Timothy Thornton and Michael Wu

Summer Institute in Statistical Genetics 2015

Lecture Outline

Yet more on rare variants...

- Gene-Environment Interaction Testing
- Meta-analysis
- Additional Concerns

Rare Variant by Environment Interactions

Gene-Environment Interactions $(G \times E)$



Complex diseases are caused by interplay of genes & environment. Identification of $G \times E$ aids in disease prevention.

Rare Variant by Environment Interactions

GxE Association Testing

Objective: Identify statistical interactions (synergism/antogonism) between environmental variable and rare variants in sequencing studies

Standard Approach:

- Test SNV individually
- Regress outcome on single variant, environment, and interaction
- Under-powered for rare variant analysis! (possibly worse than main effects)

How do we conduct region based analysis of GxE interactions?

Rare Variant by Environment Interactions

Motivation



- Circulating levels of adiponectin are highly heritable and associated with many conditions.
- SNVs at the adiponectin-encoding gene, ADIPOQ, are associated with adiponectin levels.
- Dataset consists of adiponectin levels and rare SNVs (MAF < 5%) within the ADIPOQ gene.

Rare Variant by Environment Interactions

Notation

Consider the following generalized linear model:

$$g(\mu_i) = \mathbf{X}_i^{\mathsf{T}} \alpha_1 + \alpha_2 E_i + \mathbf{G}_i^{\mathsf{T}} \alpha_3 + E_i \mathbf{G}_i^{\mathsf{T}} \beta$$

= $\widetilde{\mathbf{X}}_i^{\mathsf{T}} \alpha + \mathbf{S}_i^{\mathsf{T}} \beta.$

- Outcome: Y_i , has distribution from exponential family and $\mu_i = E(Y_i | \widetilde{\mathbf{X}}_i)$.
- ► q non-genetic covariates: X_i.
- environmental factor: E_i .
- group of p variants: $\mathbf{G}_i = (G_{i1}, \cdots, G_{ip})^{\mathsf{T}}$.
- $p \ G \times E$ interaction terms: $\mathbf{S}_i = (E_i G_{i1}, \cdots, E_i G_{ip})^{\mathsf{T}}$.

We are interested in testing if there is any $G \times E$:

$$H_0: \boldsymbol{\beta} = \boldsymbol{0}.$$

Rare Variant by Environment Interactions

Collapsing tests

Intuition behind collapsing tests

- General problem: p is large and G_1, \dots, G_p are rare.
- Solution: for each individual *i*, summarize rare SNV-set (G_{i1}, · · · , G_{ip}) using a single summary variable and conduct inference using this single summary variable.
- ► For example, define "collapsing" variable as $G_i^* = \sum_{k=1}^{p} G_{ik}$ = Total No. of rare alleles.

Collapsing Tests for Interactions

To test for main effects:

$$H_{1m}: g(\mu_i) = \alpha_1^* + \alpha_2^* E_i + \alpha_3^* G_i^*$$
$$H_{0m}: \alpha_3^* = 0$$

Can we use it to test for interactions?

$$H_{1x}: g(\mu_i) = \alpha_1^* + \alpha_2^* E_i + \alpha_3^* G_i^* + \beta^* E_i G_i^*$$
$$H_{0x}: \beta^* = 0$$

Rare Variant by Environment Interactions

Bias analysis for Collapsing $G \times E$ tests

Intuition

Null model has to be correctly specified for valid inference. Collapsing $G \times E$ tests may not give valid inference as main effects of the SNVs may not be sufficiently accounted for.

Continuous Outcome: No, even if $\boldsymbol{G} \perp \boldsymbol{E}$.

- G and E are independent: Model for mean of Y is valid; Model for variance of Y is not valid.
- G and E not independent: Model for mean of Y is not valid; Model for variance of Y is not valid.

Bias analysis for Collapsing $G \times E$ tests

Binary Outcome: Yes if disease is rare and $\boldsymbol{G} \perp \boldsymbol{E}$.

- G and E are independent: Model for mean of Y is valid; Model for variance of Y is valid approximately.
- G and E not independent: Model for mean of Y is <u>not</u> valid; Model for variance of Y is valid approximately.

Rare Variant by Environment Interactions

iSKAT: Model

To test if there is any $G \times E$ ($H_0 : \beta = \mathbf{0}$):

$$H_0: \text{logit}\left[P(Y_i = 1 | E_i, \mathbf{X}_i, \mathbf{G}_i)\right] = \mathbf{X}_i^{\mathsf{T}} \boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2 E_i + \mathbf{G}_i^{\mathsf{T}} \boldsymbol{\alpha}_3$$

$$H_A : \text{logit} \left[P(Y_i = 1 | E_i, \mathbf{X}_i, \mathbf{G}_i) \right] = \mathbf{X}_i^{\mathsf{T}} \boldsymbol{\alpha}_1 + \mathbf{G}_i^{\mathsf{T}} \left(\boldsymbol{\alpha}_3 + E_i \boldsymbol{\beta} \right) + \alpha_2 E_i$$

In principle, we can do the same thing as with SKAT, but ...

Difficulties

Need to fit null model:

- Need to estimate main effect of variants
- Lots of variants
- LD and rarity make fitting difficult

Modifications are necessary.

iSKAT: Extension of SKAT for GxE

Rare Variant by Environment Interactions

iSKAT: Test Statistic

- Assume (β₁, · · · , β_p)^T are random and independent with mean zero and common variance τ.
- Testing H_0 reduces to testing H_0 : $\tau = 0$.
- Following Lin (1997), the score test statistic is

$$\mathcal{T} = (\mathbf{Y} - \widehat{\mu})^{\mathsf{T}} \, \mathbf{S} \mathbf{S}^{\mathsf{T}} \, (\mathbf{Y} - \widehat{\mu}) = [\mathbf{Y} - \mu\left(\widehat{lpha}
ight)]^{\mathsf{T}} \, \mathbf{S} \mathbf{S}^{\mathsf{T}} \left[\mathbf{Y} - \mu\left(\widehat{lpha}
ight)
ight].$$

• $\widehat{\mu} = \mu\left(\widehat{lpha}
ight)$ is estimated under the null model,

$$g(\mu_i | \boldsymbol{X}_i, \boldsymbol{E}_i, \boldsymbol{G}_i) = \boldsymbol{X}_i^{\mathsf{T}} \boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2 \boldsymbol{E}_i + \boldsymbol{G}_i^{\mathsf{T}} \boldsymbol{\alpha}_3 = \widetilde{\boldsymbol{X}}_i^{\mathsf{T}} \boldsymbol{\alpha}.$$

- Use ridge regression to estimate α , impose a penalty only on α_3 .
- Under H_0 , $T \sim \sum_{\nu=1}^{p} d_{\nu} \chi_1^2$ approximately.
- Invert characteristic function to get p-value (Davies, 1980).

Rare Variant by Environment Interactions

iSKAT

Consider a GLMM framework:

$$H_0: g(\mu_i) = \alpha_1 + \alpha_2 E_i + G_i^{\mathsf{T}} \alpha_3$$
$$H_1: g(\mu_i) = \alpha_1 + \alpha_2 E_i + G_i^{\mathsf{T}} \alpha_3 + \boxed{E_i G_i^{\mathsf{T}} \beta}$$

- Let β_j ~ F(0, w²_jτ) and let β have exchangeable correlation structure with pairwise correlation ρ.
- $\rho = 0$ and $\rho = 1$ correspond to H_{1b} and H_{1a} respectively.
- For a fixed ρ , a score test statistic for testing $H_0: \tau = 0$ is:

$$\begin{aligned} Q_{\rho} &= (\boldsymbol{Y} - \widehat{\boldsymbol{\mu}})^{\mathsf{T}} \, \boldsymbol{SW} \left[\rho \mathbf{1} \mathbf{1}^{\mathsf{T}} + (1 - \rho) \boldsymbol{I} \right] \, \boldsymbol{W} \boldsymbol{S}^{\mathsf{T}} \left(\boldsymbol{Y} - \widehat{\boldsymbol{\mu}} \right) \\ &= \boxed{\rho Q_{1a} + (1 - \rho) \, Q_{1b}}. \end{aligned}$$

Find optimal ρ to maximize Q_{ρ} : $Q_{iSKAT} = \min_{0 \le \rho \le 1} p_{\rho}$, where p_{ρ} is the p-value computed based on Q_{ρ} .

Rare Variant by Environment Interactions

Size Simulations: iSKAT vs. Collapsing $G \times E$ test

- Bootstrap dataset samples to obtain genotypes/covariates.
- Main effects of SNVs chosen to mimic dataset.
- Simulate outcome under null hypothesis to investigate the size.

Collapsing $G \times E$ test







Rare Variant by Environment Interactions

Data Application: Adiponectin Levels

- Adiponectin levels are associated with many diseases.
- SNVs at the adiponectin-encoding gene, ADIPOQ, are associated with adiponectin levels.
- Adiponectin levels from 1945 individuals.
- ▶ 11 rare SNVs within the exon region of ADIPOQ.
- Test for $G \times E$.

	p-value		
iSKAT:	0.037		
ho = 0:	0.23		
ho = 1:	0.022		



Rare variant Meta-analysis

- Meta-analysis is an effective approach to combine data from multiple studies.
- Rare variant meta-analysis: desirable properties
 - Use summary statistics
 - Same power as mega-analysis (joint analysis)
 - Account for varying levels of heterogeneity of genetic effects across studies.

Rare Variant Meta-Analysis References

- Lee S, Teslovich T, Boehnke M, and Lin X (2013) General Framework for Meta-analysis of Rare Variants in Sequencing Association Studies. AJHG 93, 42-53
- Hu Y et al.(2013) Meta-analysis of Gene-Level Associations for Rare Variants Based on Single-Variant Statistics. AJHG 93, 236-248
- Lumley T et al.(2013) Meta-analysis of a rare-variant association test manuscript, http://stattech.wordpress.fos.auckland.ac.nz/files/2012/11/skat-meta-paper.pdf
- Liu DJ, et al.(2014) Meta-analysis of gene-level tests for rare variant association. Nat Genet 46, 200-204

Multi-Study Model

For the
$$k^{th}$$
 study $(k = 1, \dots K)$,

- Genotype $\mathbf{G}_{ki} = (g_{ki1}, \dots, g_{kip})'$
- ► Covariates X_{ki} = (x_{ki1},..., x_{kiqk})'
- Model:

$$g(\mu_{ik}) = \mathbf{X}_{ki} \boldsymbol{\alpha}_k + \mathbf{G}_{ki} \boldsymbol{\beta}_k$$

• Test H₀:
$$\beta_k = 0$$
 ($k = 1, \cdots, K$)

Meta-Analysis for Rare Variants in Sequencing Association Studies

Challenges

- Jointly analyze multiple SNPs in a region.
- Hard to estimate β_k for rare variants
- Wald-based meta-analysis for vector β_k is challenging

Meta-Analysis for Rare Variants in Sequencing Association Studies

Idea of Meta-SKAT family tests:

- Work with the scores of β_k by fitting only null models.
- Assume a distribution for β_{kj} $(j = 1, \dots, p)$
- Perform variance component score test by allowing homogeneous and heterogeneous genetic effects across studies.

Single study k

Score statistic of variant j

$$S_{kj} = \sum_{i=1}^{n} g_{ijk} (y_{ij} - \widehat{\mu}_{ij}) / \widehat{\phi}_k$$

$$Q_{SKAT} = \sum_{j=1}^{p} (w_{jk}S_{kj})^2, \quad Q_{Burden} = \left(\sum_{j=1}^{p} w_{jk}S_{kj}\right)^2$$

Single study k

SKAT-O (combined approach):

$$T = min_{0 \le \rho \le 1} P_{\rho}$$

where P_{ρ} is the p-value of

$$Q_{
ho} = (1 -
ho) Q_{SKAT} +
ho Q_{Burden}$$

Input Summary Statistics for meta-analysis

Input summary statistics from each study

- MAF
- S_{kj} : score statistic of each marker
- Between-variant relationship matrix $(p \times p)$

$$\mathbf{\Phi}_k = \mathbf{G}'_k \mathbf{P}_k \mathbf{G}_k,$$

where
$$\mathbf{P}_k = \mathbf{V}_k^{-1} - \mathbf{V}_k^{-1} \mathbf{X}_k (\mathbf{X}_k' \mathbf{V}_k^{-1} \mathbf{X}_k)^{-1} \mathbf{X}_k' \mathbf{V}_k^{-1}$$

Homogeneous genetic effects

Assume the same SNP effects across studies.

$$\bullet \ \beta_1 = \beta_2 = \cdots = \beta_K = \beta$$

- $E(\beta_j) = 0$, $var(\beta_j) = w_j \tau$ and $cor(\beta_j, \beta_{j'}) = \rho$.
- Derive the VC score test for H_0 : $\tau = 0$.
- Multivariate score-based analog of univariate fixed effect meta-analysis

Meta-SKAT: Homogeneous genetic effects

Meta-SKAT assuming homogeneous genetic effects:

$$Q_{hom_meta_SKAT} = \sum_{j=1}^{p} \left(\sum_{k=1}^{K} w_{kj} S_{kj} \right)^2$$

Meta-Burden:

$$Q_{meta_Burden} = \left(\sum_{j=1}^{p}\sum_{k=1}^{K}w_{kj}S_{kj}
ight)^2$$

Meta-SKAT-O:

$$\mathcal{Q}_{ extsf{hom_meta}}(
ho) = (1-
ho) \mathcal{Q}_{ extsf{hom_meta_SKAT}} +
ho \mathcal{Q}_{ extsf{meta_Burden}}$$

Meta-SKAT: Homogeneous genetic effects

- ► Test statistics are essentially identical to those of the mega analysis SKAT and burden test
 ⇒ As powerful as mega-analysis
- P-values can be computed using the Davies method.
 ⇒ Fast computation
- SKAT-O can be conducted with adaptively selecting ρ .

Meta-SKAT: Heterogeneous genetic effects

- Assume genetic effects vary between studies
 - ▶ β_1, \cdots, β_K are iid
 - $E(\beta_{kj}) = 0$, $var(\beta_j) = w_{kj}\tau$ and $cor(\beta_{kj,kj'}) = \rho$.
- Multivariate score-based analog of the univariate random effect model meta-analysis.
- P-values can be calculated analytically
- Useful for meta analysis of studies of the same ethnicity or different ethnicities.

Meta-SKAT: Heterogeneous genetic effects

Meta-SKAT assuming heterogeneous genetic effects:

$$Q_{het_meta_SKAT} = \sum_{j=1}^{p} \sum_{k=1}^{K} (w_{kj}S_{kj})^2$$

Meta-SKAT-O:

 $Q_{hom_meta}(
ho) = (1 -
ho)Q_{het_meta_SKAT} +
ho Q_{meta_Burden}$

Meta-SKAT for multi-ethnicities:

- Multi-ethnic studies:
 - within-group homogeneity and between-group heterogeneity
 - $\beta_k = \beta_l$ for the same group and $\beta_k \perp \beta_l$ for the different groups
- Meta-SKAT with B ancestry groups

$$Q_{het_meta_SKAT} = \sum_{j=1}^{p} \sum_{b=1}^{B} \left(\sum_{k=k_{b-1}+1}^{k_{b}} w_{kj} S_{kj} \right)^{2}$$

Meta-SKAT-O:

$$Q_{hom_meta}(
ho) = (1-
ho) Q_{het_meta_SKAT} +
ho Q_{meta_Burden}$$

Simulation Studies

- 3kb randomly selected regions
- Three scenarios:
 - ► 1: homogeneous
 - 2: moderately heterogeneous (studies share 50 % of causal variants)
 - 3: two different ancestry groups (study 1,2 : EUR and study 3: AA)

Powers comparison: Meta vs Mega (Joint)



Simulation Results, $\beta + /- = 100/0$









Causal Percent





Analysis of the lipid data (LPL gene)

- ▶ 11,000 samples from 7 studies.
- Adjusted for the index SNP
- For HDL and TG, Het-Meta-SKAT-O achieved the smallest p-values

Traits	Hom-Meta	Het-Meta	Meta-	Meta-
	SKAT-O	SKAT-O	Fisher	Burden
HDL LDL TG	$\begin{array}{c} 2.5\times 10^{-2} \\ 1.00 \\ 5.3\times 10^{-3} \end{array}$	$\begin{array}{c} 1.2 \times 10^{-4} \\ 4.0 \times 10^{-1} \\ 2.8 \times 10^{-5} \end{array}$	$\begin{array}{c} 1.7\times 10^{-2}\\ 3.9\times 10^{-1}\\ 6.0\times 10^{-4} \end{array}$	$\begin{array}{c} 3.5\times 10^{-1} \\ 2.1\times 10^{-2} \\ 7.7\times 10^{-2} \end{array}$

Summary of Meta-Anlaysis for Rare Variants

- Based on study-specific summary score statistics
- As powerful as the joint analysis
- Flexible to accommodate a wide range of heterogeneity of genetic effects

Additional Concerns

- Quality control:
 - Are the observed variants really variants?
 - Batch effects
 - Some standard pipelines now in place
- Population stratification:
 - Common strategy: just use same PCs from common variant analysis to correct for PS
 - Some evidence that rare variants require special accommodation (much larger number of PCs)
- Accommodating common variants:
 - What do you do with common variants?
 - (a) Assess joint effect with rare variants
 - (b) Adjust for effect of common variants

Additional Concerns

- Prediction
 - In a new population (sample), we're unlikely to see the same variants and we're likely to see a lot of variants not previously observed
- Prioritization of individual variants
 - How to choose individual causal variants?
 - Some work on variable selection methods, but no ability to control type I error.
 - Bioinformatics and functionality tools may be useful
- Incorporation of functional information and other genomic data

Additional Concerns

- Design Choices
 - Want to enrich for variants (extreme phenotypes)
 - Some of these designs require specialized methods
 - Stuck with the design chosen
- Dealing with admixed populations
- Related individuals
- Tim: what is a "rare variant"?
- (Statistically) complex phenotypes