# Estimating Relatedness in Homogenous Populations 

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Lecture 7: Part I

## Incomplete Genealogy

- Many statistical methods for genetic data, e.g. linkage and association methods, are based on assumptions of independent samples or samples with known relationships.



## Incomplete Genealogy

- Misspecified and cryptic relationships can invalidate many of these methods.



## Identifying Relative Pairs

- In principle, could determine the relationship between two individuals by simply looking at the percentage of IBD sharing in the genome for the two
- parent-offspring sharing: $50 \%$ of genome
- sibs: $50 \%$ of genome (on average)
- avuncular: $25 \%$ of genome (on average)
- However, we do not directly observe IBD sharing. We only observe DNA sequences.


## Genome Screen Data to Identify Relative Pairs

- It is now common to have genome screen data on hundreds of thousands of genetic markers.
- Genome screen data can be used to infer genealogical relationships.
- Example: Suppose we are interested in identifying the relationship between two individuals and assume for now that haplotype phase is known.
- Observed sequence on a chromosome from individual 1:
...TATACGTGCACCTGGATTACAGATTACAGATTACAGATTACATTGCATCGATCGAA...
- Observed sequence on a chromosome from from individual 2:
...GGATCCTGAACCTAGATTACAGATTACAGATTACAGATTACAATGCTTCGATGGAC...
- If haplotype phase is known, blocks of identical DNA sequences can be used to infer relationships.


## Genome Screen Data to Identify Relative Pairs

- Stanley F Nelson (UCLA Department of Human Genetics): IBD sharing between relatives: rapid drop in number of blocks yet size drops asymptotically:
- 1 st cousins: $\mathrm{n}=20-30$, average size $\sim 20-30 \mathrm{mb}$
- 2nd cousins: $n=5-8$, average size $\sim 20 \mathrm{mb}$
- 3rd cousins: $\mathrm{n}=1-3$, average size $\sim 18 \mathrm{mb}$
- 4th cousin: $\mathrm{n}=0-1$, average size $\sim 16 \mathrm{mb}$
- 5th cousins: $\mathrm{n}=0-1$, average size $\sim 14 \mathrm{mb}$
- 6th cousins: $\mathrm{n}=0-1$, average size $\sim 12 \mathrm{mb}$


## Hidden Markov Model for Identifying Relative Pairs

- McPeek and Sun (2000) developed approximate likelihood method to identify relative pairs for close relationships
- Stankovich et al. (2005) extended method for more distantly related pairs (degree 13: 6th cousin). Software is GBIRP
- Uses a 2-state Hidden Markov model for IBD status (yes/no) to approximate the likelihood
- Likelihood is a function of the distance between genetic markers, frequency of alleles between the markers, and relationship of individuals


## Hidden Markov Model for Identifying Relative Pairs

- Find pairwise relationship that maximizes the log likelihood ratio for the observed genome screen data $\left(g_{1}, g_{2}\right)$ over various types of relationships (up to 6th cousins)

$$
\log \frac{P\left(g_{1}, g_{2} \mid \text { related }\right)}{P\left(g_{1}, g_{2} \mid \text { unrelated }\right)}
$$

- High power to identify relationships up to degree eight (third cousins once removed)
- Typical error in degree for relationship $\leq$ eight is 1


## GBIRP Results for Known Relationships



## IBD Sharing Probabilities

- IBD sharing probabilities are another measure of relatedness for pairs of individuals
- For any pair of outbred individuals $i$ and $j$, let $\delta_{k}$ be the probability that $i$ and $j$ share $k$ alleles IBD at a locus where $k$ is 0,1 , or 2 .

IBD Sharing Probabilites for Outbreds

| Relationship | $\delta_{2}$ | $\delta_{1}$ | $\delta_{0}$ |
| :--- | :---: | :---: | :---: |
| Parent-Offspring | 0 | 1 | 0 |
| Full Siblings | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |
| Half Siblings | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| Uncle-Nephew | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| First Cousins | 0 | $\frac{1}{4}$ | $\frac{3}{4}$ |
| Double First Cousins | $\frac{1}{16}$ | $\frac{6}{16}$ | $\frac{9}{16}$ |
| Second Cousins | 0 | $\frac{1}{16}$ | $\frac{15}{16}$ |
| Unrelated | 0 | 0 | 1 |

## Estimating IBD Sharing Probabilities: EM Algorithm

- It is often not be possible to determine exactly how many alleles a pair share IBD.
- Can estimate IBD sharing probabiliting using genetic marker data across the genome.
- Choi, Wijsman, and Weir (2009) proposed using an EM algorithm to estimate the IBD probabilities for this problem.


## Estimating IBD Sharing Probabilities: EM Algorithm

- Suppose the data consists of $N$ genetic markers accross the genome
- Assume for now that at we observe IBD sharing at each marker for individuals $i$ and $j$ in the sample
- Let $X_{k}$ be the number of markers for which $i$ and $j$ share $k$ alleles IBD, and let let $\delta_{k}$ be the probability that $i$ and $j$ share $k$ alleles IBD at a merek where $k$ is 0 , 1 , or 2 ..
- If the IBD sharing process at the markers is observed, what would the likelihood function be?


## Estimating IBD Sharing Probabilities: EM Algorithm

- The likelihood function for the IBD sharing process would have the following multinomial distribution

$$
L\left(X_{0}, X_{1}, X_{2}\right)=\frac{N!}{X_{0}!X_{1}!X_{2}!} \delta_{0}^{X_{0}} \delta_{1}^{X_{1}} \delta_{2}^{X_{2}}
$$

where $X_{k}=\sum_{r=1}^{N} I\{i$ and $j$ share $k$ alleles IBD at marker $r\}$

- Could estimate the $\delta_{k}$ 's using the $X_{k}$ 's, which are the sufficient statistics: The MLE is $\hat{\delta}_{k}=\frac{X_{k}}{N}$ for $k=0,1,2$.
- The IBD process, however is not observed.
- What is the complete data and what is the observed data?


## Expectation Step of EM Algorithm

- The $X_{k}$ values are the unobserved complete data.
- The observed data is the genotype data for individuals $i$ and $j$ at the $N$ markers, and the $X_{k}$ values are the missing data
- The E step of the EM algorithm calculates the expected value of $X_{k}$ conditioned on the observed genotype data.
- Remember that initial values for the $\delta_{k}$ 's need to be given for the EM algorithm.
- Let $\delta^{0}=\left(\delta_{0}^{0}, \delta_{1}^{0}, \delta_{2}^{0}\right)$ be the initial values.
- Let $\mathbf{G}=\left(G_{1}, \ldots G_{r}, \ldots G_{N}\right)$, where $G_{r}=\left(G_{i r}, G_{j_{r}}\right)$ is the genotype data at marker $r$ for $i$ and $j$.


## Expectation Step of EM Algorithm

- $X_{2}=\sum_{r=1}^{N} I\{i$ and $j$ share 2 alleles IBD at marker $r\}$
- $E\left[X_{2} \mid \mathbf{G}, \delta^{0}\right]=$

$$
\begin{aligned}
& \sum_{r=1}^{N} E\left[I\{i \text { and } j \text { share } 2 \text { alleles IBD at marker } r\} \mid \mathbf{G}, \delta^{0}\right] \\
= & \sum_{r=1}^{N} E\left[I\{i \text { and } j \text { share } 2 \text { alleles IBD at marker } r\} \mid G_{r}, \delta^{0}\right] \\
= & \sum_{r=1}^{N} P\left(i \text { and } j \text { share } 2 \text { alleles IBD at marker } r \mid G_{r}, \delta^{0}\right) \\
= & \sum_{r=1}^{N} \frac{P\left(i \text { and } j \text { share } 2 \text { alleles IBD at marker } r, G_{r} \mid \delta^{0}\right)}{P\left(G_{r} \mid \delta^{0}\right)}
\end{aligned}
$$

## Expectation Step of EM Algorithm

- The numerator of the summand is $P\left(i\right.$ and $j$ share 2 alleles IBD at marker $\left.r, G_{r} \mid \delta^{0}\right)$
$=P\left(G_{r} \mid i\right.$ and $j$ share 2 alleles IBD at marker $\left.r, \delta^{0}\right) \times$ $P\left(i\right.$ and $j$ share 2 alleles IBD at marker $\left.r \mid \delta^{0}\right)$
$=P\left(G_{r} \mid i\right.$ and $j$ share 2 alleles IBD at marker $\left.r, \delta^{0}\right) \delta_{2}^{0}$
- $P\left(G_{r} \mid i\right.$ and $j$ share 2 alleles IBD at marker $\left.r\right)$ will be based on the population allele frequency distribution at marker $r$.


## Expectation Step of EM Algorithm

- For simplicity, assume that marker $r$ is a SNP with the 2 allelic types labeled " 0 " and " 1 ""
- Let $p_{r}$ be the frequency of allelic type 1 in the population at marker $k$, where $0<p_{r}<1$.
- If the genotype of $i$ is $(1,1)$ and the genotype of $j$ is $(1,1)$ at marker $r$, then
$P\left(G_{r} \mid i\right.$ and $j$ share 2 alleles IBD at marker $r$ ) $=p_{r}^{2}$ (if HWE is assumed).
- What is the probability if the genotype of $i$ is $(1,2)$ and the genotype of $j$ is $(2,2)$ at marker $r$ ?
- What is the probability if the genotype of $i$ is $(1,2)$ and the genotype of $j$ is $(1,2)$ at marker $r$ ?


## Expectation Step of EM Algorithm

- From these probabilities, we can obtain $E\left[X_{2} \mid \mathbf{G}, \delta^{0}\right]=$

$$
\sum_{r=1}^{N} \frac{P\left(i \text { and } j \text { share } 2 \text { alleles IBD at marker } r, G_{r} \mid \delta^{0}\right)}{P\left(G_{r} \mid \delta^{0}\right)}
$$

- Can similarly obtain $E\left[X_{1} \mid \mathbf{G}, \delta^{0}\right]$ and $E\left[X_{0} \mid \mathbf{G}, \delta^{0}\right]$, where

$$
X_{1}=\sum_{r=1}^{N} I\{i \text { and } j \text { share } 1 \text { alleles IBD at marker } r\}
$$

and

$$
X_{0}=\sum_{r=1}^{N} I\{i \text { and } j \text { share } 0 \text { alleles IBD at marker } r\}
$$

## Maximization Step of EM Algorithm

- The M step involves maximizing the expected value of the log-likelihood (obtained in the E step) with respect to the $\delta_{k}$ parameters.
- The MLE is:
- $\hat{\delta}_{0}=\frac{E\left[X_{0} \mid \mathbf{G}, \delta^{0}\right]}{E\left[X_{0} \mid \mathbf{G}, \delta^{0}\right]+E\left[X_{1} \mid \mathbf{G}, \delta^{0}\right]+E\left[X_{2} \mid \mathbf{G}, \delta^{0}\right]}$
- $\hat{\delta}_{1}=\frac{E\left[X_{1} \mid \mathbf{G}, \delta^{0}\right]}{E\left[X_{0} \mid \mathbf{G}, \delta^{0}\right]+E\left[X_{1} \mid \mathbf{G}, \delta^{0}\right]+E\left[X_{2} \mid \mathbf{G}, \delta^{0}\right]}$
- $\hat{\delta}_{2}=\frac{E\left[X_{2} \mid \mathbf{G}, \delta^{0}\right]}{E\left[X_{0} \mid \mathbf{G}, \delta^{0}\right]+E\left[X_{1} \mid \mathbf{G}, \delta^{0}\right]+E\left[X_{2} \mid \mathbf{G}, \delta^{0}\right]}$
- The next step is to set $\delta^{1}=\hat{\delta}$ and then return to the E step of the algorithm.
- Continue iterating between the E and M step until the $\hat{\delta}^{i}$ values converge.


## Estimating IBD Sharing Probabilities: Method of Moments

- Purcell et al. (2007) proposed a method of moments estimator for IBD sharing probabilities
- Estimate IBD sharing probabilities based on IBS sharing for pairs of individuals
- Implements the IBD sharing method of moments estimator in their software package PLINK


## Estimating Kinship Coefficients

- Kinship coefficients can also be used to quantify relationships between two individuals.

| Table: Kinship Coefficients |  |
| :--- | :---: |
| Relationship | $\phi$ |
| Parent-Offspring | $1 / 4$ |
| Full Siblings | $1 / 4$ |
| Half Siblings | $1 / 8$ |
| Uncle-nephew | $1 / 8$ |
| First Cousins | $1 / 16$ |
| Double First Cousins | $1 / 8$ |
| Second Cousins | $1 / 64$ |
| unrelated | 0 |

- Note that $\phi=\frac{1}{2} \delta_{2}+\frac{1}{4} \delta_{1}$


## Estimating Kinship Coefficients

- Thornton and McPeek (2010) propose a method to estimate kinship coefficients using genetic marker data
- Consider once again a marker $r$ with 2 allelic types labeled " 0 " and " 1 "
- Let $p_{r}$ be the frequency of allelic type 1 , where $0<p_{r}<1$.
- Consider two individuals $i$ and $j$. For individual $i$, let $Y_{i_{r}}=\frac{1}{2}$ $\times$ (the number of alleles of type 1 in individual $i$ at marker $r$ ). So the value of $Y_{i_{r}}$ is $0, \frac{1}{2}$, or 1 . Similarly define $Y_{j_{r}}$ for individual $j$.
- It can be shown that $\operatorname{Cov}\left(Y_{i_{r}}, Y_{j_{r}}\right)=p_{r}\left(1-p_{r}\right) \phi_{i j}$, where $\phi_{i j}$ is the kinship coefficient for $i$ and $j$.
- Rearrange terms to see that $\phi_{i j}=\frac{\operatorname{Cov}\left(Y_{i r}, Y_{j r}\right)}{p_{r}\left(1-p_{r}\right)}$


## Estimating Kinship Coefficients

- This relationship will hold for markers across the genome (with the allele frequency distribution changing for each marker).
- Can use data across the genome to estimate kinship coefficients for pairs of individuals
- Let $N$ be the total number of markers in the data.
- For any pair of individuals $i$ and $j$, can estimate $\phi_{i j}$ with

$$
\hat{\phi}_{i j}=\frac{1}{N} \sum_{r=1}^{N} \frac{\left(Y_{i_{r}}-\hat{p}_{r}\right)\left(Y_{j_{r}}-\hat{p}_{r}\right)}{\hat{p}_{r}\left(1-\hat{p}_{r}\right)}
$$

where $\hat{p}_{r}$ is an allele frequency estimate for the type 1 allele at marker $r$

## Estimating Kinships Using GAW 14 COGA Data

- The Collaborative Study of the Genetics of Alcoholism (COGA) provided genome screen data for locating regions on the genome that influence susceptibility to alcoholism.
- There were a total of 1,009 individuals from 143 pedigrees with each pedigree containing at least 3 affected individuals. Individuals labeled as "white, non-Hispanic" were considered.
- 10K SNP array (10,081 SNPs) on 22 autosomal chromosomes
- Estimated kinship coefficients using genome-screen data


## Estimating Kinships Using COGA Data



Hist w/ True Kinship $=.0625$


Hist w/ True Kinship $=\mathbf{0 1 5 6 2 5}$


Hist w/ True Kinship $=.03125$


Hist w/ True Kinship $=0$


## Estimating Kinships Using COGA Data

- From the given pedigrees, two pairs of individuals that should have a kinship coefficient of .25 appear to be unrelated (estimated kinship coefficients of -0.006 and -0.003 , respectively)
- Two pairs of individuals that should have a kinship coefficient of . 125 appear to be unrelated (estimated kinship coefficients of -0.003 and 0.002 , respectively)
- 9 pairs of "unrelated" individuals have a kinship coefficient around .125
- 2 pairs of "unrelated" individual have a kinship coefficient around . 25


## References

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# Estimating Relatedness in Populations with Admixed Ancestry 

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Summer Institute in Statistical Genetics 2014 Module 10
Lecture 7: Part II

## Relatedness Inference in Structured Populations

- Popular algorithms for relationship inference are based on a strong assumption of population homogeneity
- This assumption is often untenable. GWAS often have cryptic population structure (or ancestry differences among the sample individuals)
- In samples with population structure, relationship estimation methods that assume homogeneity can give extremely biased results
- The degree of relatedness among related and unrelated sample individuals with similar ancestry are systematically inflated


## Structured Populations with Distinct Ancestral Subpopulations

- Manichaikul A et al. (2010) propose an estimator, KING-robust, which stands for Kinship-based INference for Genome-wide association studies
- Estimates kinship coefficients in for individuals from ancestrally distinct subpopulations
- KING-robust estimates kinship coefficients for a pair of individuals by using the shared genotype counts as a measure of the genetic distance between the pair.
- Method does not require allele frequency estimates at the marker: is based on allele sharing counts for individuals
- Gives biased kinship estimates for individuals with different ancestry


## Admixed Populations

- Genetic models used to identify related individuals from large scale genetic data often make simplifying assumptions about population structure - either random mating or simple structures.
- In reality, human populations do not mate at random nor are there simple endogamous subgroups.
- While GWAS have primarily examined populations of European ancestry, more recent studies involve admixed populations.
- A number of populations, including the two largest minority populations in the United States, Hispanics and African Americans, are known to have ancestral admixture of chromosomes from different continents.


## Ancestry Admixture

- Consider two admixed parents, where each are admixed from different ancestral populations.
- In the picture below, positions on the chromosomes that are the same color are from the same ancestral population.



## Relatedness Inference in Admixed Samples

- Thornton et al. (2012) proposed REAP (Relatedness Estimation in Admixed Populations) for relatedness inference in samples from populations with admixed ancestry
- Consider the problem of estimating relatedness in a set $N$ of outbred individuals who are sampled from a population with admixture from $K$ subpopulations
- Let $\mathbf{q}^{s}=\left(q_{1}^{s}, \ldots, q_{K}^{s}\right)^{T}$ denote the vector of subpopulation-specific allele frequencies at SNP $s$, where $q_{k}^{s}$ is the allele frequency of SNP $s$ in subpopulation $k, 1 \leq k \leq K$.
- Define $\mathbf{a}_{i}=\left(a_{i 1}, \ldots, a_{i K}\right)^{T}$ to be the genome-wide ancestry vector for $i \in N$, where $a_{i k}$ is the proportion of ancestry from subpopulation $k$ for $i, a_{i k} \geq 0$ for all $k$, and $\sum_{k=1}^{K} a_{i k}=1$.


## Estimating Relatedness in an Admixed Population

- Let $Y_{i}^{s}$ be the genotype variable for individual $i$, where $Y_{i}^{s}=\frac{1}{2} \times$ (the number of alleles of type 1 at SNP $s$ in individual $i$ ). Similarly define $Y_{j}^{s}$ for individual $j$.
- Conditional on $\mathbf{q}^{s}$, we assume alleles of an outbred individual $i$ are independent, identically-distributed (i.i.d.) Bernoulli random variables, a modeling assumption made by other commonly-used models of population structure (Balding-Nichols model with admixture).
- We denote $\mu_{i}^{s}=E\left[Y_{i}^{s} \mid \mathbf{a}_{i}, \mathbf{q}^{s}\right]$ to be the expected value of $Y_{i}^{s}$ conditional on $\mathbf{q}^{5}$ and $\mathbf{a}_{i}$ where

$$
\mu_{i}^{s}=\mathbf{a}_{i}^{T} \mathbf{q}^{s}=\sum_{k=1}^{K} a_{i k} q_{k}^{s},
$$

- The variance of $Y_{i}^{s}$ conditional on $\mathbf{q}^{s}$ and $\mathbf{a}_{i}$ is $5 \mu_{i}^{s}\left(1-\mu_{i}^{s}\right)_{\overline{\bar{E}}}$


## Estimating Kinship Coefficients: Admixed Population

- For $i$ and $j$ from a homogenous populations, it can be shown that $\phi_{i j}=\frac{1}{2} \rho_{Y_{i} Y_{j}}$ for $i$ and $j$, where $\rho_{Y_{i} Y_{j}}$ is the correlation of $Y_{i}^{s}$ and $Y_{j}^{s}$.
- For estimating $\phi_{i j}$ in structured populations with admixture, we propose to similarly calculate the correlation of $Y_{i}^{s}$ and $Y_{j}^{s}$
- Propose using a correlation that is calculated conditional on the admixture ancestry proportions of $i$ and $j$ as well as the subpopulation allele frequencies.


## Estimating Kinship Coefficients: Admixed Population

- The conditional correlation that we estimate for inference on $\phi_{i j}$ is $\rho_{Y_{i} Y_{j} \mid \mathbf{a}_{i}, \mathbf{a}_{j}, \mathbf{q}^{s}}$, which is the correlation of $Y_{i}^{s}$ and $Y_{j}^{s}$ conditional on $\mathbf{a}_{i}, \mathbf{a}_{j}$, and $\mathbf{q}^{s}$.
- When genome-screen data is available for $i$ and $j$ we estimate $\phi_{i j}$ in the presence of population structure with admixture with the REAP estimator

$$
\hat{\phi}_{i j}^{A}=\frac{1}{2} \hat{\rho}_{Y_{i} Y_{j} \mid \mathbf{a}_{i}, \mathbf{a}_{j}, \mathbf{q}^{s}}
$$

where

$$
\hat{\rho}_{Y_{i} Y_{j} \mid \mathbf{a}_{i}, \mathbf{a}_{j}, \mathbf{q}^{s}}=\frac{1}{\left|\mathcal{S}_{i j}\right|} \sum_{s \in \mathcal{S}_{i j}} \frac{\left(Y_{i}^{s}-\hat{\mu}_{i}^{s}\right)\left(Y_{j}^{s}-\hat{\mu}_{j}^{s}\right)}{\sqrt{.5 \hat{\mu}_{i}^{s}\left(1-\hat{\mu}_{i}^{s}\right)} \sqrt{.5 \hat{\mu}_{j}^{s}\left(1-\hat{\mu}_{j}^{s}\right)}}
$$

## Estimating IBD Sharing Probabilities: Admixed Populations

- Can also extend estimating IBD sharing probabilities in admixed populations.
- Define $Z_{i j}^{s}$ as before to be an indicator for $i$ and $j$ sharing 0 alleles IBD at SNP s
- Can use the conditional expectation of $Z_{i j}^{s}$ given $\mathbf{a}_{i}, \mathbf{a}_{j}, \mathbf{q}^{s}$ to obtain a method of moments estimator for $\delta_{i j}^{0}$ in the the presence of admixture.
- For any pair of individuals $i$ and $j$ from an admixed population, we have that

$$
E\left(Z_{i j}^{s} \mid \mathbf{a}_{i}, \mathbf{a}_{j}, \mathbf{q}^{s}\right)=\left[\left(\mu_{i}^{s}\right)^{2}\left(1-\mu_{j}^{s}\right)^{2}+\left(1-\mu_{i}^{s}\right)^{2}\left(\mu_{j}^{s}\right)^{2}\right] \delta_{i j}^{0}
$$

## Estimating IBD Sharing Probabilities: Admixed Populations

- Let $\mathcal{S}_{i j}$ be the set of markers in the genome screen for which both $i$ and $j$ have nonmissing genotype data.
- Our REAP method of moments for $\delta_{i j}^{0}$ in the presence of admixture is

$$
\hat{\delta}_{i j}^{0^{A}}=\frac{\sum_{s \in \mathcal{S}_{i j}} Z_{i j}^{s}}{\sum_{s \in \mathcal{S}_{i j}}\left[\left(\hat{\mu}_{i}^{s}\right)^{2}\left(1-\hat{\mu}_{j}^{s}\right)^{2}+\left(1-\hat{\mu}_{i}^{s}\right)^{2}\left(\hat{\mu}_{i}^{s}\right)^{2}\right]}
$$

## Estimating IBD Sharing Probabilities: Admixed Populations

- The remaining two IBD sharing probabilities, $\delta_{i j}^{1}$ and $\delta_{i j}^{2}$, can be written as a function of $\delta_{i j}^{0}$ and $\phi_{i j}$
- Estimate $\delta_{i j}^{1^{A}}$ with $\hat{\delta}_{i j}^{1^{A}}=2-2 \hat{\delta}_{i j}^{0^{A}}-4 \hat{\phi}_{i j}^{A}$
- Estimate $\delta_{i j}^{2^{A}}$ with $\hat{\delta}_{i j}^{2^{A}}=\hat{\delta}_{i j}^{0^{A}}+4 \hat{\phi}_{i j}^{A}-1$.


## Simulation Studies: Relatedness and Population Structure

- Perform simulation studies, in which population structure and related individuals are simultaneously present
- The population structure settings used in the simulation studies are based on the Balding-Nichols model.
- For each SNP, an ancestral population allele frequency $p$ was drawn from the uniform distribution on $[0.1,0.9]$.
- We set $F_{S T}=.2$ in the Balding-Nichols model to simulate two highly divergent subpopulations.


## Simulation Studies: Relatedness and Population Structure

- We consider population structure settings where individuals from an admixed population formed from two divergent subpopulations.
- Population structure setting 1 has individuals sampled from an admixed population formed from ancestral populations and where there is assortative mating.
- Population structure setting 2 has individuals sampled from an admixed population formed from ancestral populations where there is random mating
- We sample 400 individuals from 20 outbred pedigrees containing 1st, 2nd, 3rd, and 4th-degree relationships.


## Pedigree Configuration



## Simulation Studies: Relatedness and Population Structure

- For each of the two population structure settings we generate genotype data for 10,000 random SNPs.
- Genome-wide ancestry estimates used by REAP for the sample individuals were obtained by the frappe software program
- frappe implements an EM algorithm for simultaneously inferring each individuals ancestry proportion and allele frequencies in the ancestral populations.


## Setting 1: Admixture from Two Ancestral Populations and Assortative Mating

Admixture from Two Ancestral Populations: REAP Estimators


Admixture from Two Ancestral Populations : Homogenous Estimators


## Setting 2: Admixture from Three Ancestral Populations and Random Mating

Admixture from Three Ancestral Populations: REAP Estimators


Admixture from Three Ancestral Populations: Homogenous Estimators


## Estimating Kinship: HapMap Mex Sample

- Estimate estimating kinship coefficients and IBD sharing probabilities in the HapMap Mexicans in Los Angeles (MXL) sample of release 3 of phase III..
- Used frappe to estimate genome-wide ancestry for the 86 individuals in the sample
- We set the number of ancestral populations $K=3$
- HapMaP YRI for African ancestry
- HapMap CEU samples for northern and western European ancestry
- HGDP Native American samples for Native American ancestry.

HapMap MXL Estimated Ancestry


## HapMap MXL: REAP Estimators



## Reconstructed HapMap MXL Extended Pedigree



## Women's Health Initiative

- The Womens Health Initiative (WHI) is a national health study focusing on strategies for preventing chronic diseases in postmenopausal women.
- A total of 161,808 women aged $50-79$ yrs. old were recruited from 40 clinical centers in the US between 1993 and 1998.
- The WHI cohort included
- Two clinical trials of postmenopausal hormone therapy (estrogen alone and estrogen plus progestin)
- A clinical trial of calcium and vitamin D supplements, and a dietary modification trial.


## Genetic analysis of WHI-SHARe Minority Cohort

- Minority populations have largely been underrepresented in genetic studies despite bearing a disproportionately high burden for disease.
- WHI study opens up tremendous new possibilities for the identification of genetic risk factors associated with a number of clinical outcomes in the two largest minority populations in the U.S.
- The WHI SNP Health Association Resource (SHARe) minority cohort includes 8421 self-identified African American women from and 3587 self-identified Hispanic women
- 909,622 single nucleotide polymorphisms (SNPs) across the genome


## Ancestry Estimation: WHI-SHARe data

- Used frappe to estimate genome-wide ancestry of every individual in the sample
- We set the number of ancestral populations $K=4$
- HapMaP YRI for African ancestry
- HapMap CEU samples for northern and western European ancestry
- HGDP Native American samples for Native American ancestry.
- HGDP East Asian samples for East Asian Ancestry


## Relatedness Inference in WHI-SHARe

- No available genealogical information for the WHI-SHARe sample
- Used REAP to estimated relationships for all possible pairs:

$$
\binom{12008}{2}=7,209,028
$$

- Obtained estimates for kinship coefficients and IBD sharing probabilities


## WHI-SHARe African Americans

WHI-SHARe African Americans: Close Relatives


## WHI-SHARe Hispanics

WHI-SHARe Hispanics: Close Relatives


## Relatedness Inference in WHI-SHARe

- Also used the PLINK software (Purcell et al., 2007) method of moments kinship coefficient estimator: 8,932 pairs are identified to be either first or second degree relatives
- Our REAP kinship estimator that adjusts for individual specific ancestry identifies 344 individuals with kinship coefficients that are consistent with either first or second degree relatives


## Relatedness Inference in WHI-SHARe

- Interestingly, there are individuals who are identified as second- and third-degree relative pairs by REAP but who have a different self-reported race/ethnicity, e.g. one individual is a self-report African American and the other is a self-report Hispanic.
- An advantage of the REAP approach is that robust relatedness estimates can be obtained for all individuals, even for individuals who have different admixed ancestry distributions and self-identify in different ethnic or nationality groups.


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