Identity by Descent, the Kinship Coefficient, and the Coefficient of Fraternity

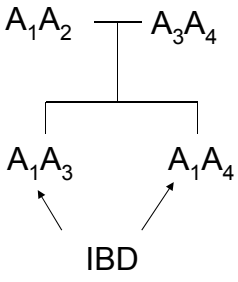
Summer Institute in Statistical Genetics 2014

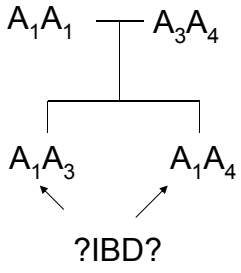
Module 10

Topic 6

Identical by Descent (IBD)

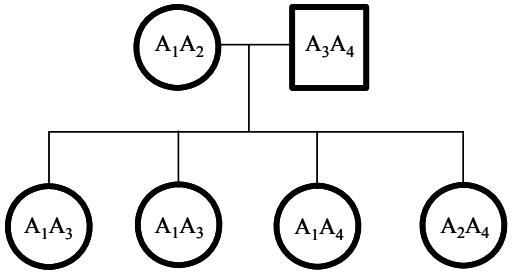
Two alleles are IBD if they are copies of the same ancestral gene. (The definition of "ancestor" is somewhat dependent on your pedigree.)





Exercise 1

For the following nuclear family, there are six pairs of siblings. For each pair, how many alleles are shared IBD?

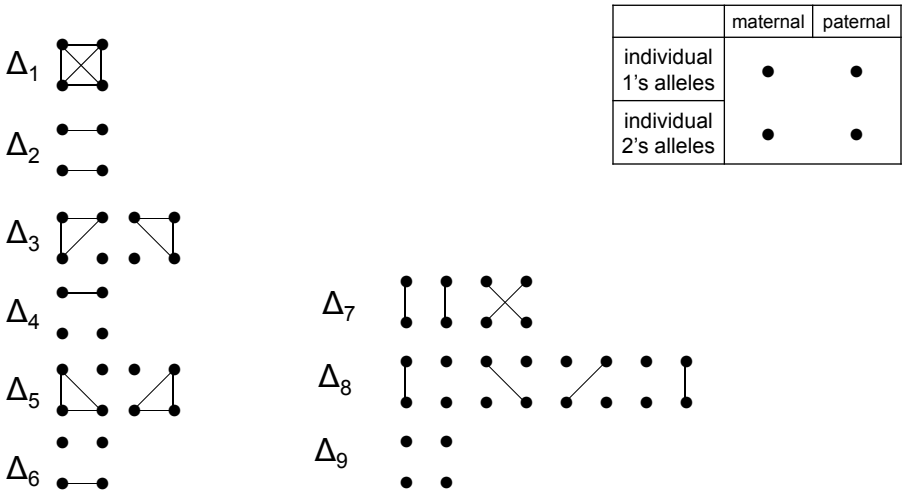


Consider a single locus in two individuals. There are four alleles. We can use a schematic to describe IBD status. Start with a grid of four nodes:

	maternal	paternal
individual 1's alleles	•	•
individual 2's alleles	•	•

Connect a pair of nodes with a line if the alleles are IBD. There are 15 possible configurations of IBD (next slide). We do not usually care about the distinction between maternal and paternal alleles, so the 15 possible configurations of IBD reduce to nine "identity states."

Part 6



For a given pair of individuals there are probabilities, Δ_1 through Δ_9 associated with each of the nine states. Δ_i is the probability the two individuals have IBD status in state i.

The first six states only occur if there is inbreeding. In large, random-mating populations the probability of the first six states is essentially 0. That is, Δ_1 through Δ_6 are 0. Δ_7 through Δ_9 are the probabilities of interest.

Note that Δ_7 can be characterized as the probability that two individuals share <u>both</u> of their alleles IBD. Δ_7 is also called the **coefficient of fraternity**.

θ_{ij} is the **kinship coefficient**, also called the **coefficient of coancestry**, and is the probability that for a given genetic locus a randomly selected allele from individual i and a randomly selected allele from individual j are identical by descent (IBD).

The coefficients of kinship and fraternity characterize the IBD sharing for a pair of relatives. The more distant two relatives are, the less likely it is that they will have alleles IBD. The table on the next slide summarizes these coefficients for a number of relationships.

Coefficients of kinship and fraternity for common relationships in a non-inbred population:

Relationship	θ	Δ_7
Parent-offspring		
Grandparent-grandchild	1/8	0
Great grandparent-great grandchild	1/16	0
Half sibs	1/8	0
Full sibs, dizygotic twins		
Uncle(aunt) – nephew(niece)	1/8	0
First cousins	1/16	0
Double first cousins	1/8	1/16
Second cousins	1/64	0
Monozygotic twins		

Exercise: Fill in the missing values in the table

- Monozygotic ("identical") twins

Δ_7 =P(share both alleles IBD)
θ=P(a randomly chosen allele from a locus is IBD)

For identical twins, Δ_7 =1 since they are genetically identical
θ=1/2

- Parent-offspring

Parent-offspring share exactly 1 allele IBD (assuming there is no inbreeding so parents do not share alleles IBD)

$\Delta_7 = 0$

$\theta = 1/2 \times 1/2 = 1/4$

- Full siblings

Without loss of generality, label the father's alleles A and B and label the mother's alleles C and D. Without loss of generality, say the first child is A C. Then we consider four cases for the second child:

Case 0: Second child is BD

Case 1: Second child is AD or BC

Case 2: Second child is AC

There are four equally probable possibilities for the second child, and for one of them there are two alleles IBD, so $\Delta_7 = ¼$.

To calculate θ, we want P(E), where E is the event that a randomly chosen allele from the locus for each individual is IBD. $\Theta = P(E)$

$= P(E|\text{Case 2})P(\text{Case 2}) + P(E|\text{Case 1})P(\text{Case 1}) + P(E|\text{Case 0})P(\text{Case 0})$

$= ½ \times ¼ + ½ \times ½ \times ½ + 0 \times ¼ = 1/8 + 1/8 = 1/4$

Inbreeding

- Two individuals that have IBD alleles are **related**.
- We defined θ = probability that an allele taken at random from one individual is IBD to an allele from the same locus taken at random from another individual (coancestry coefficient).
- The **inbreeding coefficient** of an individual is the coancestry of its parents.