A SAMPLE ANALYSIS

From Cobb, P. (1988) The tension between theories of learning and instruction in mathematics education. Educational Psychologist, 23, 87-103.

Melissa, the first-grade student of the following episode, routinely was able to solve missing addend sentences such as 8 + ___ = 12. She consistently solved these tasks and addition and subtraction sentences by using finger patterns associated with number words up to 15. These patterns could include both visible and imaginary fingers. Consider, for example, her solution to a subtraction task in which she was asked to find how many marbles remained in a cup given that there were initially 14 marbles and 11 had been removed. Melissa put up all 10 fingers simultaneously as she said, "Fourteen." She then pointed to four locations to the right of her right hand as she whispered "1, 2, 3, 4" and continued "5, 6, . . . , 11" as she sequentially closed seven fingers. Finally, she looked at her remaining three fingers and answered, "Three." For this task and in solutions to addition and missing addend tasks, Melissa seemed to visualize a third hand. This inference is consistent with limitations in her methods that became apparent when she was asked to solve tasks that involved a total of 16 or more items. In each case, she was unsuccessful and explained that she did not have enough fingers.

Melissa's reliance on direct modeling solutions led to the inference that she could only express her relatively sophisticated concept of number by creating a collection of visible and imagined items. Previous investigations indicate that children inferred to be at about the same conceptual level as Melissa can also express their number concepts by imagining themselves counting (Cobb, 1985; Steffe et al., 1983). To give meaning to the first addend of 8 + 5 = ___, for example, the child might take the activity of counting "1, 2, . . . , 8" as having been completed without ever imagining a collection of items and solve the sentence by counting-on "8, 9, 10, 11, 12, 13." Therefore, the teacher speculated that Melissa would only have to express her number concept in terms of imagined counting activity rather than collections of items in order to overcome the limitations of her direct modeling methods. On the basis of this hypothesis, the teacher made several highly directive interventions to investigate whether Melissa could learn to solve subtraction sentences by counting backwards (e.g., solve 17 − 4 = by counting "17, 16, 15, 14, 13"). This method was chosen because it is children's first natural method to subtraction sentences beyond direct modeling and because children inferred to be at the same conceptual level as Melissa frequently construct this method without direct instruction (Steffe et al., 1983).

The teacher first checked that Melissa had developed the prerequisite ability of reciting the standard backward number word sequence starting at 20. He then asked her to say how many marbles remained in a cup when he removed them one at a time. She answered appropriately on each occasion, and the activity was then repeated with the variation that she was also required to put up a finger each time a marble was removed. Melissa again responded appropriately. The teacher then presented the sentence 15 − 3 = ___, but Melissa solved it by using her finger pattern method. Finally, he asked her to solve it by counting backwards and she started to do so before saying, "Okay, I know it—I just can't get it in my mind."

The instruction provided up to this point was a form of direct training in which the teacher assessed the child's methods, chose the next method in the developmental sequence as the goal of instruction, assessed prerequisite skills, and attempted to simplify the target method as much as possible. Unfortunately, something seemed to be going wrong. Melissa's final comment and her general demeanor indicated to the teacher that she saw no point in trying to solve the task by counting backwards. He therefore inferred that if he persisted he might merely train her to behave as he desired rather than encourage her to express her number concept in a novel way. In other words, Melissa's primary goal might become to do exactly what she was told and thus get out of an unpleasant situation as quickly as possible. On the basis of these inferences, the teacher engaged her in an alternative activity for 10 min before presenting further subtraction sentences. No hints or prompts to count backwards were given in the remainder of the session.

Melissa solved the first subtraction sentence presented, 13 − 4 = ___, by using her finger pattern method. The teacher then presented the sentence 21 − 4 = ___. Because 21 was beyond the range of her finger patterns the teacher hypothesized that she might count backwards. She muttered quietly to herself for 50 sec before whispering "20, 19, 18, 17." Finally, the teacher asked her to solve 32 − 5 = ___ and she struggled with the problem for 2½ min before counting backwards "31, 30, 29, 28, 27" as she sequentially closed five fingers.

The teacher's prior direct instruction clearly influenced Melissa's production of these two backward-counting solutions, but three observations suggest that she did not merely recall what she had been told to do. Instead, she constructed a backward counting method that expressed her concepts. First, she did not use a new method until she was in a situation where her finger pattern methods did not work. The tasks 21 − 4 = ___ and 32 − 5 = ___ were genuine problems for her. As she had previously explained, "I'm really used to having something to help me and stuff like little numbers with my hands and big numbers with the little cubes that we have [in class]." This contrasts with the situation in which the teacher attempted to train her to count backwards. Second, the time it took her to solve the two sentences, particularly 2½ min to solve 32 − 5 = ___ after she had just counted backwards to solve 21 − 4 = ___, suggests that her difficulty was not merely one of recall. Third, the methods she used differed from the one that she had been taught. She did not put up fingers or use any other observable keeping-track procedure to solve 21 − 4 = ___, and she closed fingers as she counted to solve 32 − 5 = ___. In fact, her problem with 32 − 5 = ___ appeared to be to find a way to keep track of her backward-counting activity.

What did Melissa learn in this episode? How did she learn it?
What evidence do you have for your conclusions?