depreciation rate. The corresponding growth rates are illustrated in Figures 1b and 1c. After the initial decline in scale-adjusted knowledge, the increase in capital raises the return on investing in knowledge, relative to the return on capital. The central planner then begins to invest in knowledge accumulation and the speed of capital accumulation slows dramatically during the last stages of transition. Output, being a composite of capital, knowledge, and labor allocation, grows at a rate that reflects the dynamics of all three quantities.

The only long-run effect of this shock is to raise the long-run level of scale-adjusted capital, $\tilde{k}$, and output, $\tilde{y}$, proportionately; everything else, including the long-run scale-adjusted stock of knowledge, remain unchanged.[20] However, capital, output, and technology exhibit very different convergence properties in terms of the time paths of speeds and growth rates. Figure 1d illustrates the convergence of different variables to different growth rates, and while the speeds of convergence exceed the asymptotic rate of convergence for capital and output, the speed for technology initially falls below its long-run rate and recovers only later.[21] Immediately after the shock, the convergence of capital exceeds 10 percent, output adjusts with approximately 10 percent and the speed of convergence of technology is negative. Within about 20 periods the economy attains normal convergence rates of between 1 and 6 percent.

It is important to note that the growth rate of capital falls monotonically toward its long-run equilibrium value, while the growth rate of knowledge overshoots it long-run value. This is because of the relatively high rate of return to investing in knowledge during this phase of the transition. Hence we provide an example where convergence speeds of per capita outputs across countries need not be correlated with the convergence of technology, either in the short run or in the long run. This may explain part of the puzzle discussed by Bernard and Jones (1996a), with respect to the contrasting convergence properties of output and technology among OECD economies. Thus, this simulation provides a simple and instructive example of the significance of two stable roots and the importance of two-dimensional transition paths.

5. Concluding Remarks

The determinants of long-run growth rates and the characteristics of economies' transitions to their balanced growth paths are central to theories of economic growth. The convergence characteristics of the one-sector neoclassical and the two-sector endogenous growth models imply a uniform speed of convergence, both through time and between variables. As a consequence, much empirical research has focused on monotonic convergence in per capita output only, with the implication that this was an adequate representation of the economy-wide speed of convergence. Recent work has called into question the empirical validity of this approach. In this paper we have introduced a hybrid two-sector non-scale growth model that possesses features of both neoclassical and endogenous growth models. We have shown that the non-scale model easily accounts for reasonable speeds of convergence, in contrast to basic neoclassical models, but it can also account for conditional convergence, in contrast to endogenous growth models.

The crucial determinant of the asymptotic speed of convergence is the larger of the two stable eigenvalues, m_1. Indeed, it is the magnitude of m_1 that accounts for the difference in convergence speeds between the non-scale model and basic endogenous growth models. The critical difference between the two is that the latter assume *constant* returns to scale in the reproducible factors, *A* and *K*. Under this assumption, the basic Lucas model yields one eigenvalue equal to zero, while the other root is typically around -0.10, and implies an excessive speed of convergence. By contrast, the examples we have presented have assumed *decreasing* returns to scale in *A* and *K*. Given our assumptions on the parameters, this reduces the larger stable eigenvalue to around -0.02, which then forms the basis for our inferred speed of convergence. As we move the production functions toward constant returns to scale in *A* and *K* (though stability considerations preclude setting $h_A = 1$), we find that m_1 converges to zero and that the model behaves more like an endogenous growth model. Hence, it appears that an important driving force for our results is the assumption (which we view as plausible) that there are decreasing returns to scale in the reproducible factors, though there may be increasing returns to scale overall.[22]

Appendix

This Appendix briefly indicates the steps used to derive the linearized dynamic system (4). The optimality and transversality conditions to the central planning problem are:

$$C^{-g} = nN^{1-g} \tag{A.1a}$$

$$ns_N(Y/q) = mh_N(J/(1-q)) \tag{A.1b}$$

$$s_K(Y/K) - d_K = r - (\dot{n}/v) \tag{A.1c}$$

$$(n/m)s_A(Y/A) + h_A(J/A) - d_A = r - (\dot{m}/m) \tag{A.1d}$$

$$\lim_{t\to\infty} nKe^{-rt} = \lim_{t\to\infty} mAe^{-rt} = 0 \tag{A.1e}$$

where n, m are the respective shadow values of physical capital and knowledge.

Substituting (3a) and (3b) into (A.1b), we may express the labor allocation condition (2b) as:

$$a_F q s_N q^{s_N-1} a^{s_A} k^{s_K} = a_J h_N (1-q)^{h_N-1} a^{h_A} \tag{A.2}$$

yielding the solution, $q = q(q, a, k)$, having the properties indicated in the text. Similarly, substituting (3a) and (3b) into (A.1c) and (A.1d) implies:

$$\dot{n}/n = r + d_K - s_K a_F q^{s_N} a^{s_A} k^{s_K-1} \tag{A.3a}$$

$$\dot{m}/m = r + d_A - a_J(1-q)^{h_N} a^{h_A-1}\left[h_A + (s_A h_N / s_N)(q/(1-q))\right] \tag{A.3b}$$

Taking the time derivative of (A.1a) and combining with (A.2a), the growth rate of aggregate consumption is given by

$$\dot{C}/C = (1/g)\left[s_K a_F q^{s_N} a^{s_A} k^{s_K-1} - ((1-g)n + d_K + r)\right] \tag{A.3c}$$

Taking the time derivatives of *k*, *a*, *q*, and *c*, and combining with (1d), (1c), (A.3a) - (A.3c), the dynamic system can be expressed in terms of the redefined stationary variables by:

$$\dot{k} = k\left[q^{s_N} a_F a^{s_A} k^{s_{K-1}} - b_K n - d_K - (c/k)\right] \qquad \text{(A.4a)}$$

$$\dot{a} = a\left[(1-q)^{h_N} a_J a^{h_A - 1} - b_A n - d_A\right] \qquad \text{(A.4b)}$$

$$\dot{q} = q\left\{a_J (1-q)^{h_N} a^{h_A - 1}\left[h_A + \frac{s_A h_N}{s_N}\frac{q}{1-q}\right] - s_K a_F q^{s_N} a^{s_A} k^{s_K - 1} - (b_A - b_K)n - (d_A - d_K)\right\} \qquad \text{(A.4c)}$$

$$\dot{c} = (c/g)\left\{s_K a_F q^{s_N} a^{s_A} k^{s_K - 1} - (r + d_K) + \left[g(1 - b_K) - 1\right]n\right\} \qquad \text{(A.4d)}$$

where q is determined by (A.2).

The steady state to this system. denoted by ~ superscripts, can be summarized by:

$$\left(\tilde{y}/\tilde{k}\right) - \left(\tilde{c}/\tilde{k}\right) = b_K n + d_K \qquad \text{(A.5a)}$$

$$\left(\tilde{j}/\tilde{a}\right) = b_A n + d_A \qquad \text{(A.5b)}$$

$$\left[h_A + \left(s_A h_N / s_N\right)\left(\tilde{q}/(1-\tilde{q})\right)\right]\left(\tilde{j}/\tilde{a}\right) - b_A n - d_A = s_K\left(\tilde{y}/\tilde{k}\right) - b_K n - d_K \qquad \text{(A.5c)}$$

$$s_K\left(\tilde{y}/\tilde{k}\right) - b_K n - d_K = r + (1-g)(1-b_K)n \qquad \text{(A.5d)}$$

Linearizing (A.4) about (A.5) yields equations (4) of the text.

References

Adams, J.D. (1990). “Fundamental Stocks of Knowledge and Productivity Growth.” *Journal of Political Economy* 98, 673-702.

Aghion, P. and P. Howitt. (1998). *Endogenous Growth Theory*. Cambridge, MA:MIT Press.

Backus, D., P. Kehoe and T. Kehoe. (1992). “In Search of Scale Effects in Trade and Growth.” *Journal of Economic Theory* 58, 377-409.

Barro, R.J., (1991). “Economic Growth in a Cross-Section of Countries.” *Quarterly Journal of Economics* 106, 407-443.

Barro, R.J., N.G. Mankiw, and X. Sala-i-Martin. (1995). “Capital Mobility in Neoclassical Models of Growth.” *American Economic Review* 85, 103-115.

Barro, R.J., and X. Sala-i-Martin. (1995). *Economic Growth*. NY: McGraw-Hill.

Barro, R.J. and X., Sala-i-Martin. (1997). “Technological Diffusion, Convergence, and Growth.” *Journal of Economic Growth* 2, 1-26.

Bernard, A.B. and C.I. Jones. (1996a). “Technology and Convergence.” *Economic Journal* 106, 1037-1044.

Bernard, A.B. and C.I. Jones. (1996b). “Comparing Apples and Oranges: Productivity Consequences and Measurement Across Industries and Countries.” *American Economic Review* 86, 1216-1238.

Bond, E.W., P. Wang, and C.K. Yip. (1996). “A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics.” *Journal of Economic Theory* 68, 149-173.

Caballero, R.J. and A.B. Jaffee. (1993). “How High are the Giants' Shoulders: An Empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth.” *Macroeconomics Annual* 8, 15-74.

Caselli, F. G. Esquivel, and F. Lefort. (1996). “Reopening the Convergence Debate: A New Look at Cross-country Empirics.” *Journal of Economic Growth* 1, 363-390.

Dinopoulos, E. and P. Thompson. (1998). "Schumpeterian Growth Without Scale Effects," *Journal of Economic Growth* 3, 313-337.

Eicher, T.S. (1996). "Interaction Between Endogenous Human Capital and Technological Change." *Review of Economic Studies* 63, 127-144.

Eicher, T.S. and S.J. Turnovsky. (1998). "Convergence Speeds and Transition Dynamics in Non-Scale Growth Models," unpublished manuscript, University of Washington.

Eicher, T.S. and S.J. Turnovsky. (1999). "Non-Scale Models of Economic Growth," *Economic Journal* 109, 394-415.

Evans, P. (1997). "How Fast do Countries Converge?" *Review of Economics and Statistics* 79, 219-225.

Griliches, Z. (1988) "Productivity Puzzles and R&D: Another Nonexplanation." *Journal of Economic Perspectives*." 2, 9-21.

Grossman, G.M. and E. Helpman. (1991). *Innovation and Growth in the Global Economy*. Cambridge, MA: MIT Press.

Howitt, P. (1999). "Steady-State Endogenous Growth with Population and R&D Inputs Growing." *Journal of Political Economy* (forthcoming).

Islam, N. (1995). "Growth Empirics: A Panel Data Approach." *Quarterly Journal of Economics* 110, 1127-1170.

Jones, C.I. (1995a). "R&D Based Models of Economic Growth," *Journal of Political Economy* 103, 759-84.

Jones, C.I. (1995b). "Time-Series Tests of Endogenous Growth Models." *Quarterly Journal of Economics* 110, 495-525.

Jones, C.I. (1999). "Growth: With or Without Scale Effects." *American Economic Review, Papers and Proceedings* 89, 139-144.

Jones, C.I. and J. Williams. (1998). "Measuring the Social Returns to R&D." *Quarterly Journal of Economics* 113, 1119-1135..

Kortum, S. (1993). "Equilibrium R&D and the Decline in the Patent-R&D Ratio; US Evidence." *American Economic Review: Papers and Proceedings* 83, 450-457.

Kortum, S. (1997). "Research Patenting, and Technological Change," *Econometrica* 65, 1389-1419.

Lucas, R.E. (1988). "On the Mechanics of Economic Development." *Journal of Monetary Economics* 22, 3-42.

Mankiw, N.G., D. Romer, and D. Weil. (1992). "A Contribution to the Empirics of Economic Growth." *Quarterly Journal of Economics* 107, 407-38.

Mulligan, C.B. and X. Sala-i-Martin. (1993). "Transitional Dynamics in Two-Sector Models of Endogenous Growth." *Quarterly Journal of Economics* 108, 739-773.

Ortigueira, S. and M.S. Santos. (1997). "On the Speed of Convergence in Endogenous Growth Models." *American Economic Review* 87, 383-399.

Peretto, P.F. (1998). "Technological Change and Population Growth," *Journal of Economic Growth* 3, 283-311.

Romer, P. (1989). "Capital Accumulation in the Theory of Long-Run Growth." in R.J. Barro (ed.), *Modern Business Cycle Theory*. Cambridge, MA: Harvard University Press).

Romer, P. (1990). "Endogenous Technological Change." *Journal of Political Economy* 98, S71-103.

Sala-i-Martin, X. (1994). "Cross-Sectional Regressions and the Empirics of Economic Growth." *European Economic Review* 38, 739-747.

Segerstrom, P. (1998). "Endogenous Growth Without Scale Effects." *American Economic Review* 88, 1290-1310.

Temple, J.R.W. (1998). "Robustness Test of the Augmented Solow Model," *Journal of Applied Econometrics* 13, 361-375.

Young, A. (1998). "Growth Without Scale Effects," *Journal of Political Economy* 106, 41-63.

ENDNOTES

[1] See Lucas (1988), Bond, Wang and Yip (1996), Ortigueira and Santos (1997).

[2] *Non-scale* refers to the characteristic that variations in the size or scale of the economy do not permanently alter its long-run equilibrium *growth rate,* see Young (1998).

[3] Examples of this type of non-scale growth model include Jones (1995a), Kortum (1997), Segerstrom (1998), and Eicher and Turnovsky (1999).

[4]See, for example, Jones (1995b), for evidence in the case of taxes, and Backus, Kehoe and Kehoe (1992 for more general measures of scale effects.

[5] Examples of the second form of non-scale model include: Young (1988), Peretto (1998), Aghion and Howitt (1998), Dinopoulos and Thompson (1998), and Howitt (1999).

[6]Mulligan and Sala-i-Martin (1993) provide a comprehensive qualitative analysis of the transitional dynamics of the two-sector Lucas (1988) model. Their model does not rely on technological change, but human capital accumulation. The analytical analysis shows that all accumulated factors must move together during transition and the paper does not focus explore the implied speed of convergence. This model requires exactly constant returns to privately accumulated factors in all sectors. One of the strength of our approach is that this restriction can be significantly relaxed.

[7]This technology is somewhat more general than Jones (1995a), who specifies $s_A = s_N = 1 - s_K \equiv s$. Eicher and Turnovsky (1999) discuss the balanced-growth characteristics of a general two-sector technology.

[8]Intuitively one would expect the rate of depreciation of technology to be much smaller than that of physical capital, a choice reflected in our simulations. Indeed, a referee has suggested that the rate of depreciation of technology could be negative because of R&D effort or learning by doing. In any event, the specific value of the depreciation rate is unimportant. The result that technology and output may move in different directions during the transition is due to the flexibility in returns to scale permitted by the non-scale production function.

[9]Intuitively, an increase in k attracts labor to the output sector because it raises the productivity of labor in producing final output. By contrast, an increase a, being an input in both sectors, raises the productivity of labor in both sectors and causes a net shift in employment toward the sector in which knowledge is more productive.

[10]The characteristic equation for the linearized system (4) is of the form: $m^4 + p_1 m^3 + p_2 m^2 + p_3 m + p_4 = 0$, where p_i are functions of the elements a_{ij} of the matrix in (4). The determinantal condition (i) implies $p_4 > 0$, while the dynamic efficiency condition (ii) implies $p_1 < 0$. By Descartes rule of signs, necessary and sufficient conditions for the characteristic equation to have just two positive roots is that either $p_2 < 0$ or $p_3 > 0$. Conditions (ii) and (iii) noted above are examples of conditions that ensure $p_2 < 0$ and other more general, but more complicated conditions can also be found. In addition, conditions that suffice to ensure $p_3 > 0$ can also be obtained.

[11]Note that in the complete analysis of the transitional dynamics in *k*-*a* space the feedbacks from the jump variables must be taken into account; full account of this is taken in our formal analysis available on request.

[12]The generic form of the transitional path of a typical variable, *x*, for a dynamic system generated by two state variables is of the form: $x(t) = \tilde{x} + A_1 e^{m_1 t} + A_2 e^{m_2 t}$, where A_1, A_2 are appropriately determined constants.

[13] See, for example, Lucas (1988), Ortigueira and Santos (1997), and Jones (1995a).

[14] The empirical literature on research functions is sparse, especially if one requires separate elasticities for labor and technology. Adams (1990) and Caballero and Jaffee (1993) are examples of thorough empirical investigations that are ultimately unsuccessful in reporting separate elasticities for labor and technology. Kortum (1993) derives values of about .2 by extrapolating results from aggregate patent data. Jones and Williams (1998) obtain estimates between .5 and .75, these however, are a function of their assumed rate of growth and the assumed share of technology in research.

[15]Given that the elasticities of labor and capital in output are commonly assumed to be around 0.6 and 0.3, respectively, higher per capita growth rates would require greater technology spillovers in production or greater returns to scale in technology. The evidence does not seem to provide sufficient support for either. Alternatively, the long-run growth rate of output may be raised by introducing capital into the production of knowledge. However, the intersectoral allocation of capital (along with labor) complicates the dynamics further, without adding insight.

[16]Non-scale growth alone is not sufficient to generate realistic speeds of convergence. The one-sector non-scale model can be conveniently parameterized by setting $s_A = 0,\ h_x = 0,\ x = A, N, K$. In the absence of endogenous technology, its rate of convergence is roughly identical to the Lucas model.

[17]The convergence rate of around 2% was established as a benchmark by the influential work of Barro (1991), Barro and Sala-I-Martin (1992, 1995), Barro and Lee (1994), Sala-I-Martin (1994), Mankiw, Romer and Weil (1992).

[18]For example, using a panel data approach taking account of fixed effects, Islam (1995) estimates the rate of convergence to be 4.7% for nonoil countries and 9.7% for OECD countries. Caselli, Esquivel, and Lefort (1996), use a GMM estimator to correct for sources of inconsistency due to correlated country-specific effects and endogenous explanatory variables and obtain a convergence rate of around 10%. Evans (1997) using an alternative method to generate consistent estimates of convergence finds them to be around 6%. Finally, Temple (1998) finds that allowing for measurement error produces estimates of the convergence rate for OECD countries of between 1.5% and 3.6% and for nonoil countries between 0.3% and 6.7%.

[19]Empirical estimates by Griliches and others suggests estimates of s_A of between 0.06 and 0.10; see Griliches (1988).

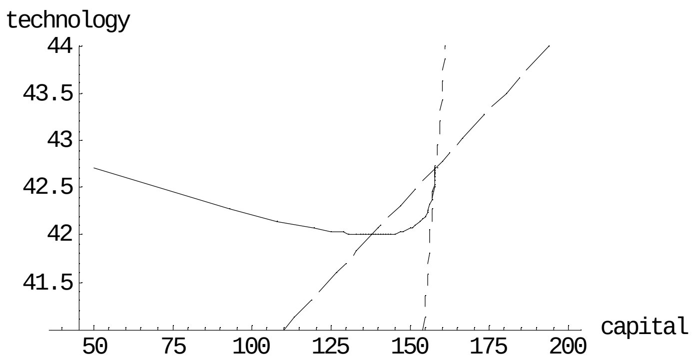
[20]The independence of the long-run stock of knowledge is a consequence of the assumption that the production function of knowledge is independent of capital. Aghion and Howitt (1998 Ch 3) and Eicher and Turnovsky (1999) allow for

capital in the technology function to analyze the consequences for growth. They show that the above independence result is not robust under these circumstances.

[21]The speed of convergence, f_x, in (5) is defined so as to be positive when x is converging and negative when it is diverging. Fig 1.d illustrates an example where per capita knowledge diverges during the initial phase of the transition.

[22]A similar result is obtained by Barro, Mankiw and Sala-i-Martin (1995).

Figure 1a
Dynamic Adjustment of Scale-Adjusted Capital and Technology



Time Profile of Per Capita Growth Rates

Figure 1b
Capital and Output

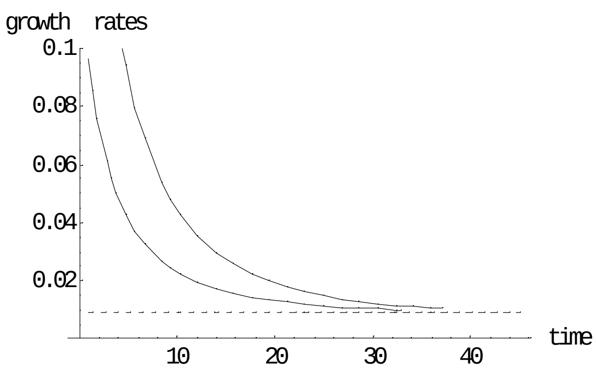


Figure 1c
Technology

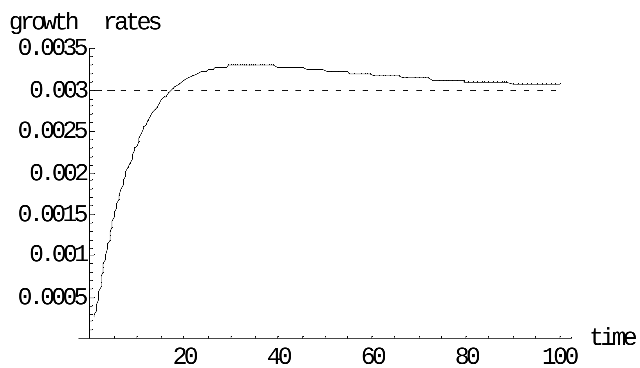


Figure 1d

Time Profile of Per Capita Convergence Speeds