Stat 311 Midterm
4 May 2007

Name:_____**KEY**________
Section:___________________

This is a closed book exam, except for the use of the normal table at the back of your book. You are allowed, however, to have one sheet (double-sided) of 8.5 x 11 paper with notes, either handwritten or typed. You may also use a calculator, although be sure to show your work. The exam consists of seven problems worth a total of 100 points. Point values for each part of a question are designated in parentheses at the beginning of the problem.

Be sure to show your work as indicated in order to receive credit.

Problem 1 (5+5+5+5 points): Consider samples A and B given below. Notice that the two samples are the same except for two values.

A:	20	60	60	70	90
B:	20	30	70	90	90

Without calculating actual values for the statistics (except perhaps for the median), explain what effect changing the two 60s to 30 and 90 will have on each of the following statistics. Your answers should be along the lines of "the mean in sample B will ____ because …" Note that the mean of sample A is 60.

a) mean

The mean in sample B remains the same because the sum of 60 + 60 = 120 was replaced with 90 + 30 = 120.

b) median

The median in sample B increases from 60 to 70 because 70 is now the middle number.

c) range

The range in sample B remains the same because the two samples have the same min and max.

d) standard deviation

The standard deviation in sample B increases because sample A has two values at the average and sample B does not have any values at the average.

Problem 2 (5+5+5 points): The survival rate during a risky operation for patients with no hope of survival is 80%. Define X to be the number of survivors in five operations.

a) What is the exact probability that four of the next five patients survive this operation?

$$P(X=4)=\binom{5}{4}(.8)^4(.2)=5(.4096)(.2)=.4096$$

b) What is the mean and standard deviation for the number of survivors among the next five patients?

$$\mu = np = 5(.8) = 4$$

$$\sigma = \sqrt{npq} = \sqrt{4(.2)} = \sqrt{.8} = 0.894$$

c) Assuming that the sample size is sufficiently large, use normal approximation with continuity correction to estimate the probability that at least four of the next five patients survive this operation.

$$P(X \geq 4) \approx P\left(Z \geq \frac{3.5-4}{0.894}\right) = P(Z \geq -0.56)$$

$$= 1 - P(Z < -0.56)$$

$$= 1 - 0.2877 = .7123$$

Problem 4 (5+5+5 points): A nationally administered test has a mean of 500 and a SD of 100.

a) If your standard score on this test was 1.8, what was your test score?

$$z = \frac{x-\mu}{\sigma} \Rightarrow 1.8 = \frac{x-500}{100} \Rightarrow 180 = x - 500 \Rightarrow x = 680$$

b) What is the probability that an individual, randomly selected from everyone who took the test, has a score less than 380 or greater than 680, assuming that the population of test scores is normally distributed?

Want $P(X < 380) + P(X > 680)$. This is the same as:

$$\begin{aligned} P\left(Z < \frac{380-500}{100}\right) + P(Z > 1.8) &= P(Z < -1.2) + P(Z > 1.8) \\ &= P(Z < -1.2) + \left[1 - P(Z < 1.8)\right] \\ &= 0.1157 + (1 - 0.9641) = 0.1157 + 0.0359 = 0.1516 \end{aligned}$$

c) What is the probability that the randomly selected individual has a score of 525?

$P(X = 525) = 0$

Problem 5 (6 points): The lengths of time, x, spent on a daily one-way commute to college by students are believed to have a mean of 22 minutes with a standard deviation of 9 minutes. If the lengths of time spent commuting are approximately normally distributed, find the time, x, that separates the 25% who spend the least time commuting from the rest of the commuters.

25% who spend the least time commuting translates to the 25^{th} percentile. This corresponds to z = -0.67.

$$-0.67 = \frac{x-22}{9} \Rightarrow -6.03 = x - 22 \Rightarrow x = 15.97 \text{ minutes}$$

Problem 6 (3+3+4+4+4 points): A computer is programmed to generate the eight single-digit integers 1, 2, 3, 4, 5, 6, 7, and 8 with equal frequency. Consider the experiment "the next integer generated" and the events

A: odd number, {1, 3, 5, 7} B: number > 4, {5, 6, 7, 8} C: 1 or 2, {1, 2}

Find the following. ***Be sure to show your work!***

a) $P\left(C^c\right)$.

$P\left(C^c\right) = 1 - P\left(C\right) = 1 - .25 = .75$

b) $P\left(A \cap B\right)$.

$P\left(A \cap B\right) = P\left(5 \text{ or } 7\right) = .25$

c) $P\left(A|B\right)$.

$$P\left(A|B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)} = \frac{.25}{.5} = .5$$

d) Are events A and B mutually exclusive? ***Explain.***

Events A and B are not mutually exclusive because their intersection is not empty;
i.e. $P\left(A \cap B\right) = P\left(5 \text{ or } 7\right) = .25$

e) Are events A and B independent? ***Explain.***

Events A and B are independent since $P\left(A|B\right) = 0.5 = P\left(A\right)$ or $P\left(A \cap B\right) = .25 = P\left(A\right)P\left(B\right)$

Problem 7 (6 points): Box 1 contains 2 red marbles and 3 green marbles. Box 2 contains 4 red marbles and 1 green marble. One marble is randomly selected from Box 1 and placed in Box 2. Then one marble is randomly selected from Box 2. What is the probability that the ball selected from Box 2 is green?

P(G from Box 2) = P(G from Box 1) P(G from Box2) + P(R from Box 1)P(G from Box 2)
= (3/5)(2/6) + (2/5)(1/6)
= 6/30 + 2/30 = 8/30 = 0.267