

BIOST 551/STAT 551: Autumn Quarter 2014

Homework 2

Due Thursday, October 23, 2014

Show All Work

1. [60 points] Consider a quantitative trait influenced by a single bi-allelic locus with alleles A_1 and A_2 . Let p and q be the allele frequencies of the A_1 and A_2 alleles, respectively.
 - (a) We showed in lecture 3 that the additive variance component $\sigma_A^2 = 2pq\alpha^2 = 2pq(\alpha_1 - \alpha_2)^2$, where α_1 and α_2 are the additive effects for alleles A_1 and A_2 , respectively, and α is the average effect of allele substitution. In lecture 5, the general formula for the additive variance of a multi-allelic locus was given as $\sigma_A^2 = 2 \sum_i p_i \alpha_i^2$. Show that the additive variance component for the bi-allelic locus can also be written in this general form, i.e., show that $2pq(\alpha_1 - \alpha_2)^2$ can be written as $2(p\alpha_1^2 + q\alpha_2^2)$.
 - (b) The breeding values for genotypes A_1A_1 , A_1A_2 , and A_2A_2 are defined to be $2\alpha_1$, $\alpha_1 + \alpha_2$, and $2\alpha_2$, respectively. We previously derived the expressions for the dominance effects of the three genotypes for a single bi-allelic locus in lecture 3. Show that the breeding values and the dominance effects at a bi-allelic locus have a covariance of 0.
2. [40 points] A quantitative trait exhibits over-dominance at a single bi-allelic locus with alleles A_1 and A_2 . The population is in Hardy-Weinberg equilibrium (HWE) and the three genotypes at the locus yield the following genotypic values in the population:

Genotype	Genotypic Value
A_2A_2	0.7
A_1A_2	1.6
A_1A_1	1.2

- (a) What allele frequencies for A_1 and A_2 would give the population its maximum mean trait value? What is this maximum mean trait value?
- (b) What allele frequencies for A_1 and A_2 would give the population its minimum mean trait value? What is this minimum mean trait value?
- (c) Calculate the additive effects, breeding values, and the additive variance component at the locus for the trait when the population is at its maximum mean trait value.
- (d) Calculate the dominance effects and the dominance variance component at the locus for the trait when the population is at its maximum mean trait value.