	Parameter	Confidence Interval
Difference between means, independent samples, unpooled	$\mu_1 - \mu_2$	$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ or $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2,df}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$, $df = \min(n_1 - 1, n_2 - 1)$
Difference between means, independent samples, pooled variance	$\mu_1 - \mu_2$	$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ or $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2,df} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$, $df = n_1 + n_2 - 2$ $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$

Often reasonable to assume the two populations have ***equal population standard deviations***, or equivalently, equal population variances

Estimate of this variance based on the combined or “pooled” data is called the **pooled variance.** The square root of the pooled variance is called the **pooled standard deviation:**

Pooled standard deviation $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$

Example: Male and Female Sleep Times

Question: How much difference is there between how long female and male students slept the previous night?

Data: The 83 female and 65 male responses from students in an intro stat class.

Task: Make a 95% CI for the ***difference*** between the two population mean sleep hours for females versus males.

Female

$n_F = 83; \ \ \bar{x}_F = 7.02; \ \ s_F = 1.75$

Male

$n_M = 65; \ \ \bar{x}_M = 6.55; \ \ s_M = 1.68$

Two **sample standard deviations** are very **similar** so we will assume equal population variances.

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{(83-1)(1.75)^2 + (65-1)(1.68)^2}{83+65-2}}$$

$$= \sqrt{2.957} = 1.72$$

Weighted average of the two sample variances

$$\text{Pooled } s.e.(\bar{x}_1 - \bar{x}_2) = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= 1.72\sqrt{\frac{1}{83} + \frac{1}{65}} = 0.285$$

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm t_{\alpha(2),df}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \ \ df = n_1 + n_2 - 2$$

$$(7.02 - 6.55) \pm t_{.025,146} \times .285$$
$$\Rightarrow 0.47 \pm 1.98 \times .285$$
$$\Rightarrow 0.47 \pm 0.564$$
$$\Rightarrow (-0.094,\ 1.034)$$

95% confidence interval contains 0 so cannot rule out that the population means may be equal.

Pooled or Unpooled Variances

- **If sample sizes are equal**, the pooled and unpooled standard errors are equal.

  If sample standard deviations are similar, assumption of equal population variance may be reasonable and the pooled procedure could be used.

- **If sample sizes are very different**, pooled test can be quite misleading unless sample standard deviations are similar. If the smaller standard deviation accompanies the larger sample size, not recommended to use the pooled procedure.

- **If sample sizes are very different**, the standard deviations are similar, and the larger sample size produced the larger standard deviation, the pooled procedure is acceptable because it will be conservative.