Chapter 8-3: Testing A Claim About a Proportion

Utilize sample information to test what the value of a population parameter may be—want to answer questions about particular values for a population parameter based on information in a sample

We will conduct tests for:

- Population proportions, p
- Population means, μ

Example
About 10% of the human population is left-handed. Suppose that a researcher speculates that artists are more likely to be left-handed than are other people in the general population. The researcher surveys 150 artists and finds that 24 of them are left-handed.

- Interested in making an inference about the proportion of a population
- Less interested in estimating the value of p, but are more interested in testing a ***hypothesis*** about its value

Method used to reach a decision is based on the rare-event concept

Define two hypotheses

Null hypothesis **(**H_0**)**---defines the status quo to the party performing the experiment; or the hypothesis that is assumed to be true unless the data provide convincing evidence that it is false

Alternative hypothesis **(**H_1**)**---that which will be accepted only if the data provide convincing evidence of its truth

For the left-handed artist example:

H_0: $p = 0.1$ or $p \leq 0.1$ (10%); the proportion of left-handed artists is about the same as that for the general population

H_1: $p > 0.1$ (10%) the proportion of left-handed artists is greater than that for the general population.

<u>How is this done?</u>

- Reasonable to use a sample proportion, $\hat{p}$, to make the inference just like we did when forming CI for p.
- The researcher will conclude that the proportion of left-handed artists is greater than 10% only when the sample proportion convincingly indicates that the population proportion exceeds 10%

- **"Convincing" evidence in favor of the alternative hypothesis will exist when the value of $\hat{p}$ exceeds 10% by an amount that cannot be readily attributed to *<u>sampling variability</u>***
- Compute a test statistic which is the *z*-value that measures the distance between the value of $\hat{p}$ and the value of p specified in the null hypothesis
- When the null hypothesis contains more than one value of p, as in H_0: $p \leq 0.1$, use the value of p closest to the values specified in the alternative hypothesis
- The idea is that if the hypothesis that $p = 0.1$ can be rejected in favor of $p > 0.1$, then $p \leq 0.1$ can certainly be rejected

Calculate z, **under the assumption that H_0 is true**:

$$z = \frac{\hat{p} - \boxed{p_0}}{\boxed{\sigma_{p_0}}} = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

$$z = \frac{\left(\frac{24}{150}\right) - 0.10}{\sqrt{.1(1-.1)/150}} = \frac{0.16 - 0.10}{\sqrt{0.10(0.90)/150}} = \frac{0.06}{0.0245} = 2.45$$

$z = 2.45$ ➔ $\hat{p}$ is 2.45 SDs above the value $p = 0.10$

How large must z be before the evidence is convincing and the null hypothesis can be rejected in favor of the alternative hypothesis?

The chance of observing $\hat{p}$ more than 1.645 SDs above 0.10 is only .05---if in fact the true mean p is 0.10. (Where does this come from?)

If the sample proportion is more than 1.645 SDs above 0.10 either

- H_0 is true and a relatively rare event has occurred (probability = .05) or
- H_1 is true and the population proportion exceeds 0.10

Since we would most likely reject the notion that a rare event has occurred, we would reject the null hypothesis and conclude that the alternative hypothesis is true.

Here we are rejecting at the 5% level.

P-value—**observed significance level**; probability of observing a value as extreme (or more extreme) than the observed test statistic in the direction of the alternative hypothesis, ***assuming that the null hypothesis is true***.

This can be expressed as a conditional probability:

$P(A|B)$, where A is the event of observing a test statistic value as extreme as that observed and B is the event the null hypothesis is true

The smaller the *P*-value, the stronger the evidence is against the null hypothesis. In many fields, $P < 0.05$ is considered to be ***statistically significant***

$P < 0.01$ often termed ***highly statistically significant***

For the left-handed artist example, the *P*-value is equal to the probability under the standard normal curve beyond $z = 2.45$, which is $1 – 0.9929 = 0.0071$.

What is the probability that this procedure will lead to an incorrect decision?

Recall that an incorrect decision, rejecting the null hypothesis when in fact it is true, is called a ***Type I decision error***.

α = P(Type I error)
= P(Rejecting the null hypothesis when in fact the null hypothesis is true)

For the left-handed artist example,

$$\alpha = P\left(z > 1.645 \text{ when in fact } p = 0.10\right) = .05$$

The ***rejection region*** is the value of the test statistic for which we will reject the null hypothesis

Let's look at a modified problem—18 of 150 surveyed artists were left-handed.

$$z = \frac{\left(\frac{18}{150}\right) - 0.10}{\sqrt{.1(1-.1)/150}} = \frac{0.12 - 0.10}{\sqrt{0.10(0.90)/150}} = \frac{0.02}{0.0245} = 0.82$$

Thus, the sample proportion lies 0.82 SDs above the hypothesized value $p = 0.10$.

What do we conclude now?

This value of z does not fall into the rejection region. Therefore, we *cannot reject* H_0 using $\alpha = .05$

P-value: 1 – 0.7939 = 0.2061

Should we accept H_0 and conclude that the proportion of left-handed artists is the same as the proportion of left-handed individuals in the population?

Recall a Type II Error---concluding that the null hypothesis is true when in fact it is false

β is often difficult to determine precisely

Rather than make a decision to accept H_0 for which the probability of β is unknown, we avoid the potential of committing a Type II error by avoiding the conclusion that the null hypothesis is true

Rather, simply state that the sample evidence is insufficient to reject H_0 (or fail to reject H_0) at $\alpha = .05$

The left-handed artist problem is an example of a one-sided or one-tailed test because the alternative hypothesis specifies that p is strictly greater than a specified value

You can also pose ***two-tailed or two-sided hypotheses***. These statistical tests are designed to show that the population parameter is either larger or smaller than some specified value.

For a specific α, the rejection region will vary depending on whether the test is one- or two-tailed

Rejection regions for common values of α

α	**Alternative Hypotheses**		
	Lower-tailed	**Upper-tailed**	**Two-tailed**
α**=.10**	$z < -1.28$	$z > 1.28$	$z < -1.645$ or $z > 1.645$
α**=.05**	$z < -1.645$	$z > 1.645$	$z < -1.96$ or $z > 1.96$
α**=.01**	$z < -2.33$	$z > 2.33$	$z < -2.58$ or $z > 2.58$

For a one-tailed test, the P-value is equal to ***the tail area*** beyond z **in the same direction** of the alternative hypothesis

For a two-tailed test, the P-value is equal to ***twice the tail area*** beyond z in the same direction of the alternative hypothesis

Need to make sure that:

1. Have a random sample from the population of interest
2. Conditions for a binomial distribution are statisfied
3. The sample size is sufficiently large to ensure approximate normality.

<u>Critical-Value Approach</u>

1. State the null and alternative hypotheses.
2. Decide on the significance level α.
3. Verify necessary data conditions, and if met, compute the value of the test statistic.
4. Determine the critical value(s).
5. If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.
6. Interpret the result of the hypothesis test--report the conclusion in the context of the situation.

<u>*P*-value approach</u>

1. State the null and alternative hypotheses.
2. Decide on the significance level α.
3. Verify necessary data conditions, and if met, compute the value of the test statistic.
4. Determine the *P*-value.
5. If $P \le \alpha$, reject H_0; otherwise, do not reject H_0.
6. Interpret the result of the hypothesis test--report the conclusion in the context of the situation.