Stat 311 Midterm   Name: KEY
7 May 2008   Section:_____

This is a closed book exam. You are allowed, however, to use the formula card that came with the textbook and to have one sheet (double-sided) of 8.5 x 11 paper with notes, either handwritten or typed. You may also use a calculator, although be sure to show your work. The exam consists of five problems worth a total of 95 points.   Point values for each part of a question are designated in parentheses at the beginning of the problem.

Be sure to show your work as indicated in order to receive credit.

Problem 1 (15 points): According to a web site that offers bee pollen for sale, "Bee pollen is effective for combating fatigue, depression, cancer, and colon disorders." Does bee pollen really prevent colon disorders? Outlined below are two possible ways one might study this question:

1. Recruit 500 women who do not have colon disorders. Randomly assign 250 to take bee pollen capsules and have the other 250 take placebo capsules that are identical in appearance. Follow both groups for 5 years.
2. Recruit 250 women who take bee pollen regularly. Match each with a woman of the same age, race, and occupation who does not take bee pollen. Follow both groups for 5 years.

a) Is study method 2) listed above an observational study or an experiment? Briefly explain. (5 points)

Method 2) is an observational study; there is no randomization of groups and no treatment applied to a group. Individuals are recruited into predefined groups and are then observed over the 5 years.

b) What is the point of the "placebo" that is referenced in study method 1)? (5 points)

The placebo is a control for a "pill affect"—to prevent confounding due to potential psychological affects due to swallowing a pill

c) Which of the two designs is likely to produce more useful data? Briefly explain. (5 points)

Since the first design is an experiment, as opposed to an observational study, it is more likely to produce useful data. Using an experiment, researchers have a better chance of attributing any difference in colon health between the two groups to the treatment, bee pollen or not.

Problem 2 (15 points): Some managers of companies use employee rankings to laud the best and let go of the worst. Suppose the distribution of rankings of employees at a large company is normal with a mean of 65 points and a standard deviation of 6 points. Answer the following questions, being sure to *show your work*.

a) What proportion of employees has a ranking above 59 points? (5 points)

$P(X > 59) = P(Z > (59-65)/6) = P(Z > -1) = 1 - P(Z < -1) = 1 - .1587 = .8413$

b) Managers at this large company were told to determine the top 20 percent, the bottom 10 percent and the remaining 70 percent in the middle, and then "weed out" (let go) those in that bottom tier. Using the provided model for rankings, what is the cut-off for an employee to be in the top 20 percent? (10 points)

Top 20% ➔ 80% below, so $P(Z < z) = .80$ ➔ $z =\sim .85$. Also $z = (x - 65)/6 = .85$
➔ $x = (.85)(6) + 65$ ➔ $x = 70.1$ points

Problem 3 (30 points): A soda production company has two machines to fill cans. Machine 1 produces 80% of the cans and machine 2 produces the rest. One out of every 25 cans filled by machine 1 is rejected for some reason, while one out of every 35 cans filled by machine 2 is rejected. Use the following events to answer parts a) through c).

Let A = event that a can is filled by machine 1   Given: $P(A) = 0.8$; $P(B) = 0.2$
Let B = event that a can is filled by machine 2
Let R = even that a filled can is rejected $P(R|A) = \dfrac{1}{25}$; $P(R|B) = \dfrac{1}{35}$

a) What is the probability that a filled can is rejected? (10 points)

$$P(R) = P(R|A)P(A) + P(R|B)P(B) = \left(\frac{1}{25}\right)(0.8) + \left(\frac{1}{35}\right)(0.2) = 0.032 + 0.0057 = 0.0378$$

b) What is the probability that a randomly selected can comes from machine 1, given that it is accepted? (10 points)

Let $\bar{R}$ = accepted

$$P(A|\bar{R}) = \frac{P(A \cap \bar{R})}{P(\bar{R})} = \frac{P(\bar{R}|A)P(A)}{1 - P(R)} = \frac{\left(\frac{24}{25}\right)(0.8)}{1 - 0.0378} = \frac{0.768}{0.9622} = 0.7982$$

c) Are the events B and R dependent or independent? Use an appropriate equation to support your answer. (10 points)

$$P(R|B) \overset{?}{=} P(R)$$

$$\frac{1}{35} = 0.0286 \neq 0.0378 \Rightarrow R \text{ and } B \text{ are dependent}$$

Problem 4 (15 points): Suppose that a student needs to buy 6 books for her history course. The probability that she will find any given book used is 0.30, and is independent from one book to the next. Define X to be the number of books that she will be able to find used.

a) What is the probability that she will find exactly 5 used books? (5 points)

$$P(X = 5) = \binom{6}{5}.3^5.7^1 = .0102$$

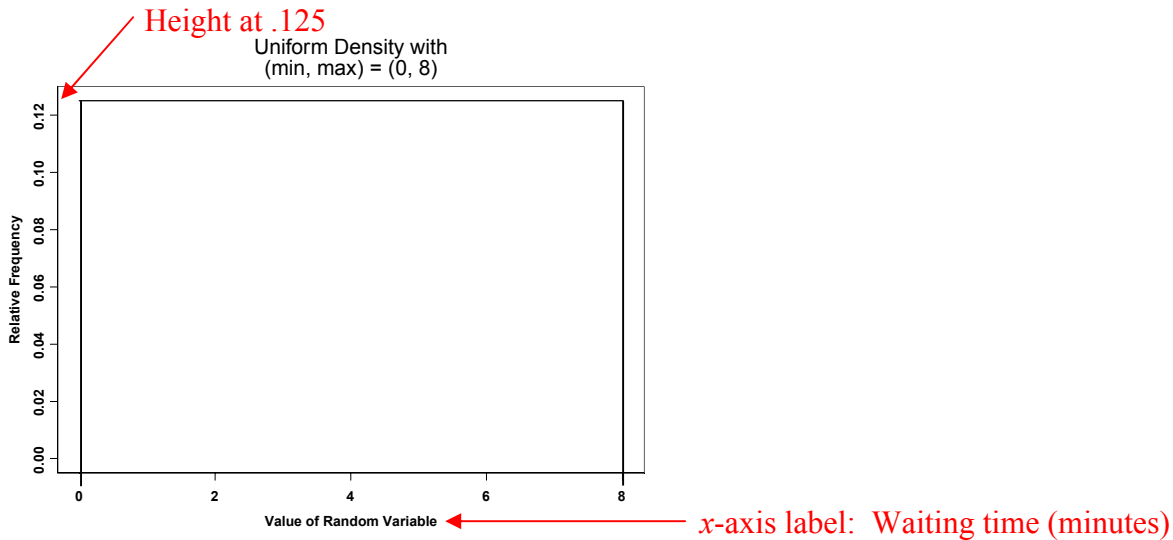b) What is the expected number of used books she will find? (5 points)

$E(X) = np = 6(.3) = 1.8$

c) What is the standard deviation for the number of used books that she will find? (5 points)

$SD(X) = \sqrt{npq} = \sqrt{6(.3)(.7)} = 1.12$

Problem 5 (20 points): Suppose the time to wait for placing an order at a drive-through window has a uniform distribution between 0 and 8 minutes.

a) Sketch the pdf for this distribution. Be sure to fully label the *x*- and *y*-axes. (5 points)

Height at .125

Uniform Density with
(min, max) = (0, 8)

Relative Frequency

Value of Random Variable

x-axis label: Waiting time (minutes)

b) What proportion of customers is expected to wait more than 7 minutes? (5 points)

P($X > 7$) = (.125) (8-7) = .125 x 1 = .125

c) Complete the sentence: About 25% of the customers are expected to wait at most _____ minutes? (5 points)

P($X < x$) = .25 ➔ (.125) ($x − 0$) = .25 ➔ x = .25/.125 = 2 minutes

d) What is the probability of waiting exactly 6 minutes? (5 points)

P($X = 6$) = 0