An Analytical Process Model of Two-party Negotiations

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There has been a call to investigate the negotiation process (Gale 1986, Shubik 1982), as it is felt that this would yield important insights beyond those obtained by outcome-oriented theories (Roth 1979). This paper proposes a new analytical process model that captures both behavioral and economic aspects related to two-party negotiations. The proposed model, inspired by Pruitt's (1981) work, explicitly incorporates concepts which are both relevant and crucial, such as the negotiators' power, concession points, aspiration level, limit, and time pressure. Based on this process model, it is possible to predict (1) conditions under which agreements will not be reached despite the existence of a zone of agreement, (2) conditions under which agreements will be reached, and (3) the patterns of the negotiators' offers and counteroffers.
(*Games; Negotiation Processes; Buyer-seller Interaction*)

1. Introduction

There has been an increased interest in the area of negotiations over the last decade from both managerial and research perspectives. This is due in part, at least from a managerial perspective, to its economic importance. Reeder et al. (1987, p. 475) observe that "most purchases by institutions, government agencies, and commercial businesses are negotiated." Angelmar and Stern (1978), for instance, estimated the annual sales of wholesalers in the United States for 1977, which are normally determined through negotiations, at an astounding 1,284 billion dollars. In this same year, the number of wholesaling establishments, which in some sense is a proxy of a lower order of magnitude for the number of negotiations, was 394,029 (Stern and El-Ansary 1982, p. 112). Graham (1985) makes a similar observation about negotiations in the international domain where U.S. merchandise sales to other countries in 1984 totaled more than $217 billion.

Negotiations can be studied from different perspectives. Raiffa (1982) suggests a framework for classifying various approaches to studying negotiations. The research perspectives identified by him are: (1) symmetrically descriptive, (2) symmetrically prescriptive, (3) asymmetrically prescriptive/descriptive, and (4) externally prescriptive or descriptive. Perspectives (1) and (4) are of special relevance to this paper. Under the first research approach "the researcher might be interested solely in describing the behavior of all negotiators, without having any interest whatsoever in prescribing how they should behave." The fourth research perspective investigates "how intervenors should behave in order to help the negotiating parties." Chatterjee (1985), in a lucid review of theories of negotiations, distinguishes between two major approaches to studying negotiations. The first approach focuses on the outcome of the negotiation and thus "abstracts from specific descriptive models of the process by which bargaining proceeds." The second approach, which does focus on the *process* of negotiation, analyzes the offers and counteroffers made by the negotiators via specific noncooperative game models.

In this paper we present a new analytical *process* model of two-party negotiations, taking a symmetrically descriptive research perspective. The major objective in developing such a model is that it can be employed by

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outside observers to obtain diagnosis and make various predictions with respect to the nature and course of the negotiations. Such information might be useful, for instance, for mediators in their attempts to help with the negotiation process. The model proposed here takes an analytical approach, and it relies on empirically grounded constructs, identified as crucial in negotiations contexts by behavioral researchers.

The paper is organized as follows. In §2 we briefly review the most relevant literature relating to negotiations in areas as diverse as economics, sociology, psychology, and management science to identify the underlying research approaches and constructs employed in the bargaining literature in order to spotlight factors which have an effect on the negotiation process. For comprehensive reviews of analytical research on negotiations readers are referred to Brams (1990), Chatterjee (1985), Kreps (1990), Raiffa (1982), Roth (1979, 1985), and Young (1991), and to Balakrishnan, Patton and Lewis (1992), Evans and Beltramini (1987), Neale and Bazerman (1985a), Pruitt (1981), and Rubin and Brown (1975) for reviews of behavioral research. We present, in §3, our new analytical process model of two-party bargaining. In §4, the implications and insights obtained from our conceptualization of the negotiation process are presented, and our approach is contrasted with that of Rubinstein (1982). In §5, a number of empirical results are provided, and issues related to further testing of the model and its implications are discussed. We conclude in §6 by suggesting directions for further research.

2. Relevant Literature Review

Since our analytical model can be characterized as a descriptive and interactive model of negotiation behavior, we begin our literature review with some empirical findings concerning descriptive negotiation behavior. The evidence in this area can be further subdivided as either anecdotal or experimental. The anecdotal works are usually those of experienced practitioners (e.g., Cohen 1980, Nierenberg 1973), relating negotiation encounters in which they participated, or those of dispassionate observers (e.g., Douglas 1962). These attempt to derive (post-hoc) reasons for success or failure of particular experiences. Based upon this anecdotal evidence, by means of a mixture of inductive and deductive reasoning, some theoretical frameworks have been advanced (Walton and Mckersie 1965, Gulliver 1979).

The experimental studies are concerned mainly with testing, in controlled settings, the role of various factors underlying behavioral theories (Clopton 1984; Neale and Bazerman 1985b; Rubin and Brown 1975; Chertkoff and Esser 1976) or analytical frameworks (Chatterjee and Lilien 1984; Neslin and Greenhalgh 1983, 1986; Roth and Malouf 1979). Issues that have been empirically examined and found to play an important role in negotiation experiences include (1) aspiration level (Bazerman et al. 1985), (2) reservation prices or limits for negotiation (Yukl 1974), (3) power (Eliashberg et al. 1986, McAlister et al. 1986), (4) time pressure (Carnevale and Lawler 1986), (5) reciprocation (Bartos 1974), (6) expectations (Oliver, Balakrishnan, and Barry 1994, Pruitt 1981) and (7) various personality traits (Greenhalgh et al. 1985).

Analytical approaches to negotiations that have focused on outcomes have studied them mainly in a static fashion. Emphasis here is placed upon explaining and/or predicting the nature of the agreement that will result. The classic examples of this approach can be found in the cooperative game-theoretic literature (Shubik 1982, Young 1975). The modeling here is axiomatic in nature: the basic goal is to develop a formal theory that prescribes the outcomes of cooperatively transformed bargaining games in which constructs such as power, equity, and efficiency are important in the context of either negotiations (Nash 1950, Kalai and Smorodinsky 1975, Myerson 1984, Gupta and Livne 1988) or arbitration (Ashenfelter and Bloom 1984, Eliashberg 1986). However, this yields little insight about the details of the process leading to the specified outcome (Gale 1986).

Noncooperative bargaining models, unlike the cooperative game-based bargaining models, construct an extensive form model of the process by which bargaining occurs. Then, all possible sequentially rational equilibria are identified, to circumscribe the optimal behavior in such a situation as well as the resulting outcomes (e.g., Rubinstein 1982). Recent research (Chatterjee 1985) has incorporated aspects of incomplete information in noncooperative nonzero-sum bargaining games. Constructs considered here include: reservation prices (Chatterjee and Samuelson 1983), type of negotiator (Harsanyi and Selten 1972) and negotiator's time preferences (Rubinstein 1987). In general, the noncooperative bargaining models (e.g., Myerson and Satterthwaite 1983) are more concerned with determining conditions under which the equilibrium solutions exist, are unique, and efficient.

There has been interest in achieving other research objectives in developing analytical models of two-party negotiations. This stream of research is evinced, for example, by the models proposed by Trifon and Landau (1974), Rao and Shakun (1974), England (1975), Fogelman-Soulie et al. (1983), and Gupta (1989). These models have deviated from the approach adopted in standard game theory. Fogelman-Soulie et al. (1983), in particular, have a well-argued exhortation for adopting alternative modeling procedures. Others express disenchantment with the applicability of some of the axioms to real-world negotiation behavior (Gale 1986, Sebenius 1992). A series of recent experiments (e.g., Guth et al. 1982; Neelin et al. 1988) suggests that even fundamental concepts such as subgame-perfect equilibrium fail not only as a ". . . point predictor of observed behavior, [but] also fail to account for observed qualitative differences" (Ochs and Roth 1989).

Some of the alternative methods of modeling the negotiation process view negotiations dynamically, with the focus on modeling the offers and counter offers (Cross 1969). Each negotiator is modeled via expectations regarding the other negotiator's response to his possible actions. These expectations, which may be incorrect, are then adjusted based on the other negotiator's actual decision (Coddington 1968). The models developed using this approach can predict the course of the negotiation process and its precise outcome if the process is specified deterministically. The model proposed in this paper is in this very spirit.

It does seem that many of the models developed fall short on the number of relevant constructs they explicitly consider, and hence, on the number of scenarios they can accommodate. Even recent promising models (e.g., Svejnar 1986) do not explicitly accommodate, for example, differing perceptions that bargainers may have about each other's relative power (Gupta 1989, Lusch 1976) and time pressure (Contini 1968, Carnevale and Lawler 1986). It is, therefore, imperative that future developmental work of analytical models specifically consider such constructs that are relevant in many negotiations contexts.

In this paper, we heed the call of researchers such as Fogelman-Soulie et al. (1983) and Sebenius (1992) and propose a new process model of two-party negotiation that is rich and tractable. (It does not suffer from computational burden of a stochastic terminal control approach as in Fogelman-Soulie et al.) The proposed process model of negotiation, which explicitly incorporates mathematically various constructs and phenomena of social psychology, such as power, concession points, aspiration levels, limits, time pressure, and personality characteristics, is described in the next section.

3. Modeling Approach

3.1. Overall Conceptual Framework

The type of scenario we wish to model involves sequential (and alternating) offers and counteroffers between two parties (e.g., seller and buyer) having conflicting preferences over a single issue (e.g., the price of an object owned by the seller) to be negotiated. That is, one party desires more of that issue and vice-versa for the other party. The parties involved are under no misperception whatsoever as to the nature and identity of the issue to be negotiated, and all communication can be summarized in the form of accepting an offer or rejecting and making a counteroffer. Additionally, the parties are ". . . hedged by a minimum and a maximum position" (Heinritz and Farrell 1984, p. 264). Related to these negotiation positions, the respective constructs target point (Walton and McKersie 1965) and limit or reservation price (Pruitt 1981 and Raiffa 1982) have also been considered in the literature.

The conceptual framework underlying our proposed model is based upon the "Basic Demand / Concession Model" of Pruitt (1981), which is static in nature. We extend it, however, to capture descriptively the social dynamics of the negotiation process. Each negotiator is accordingly represented paramorphically by a set of two opposing forces. These are termed the "resistance force" and the "concession force" (Pruitt 1981, p. 48). The resistance force, briefly, is the force acting on the negotiator which represents his / her natural disinclination to concede. Mathematically, it can be represented by a function $\rho(\cdot)$ whose argument is the level of offer considered to be made by the negotiator. The ordinate, ρ, which may be expressed in the same units as the issue under negotiation, represents the strength of the force resisting concession making. The concession force, on the other hand, represents the force prevailing on the negotiator toward making concessions. Similarly to the resistance force, this concept can be represented by a function $\phi(\cdot)$ that maps the amount of offer under consideration by the negotiator to the strength of the force pulling him/her toward making concessions.

These two forces, which have been used as primitives in the social psychology literature,[1] are akin to the gradient of avoidance (Dollard and Miller 1950) concept in psychology and psychotherapy, whereby the closer a subject is to a feared goal, the stronger the tendency to avoid it. This results in what Stevens (1963) terms the avoidance–avoidance conflict choice. In other words, a negotiator is placed in a situation wherein he or she wishes to avoid conceding from his or her *aspiration level*, and at the same time wants to avoid not reaching a settlement at his or her *concession point*. The concession point is simply the point at which the concession force vanishes. That is, it is the minimum (maximum) possible offer under consideration by the seller (buyer), and it may be a function of each party's expectations with respect to the other party. These two points together (i.e., the aspiration level and the concession point) define the range of possible offers considered by the negotiators. In such a situation, the individual constrained by the symbiotic nature of the relationship, and thereby precluded from the possibility of an escape from making a choice under conflict, seeks a compromise through negotiations. The resulting offer is what is reflected by an internal compromise position within the range of offers under consideration.

We introduce all other constructs employed in our model as well as their notations in the next subsection and discuss their interpretations.

[1] A reviewer has noted that the use of "resistance force" and "concession force" as primitives is different from the economic framework wherein the primitives are the individuals' desire to maximize their expected utilities. However, in the social-psychology framework, personality psychologists have long explained peoples' choices as a result of balancing two opposing forces. This work dates back to at least the famous ring-tossing experiments of McClelland (see Lopes 1992) and has since been adopted in the behavioral negotiations literature (Stevens 1963).

3.2. The Model, Its Constructs, and Their Notation

We will assume for the sake of exposition that the issue under negotiation is the price of an object owned by a seller. The essential conflict is that the seller *demands* a higher price. The buyer, on the other hand, would like to pay as little as possible and therefore *offers* a lower price.

The following constructs and notation are employed in our analytical model:

i = A subscript to indicate Seller ($i = S$) or Buyer ($i = B$);
τ_i = Aspiration level (or target point) of party i (\$);
μ_i = Limit (or reservation price) for party i (\$);
π_i = Party i's perceived relative power, ($\pi \in [0, \infty]$ is dimensionless);
α_i = Party i's perceived relative time pressure, ($\alpha \in [0, \infty]$ is dimensionless);
θ_i = Adjustment coefficient of party i ($-1 \le \theta_i \le 1$);
$\beta_{i,t}$ = Party i's concession point at time t (\$);
X_t = Demand made by Seller at time t (\$);
Y_t = Offer made by Buyer at time t (\$);
$\rho_S(X_t)$, $\rho_B(Y_t)$ = Strength of resistance force of party i ($i = S, B$) at time t;
$\phi_S(X_t)$, $\phi_B(Y_t)$ = Strength of concession force of party i ($i = S, B$) at time t;
t = The set of time periods or innings ($t \in N$).

Without loss of generality, we have both the Seller and Buyer making their respective demands and offers at each time period (or inning). Specifically, the Seller always goes first at each time period by making his or her demand. The Buyer then counters with his or her response of an offer each time period.

Each party's problem can then be represented analytically as choosing the maximum (minimum) level to demand (offer), subject to the constraining forces acting upon it. The problem of either party can be written in its most general form as follows:

$$\textit{Seller}: \quad \text{Max } X_t \tag{3.1}$$

subject to:

$$\rho_S(X_t) - \phi_S(X_t) = 0 \tag{3.2}$$

$$\mu_S \le \beta_{S,t} \le X_t \le \tau_S. \tag{3.2a}$$

Buyer: Min Y_t (3.3)

subject to:

$$\rho_B(Y_t) - \phi_B(Y_t) = 0 \quad (3.4)$$

$$\tau_B \le Y_t \le \beta_{B,t} \le \mu_B, \quad (3.4a)$$

and, where $t \in N$.

Functional forms for the resistance and concession forces must be chosen not only on the basis of parsimony and analytical tractability but also on behavioral and intuitive appeal. This requirement leads us to the following analytical representation of the two opposing forces.

Seller:

$$\rho_S(X_t) = \pi_S(\tau_S - X_t) \quad (3.5)$$

$$\phi_S(X_t) = \alpha_S(X_t - \beta_{S,t}). \quad (3.6)$$

Buyer:

$$\rho_B(Y_t) = \pi_B(Y_t - \tau_B) \quad (3.7)$$

$$\phi_B(Y_t) = \alpha_B(\beta_{B,t} - Y_t) \quad (3.8)$$

$$t = 0, 1, 2, \ldots.$$

The representation of the concession force (equations 3.6 and 3.8) is identical to the static formulation proposed by Pruitt (1981, p. 50). This, as he suggests, makes good sense because the distance between the seller's demand currently under consideration and his or her current concession point (i.e., $X_t - \beta_{S,t}$) spells time delay and the possibility of failing to reach agreement. Naturally, the greater the time pressure and the greater the danger of no agreement, the closer should the negotiator be to the concession point. Similarly, for the resistance force, the relative perceived power of a negotiator interacts multiplicatively with the distance of the current demand from the target point. Note that the resistance force is pulling the negotiator toward the target point. Specifically, the further the negotiator gets from his or her desired target, the greater is the resistance to making a concession. The concession and resistance forces are thus pulling the negotiator in opposing directions.

The dynamics of the concession points in each period, which precede the making of demands and offers for that period, is inspired by Raiffa (1982) and Pruitt (1981) who have argued that in the absence of sound information about the opponent, the most recently observed concession rate is often taken as a guidance to where the opponent is going. Based upon this, the negotiators' concession points are determined endogenously as:

$$\beta_{S,t} = \beta_{S,t-1} + \theta_S(Y_{t-1} - Y_{t-2}) \quad \text{for} \quad t = 2, 3, \cdots \quad (3.9)$$

$$\beta_{B,t} = \beta_{B,t-1} + \theta_B(X_t - X_{t-1}) \quad \text{for} \quad t = 1, 2, \cdots \quad (3.10)$$

given some exogenously determined $\beta_{S,0}$, $\beta_{S,1}$, and $\beta_{B,0}$.

A particularly attractive feature of our conceptualization of bargaining power is that it incorporates the endogenous aspects arising from dependence on the continuing relationship, in addition to the established viewpoint (Svejnar 1986) of power as an exogenously determined force. This allows each party to have a different perception of his or her own powers relative to those of the other party (Bacharach and Lawler 1986). In addition to other differences, our formulation does not require that the sum of the bargaining powers of the two parties to total one, neither does it call for complete information, unlike, for instance, that of Svejnar's (1986) cooperative game-theoretic model. This heeds Gupta's (1989) suggestion for specifically modeling the perceptions about the other party's power, which he notes ". . . may be more important than objective reality in determining the course and outcome of negotiations," due to factors such as uncertainty and limited cognitive abilities.

We note that equations (3.5)–(3.10) incorporate the interaction between the two parties through the concession points (β_i). Under equations (3.5)–(3.8) the optimization problems (3.1) and (3.3) degenerate to the unique intersections of equations (3.2) and (3.4). The linear functional forms that were chosen for these opposing forces thus guarantee existence and uniqueness to the negotiators' problems.[2]

[2] The parties' decisions are stated as optimization problems in (3.1) and (3.3) for the following reason. If the linearity assumption underlying equations (3.5)–(3.8) is modified to include nonlinearities, this would imply that equations (3.2) and (3.4) could result in higher order polynomials having multiple intersections. In such cases, the negotiators would choose the optimal intersection demand offer.

Equations (3.9) and (3.10) are an explication of a dynamic process. Consecutive counteroffers by the other party cause the negotiator to update his or her concession point. Specifically, the negotiators' reassessment of their concession points is linked to the other party's observed pattern of concession. Here, the adjustment coefficient θ_i captures how the negotiators adjust their concession point based on the other's actual concessions (or anti-concessions). As can be seen in Appendix A (A.8 and A.11), the adjustment coefficients become major determinants (along with α_i and π_i) of the negotiators' *tendency to reciprocate* in terms of making concessions (or anti-concessions). The coefficient of reciprocity: $\theta_i \alpha_i / (\alpha_i + \pi_i) \equiv K_i$, which is proportional to θ_i, is very similar to the parameter "b" in Bartos' (1974) reformulation of the Richardson (1960) model.

We find that, based upon the discussion advanced by Bartos (1974), when θ_i is *negative* the negotiator is considered to be "reciprocative" ($K_i < 0$), and when θ_i is *positive* the negotiator can be thought of as being "exploitative" ($K_i > 0$). That is, the sign of the coefficient of reciprocity (K_i) is determined by the sign of θ_i. The perspective taken here is that reciprocity is a personality trait rather than a strategic move made by the party (Gouldner 1960). Equation (3.9) can be interpreted as follows. In response to a concession (anti-concession) made by the Buyer, a reciprocative (exploitative) Seller will expand (narrow) the range of demands under his or her consideration. A similar interpretation applies to equation (3.10). This is consistent with Bartos, who, based on Homan's theory of distributive justice, notes in this regard that "reciprocative" type negotiators are not likely to take advantage of the other's concessions. Instead, the other's concessions may merely indicate that earlier locations of the concession point may have been too optimistic and that these need to be revised if an agreement is to be reached in any practical length of time. This is particularly true in a situation characterized by anticipation of cooperative future interaction, where the parties involved are more likely to respond helpfully to the other's requests (Deutsch and Kotik 1978).

Finally, for agreement to be possible between the two parties, it is required that the limit (e.g., reservation price) for the Seller be less than that of the Buyer. That is, only when $\mu_S < \mu_B$ does there exist a *zone of agreement* (Raiffa 1982). A specific agreement in the negotiation process is reached when either party's impending proposal is no more desirable than the previous proposal made by the other party; that is, when $Y_t \geq X_t$ (or $X_{t+1} \leq Y_t$).

4. Theoretical Implications

The seller's negotiation process, after some algebraic manipulations, can be represented by means of the following second-order difference equation (see derivation in Appendix A.1):

$$X_{t+2} - (K+1)X_{t+1} + KX_t = 0 \quad \text{for} \quad t = 0, 1, \ldots; \quad \text{and} \quad X_0, X_1 \qquad (4.1)$$

where:

$$K = K_S K_B = \frac{\alpha_S}{\pi_S + \alpha_S}\theta_S \frac{\alpha_B}{\pi_B + \alpha_B}\theta_B; \qquad (-1 \leq K \leq 1) \quad (4.2)$$

can be interpreted as the negotiation specific parameter.

A solution to the above process representation can be obtained by means of Z-transforms, and is as shown below (see Appendix A.2 for the derivations).[3]

$$X_t = \frac{X_1 - KX_0}{(1-K)} + (K)^t \left[\frac{X_0 - X_1}{(1-K)}\right] \text{for all} \quad t \in N \quad (4.3)$$

Note that the first term on the R.H.S. of (4.3) represents the steady-state component of the process, and the second term represents the transient component. (See B.6 in Appendix B.)

Next, the various implications that can be extracted from our model are presented. The results are presented

[3] To solve the second-order difference equation requires the specification of the first two demands made by the Seller (i.e., the initial conditions of (4.1)). Equation (3.9) indicates that two offers by the Buyer need to be observed for the model to be able to update the Seller's initial concession point. This means that the second demand made by the Seller (X_1) is not specified by the model, as by this time the buyer has made only one move. Hence, having observed the first two demands made by the Seller and the first offer made by the Buyer the entire trajectory is completely determined only to a mediator, who is also in a position to know the psychological parameters characterizing both parties. Derivation of the Buyer's negotiation behavior leads to a difference equation similar to (4.3) where Y_0 and Y_1 replace X_0 and X_1, respectively. For the Buyer, however, Y_1 is specified by the model ((A.19) in Appendix A.1).

in three subsections: (1) steady-state individual level predictions; (2) negotiation process dynamic results; and (3) dyadic level dynamic results.

4.1. Steady-State Individual Level Propositions

In this sub-section, we present various individual level propositions relating to the steady-state condition of the bargaining process. Without loss of generality, the propositions are presented in terms of the Seller. Selected proofs are given in Appendix B. Corresponding results hold when the parties' roles are reversed. Essentially, we investigate the impact of a change in each of the parameters upon the final demand and are stated ceterus paribus.

PROPOSITION 4.1.1. *A higher second demand results in an increased final demand.*

PROOF. See Appendix B.

PROPOSITION 4.1.2. *A less reciprocative type Seller will make a higher final demand when (a) the Buyer is "reciprocative" ($\theta_B < 0$) and Seller starts with an immediate concession ($X_1 < X_0$); or (b) the Buyer is "exploitative" ($\theta_B > 0$) and Seller's second demand is in an increased form ($X_1 > X_0$).*

PROOF. See Appendix B.

PROPOSITION 4.1.3. *Negotiating with a less reciprocative type Buyer will result in an increased final demand by Seller when (a) the Seller is "reciprocative" ($\theta_S < 0$) and starts with an immediate concession ($X_1 < X_0$); or (b) the Seller is "exploitative" ($\theta_S > 0$) and his/her second demand is in an increased form ($X_1 > X_0$).*

PROOF. Similar to Proposition (4.1.2.).

PROPOSITION 4.1.4. *As the relative perceived power of Seller increases in his or her favor the Seller's final demand will be higher when (a) both parties are of a similar reciprocity type and the Seller starts with an immediate concession ($X_0 > X_1$); or (b) both parties are of dissimilar reciprocity type and the Seller's second demand is in an increased form ($X_0 < X_1$).*

PROOF. Similar to Proposition (4.1.2).

PROPOSITION 4.1.5. *As the time pressure on Seller increases then his or her final demand will decrease when (a) both parties are of a similar reciprocity type and the Seller starts with an immediate concession ($X_1 < X_0$); or (b) both parties are of dissimilar reciprocity type and the Seller' second demand is in an increased form ($X_0 < X_1$).*

PROOF. Similar to Proposition (4.1.2).

4.2. The Negotiation Process Dynamic Results

Here, two major results pertaining to the negotiation process are presented. In particular, we shall characterize the shape of the sequence of demands under different conditions. The various propositions presented below are essentially conditions which result in a certain pattern of demands.

PROPOSITION 4.2.1. *If the conditions below hold: (1) Seller starts with a concession ($X_0 > X_1$); and (2) Both parties are of a similar reciprocity type [Sign (θ_S) = Sign (θ_B)]; then Seller's sequence of demands will follow a monotonically decreasing pattern (see Figure 1).*

PROOF. It is easy to verify that under the conditions listed above equation (B.1) will become monotonically decreasing.

PROPOSITION 4.2.2. *If the conditions below hold: (1) Seller starts with a concession ($X_0 > X_1$); (2) Both parties are of dissimilar reciprocity type [i.e., Sign (θ_S) ≠ Sign (θ_B)]; then Seller's sequence of demands will follow an oscillating pattern (see Figure 2).*

PROOF. Similar as above.

Figure 1 **Seller's Sequence of Demands (Proposition 4.2.1)**

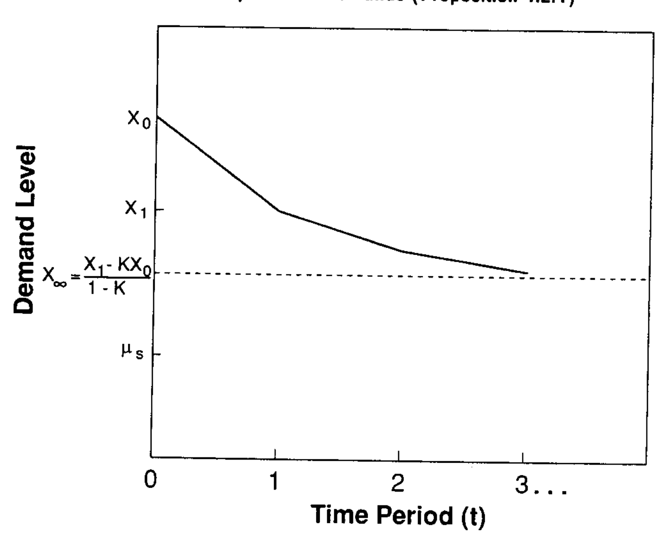


Figure 2 Seller's Sequence of Demands (Proposition 4.2.2)

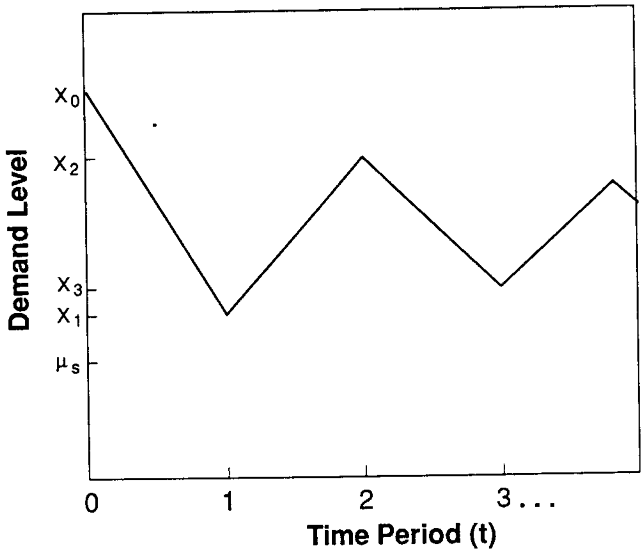
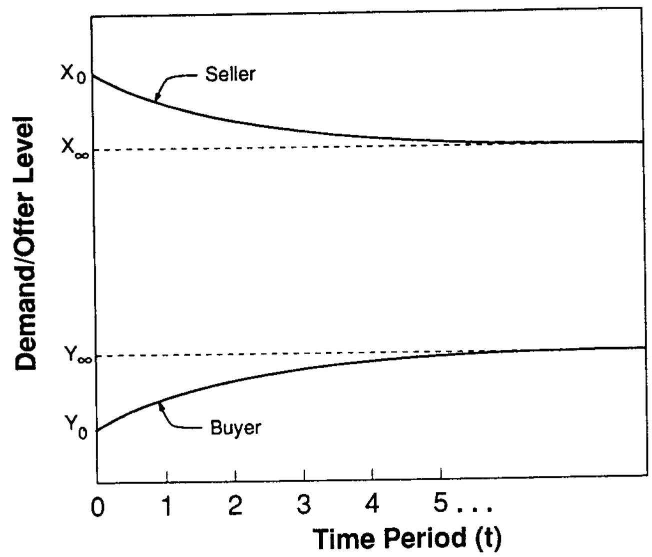


Figure 3 A Negotiation Process with No Agreement (Proposition 4.3.1)



4.3. Dyadic Level Dynamic Propositions

When the negotiations begin, we assume that there is *no* immediate agreement. In other words, the initial (conditions) set of offers–counteroffers satisfy the inequality: $Y_0 < X_0$ and $Y_1 < X_1$. This then leads to the following propositions. (See Appendix B for proofs.)

PROPOSITION 4.3.1. *If (1) Seller starts with a concession (i.e.,* $X_0 > X_1$*); (2) Buyer also starts with a concession (i.e.,* $Y_1 > Y_0$*); (3) Both parties are of a similar reciprocity type* [$Sign(\theta_S) = Sign(\theta_B)$]*; and (4)* $X_1 - KX_0 > Y_1 - KY_0$*; then there will be no agreement.*

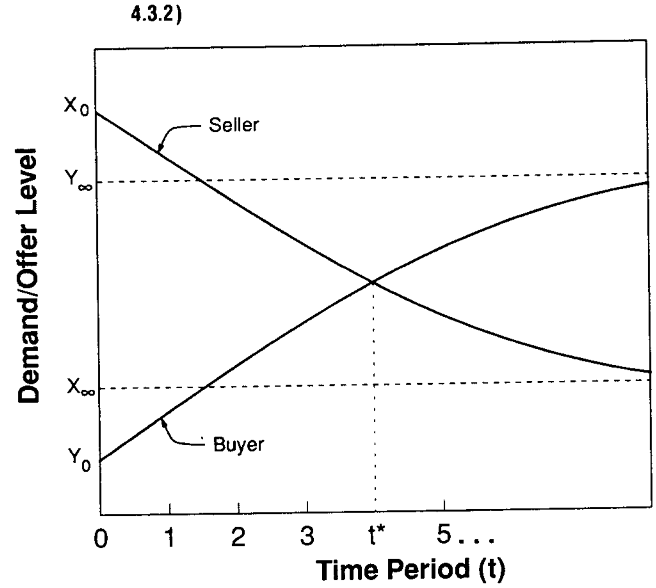
The reason for this Proposition is simple. Under the conditions listed above, the pattern of demands and counteroffers will be as shown in Figure 3. From the figure, however, it is clear that even as the steady-state condition is reached, the offers/counteroffers of each party are still unacceptable to the other.

PROPOSITION 4.3.2. *If (1) Seller starts with a concession (i.e.,* $X_0 > X_1$*); (2) Buyer also starts with a concession (i.e.,* $Y_1 > Y_0$*); (3) Both parties are of a similar reciprocity type* [$Sign(\theta_S) = Sign(\theta_B)$]*; and (4)* $X_1 - KX_0 < Y_1 - KY_0$*; then the two parties will reach an agreement at some finite time period.*

In this proposition (see Figure 4) we have stated the conditions under which an agreement will be reached when both parties are of a similar reciprocity type, and both parties start the process with a concession. The process will terminate at some finite time $0 < t^* < \infty$, when agreement is reached between the two parties, since the final demand (X_∞) of the Seller is lower than the final offer (Y_∞) of the Buyer.

The essential difference between propositions 4.3.1 and 4.3.2 lies in the directionality of the inequality of

Figure 4 A Negotiation Process Leading to an Agreement (Proposition 4.3.2)



condition 4. This condition can be interpreted as relating the negotiation-specific parameter (K) to the proportion of unresolved conflict ($([X_1 - Y_1])/[X_0 - Y_0]$) at the end of the first two offers and counteroffers. In other words, we are now able to link behavioral and situational aspects to economic factors. Also, the importance of condition 4 lies in predicting which sets of dyads are more likely to reach agreements than others. That is, all other economic factors being equal, a dyad with a *smaller* value of the negotiation specific parameter K is *less* likely to reach an agreement. This prediction of inefficiency is important, because it helps provide an answer to such questions as, for instance, raised by Neale and Bazerman (1985a), "If it is rational for a settlement to occur whenever a zone of agreement exists, why do negotiators sometimes fail to reach agreement despite the existence of a zone of agreement?"

CORROLLARY 4.3.2. *The time period at which agreement will be reached under the conditions specified in Proposition* 4.3.2 *is given by*:

$$t^* \geq \log_K \left[\frac{K(X_0 - Y_0) - (X_1 - Y_1)}{(X_0 - Y_0) - (X_1 - Y_1)} \right] \qquad (4.4)$$

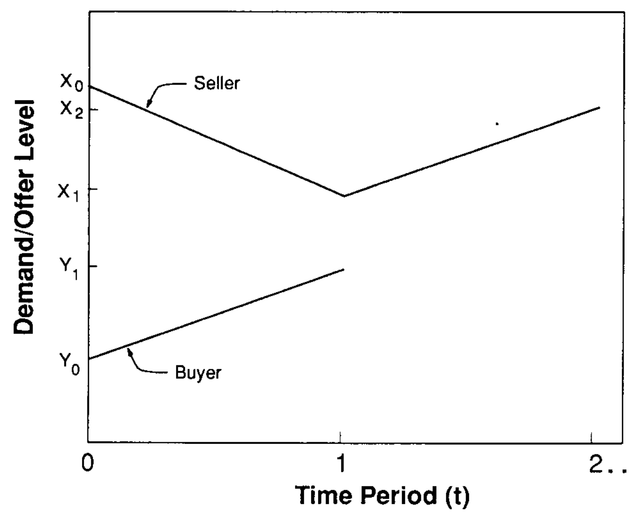
PROOF. See Appendix B.

Finally, we provide another set of conditions that lead to a stalemate.

PROPOSITION 4.3.3. *If* (1) *Seller starts with a concession* (*i.e.*, $X_0 > X_1$); (2) *Buyer is of a reciprocative type*; (3) *Seller is of a exploitative type*; (4) $X_1 - KX_0 > Y_1 - KY_0$; *then no agreement will be reached between the two parties*.

A descriptive representation of the process of negotiation specified in proposition 4.3.3 can be seen in Figure 5. Condition (4) of the proposition implies that the ultimate (steady-state) demands of either party are unacceptable to the other. The implication of the first three conditions listed above is that both parties' sequence of offers resembles a damped oscillation process. Hence, in the *long run* the two processes do not converge. The first and only opportunity for an agreement exists at the fourth move ($t = 1$) when the Buyer makes his or her second offer. If this offer Y_1 is greater than or equal to the second offer (X_1) of Seller, then an agreement is reached at this instant. Basically, what this implies is that under conditions (1) through (4), which typifies bad faith bargaining, an agreement may be reached on the fourth move or not at all.

Figure 5 **A Negotiation Process with No Agreement (Proposition 4.3.3)**



4.4. Relation to Game Theory

At this point we examine some worth noting points of similarity and differences that arise from our modeling perspective vis-a-vis that of noncooperative game theory. The classic paper by Rubinstein (1982) has since led to increased activity in the area of noncooperative bargaining models. The Rubinstein (1982) model is, as is the case with our model, an alternating offer model. The Rubinstein model goes beyond previous work by explicitly incorporating the notion of time pressure by use of discounting factors for each of the two players. Based on a relatively simple description of the bargaining game, Rubinstein is able to show that there exists a unique subgame perfect equilibrium.

There are a few major differences between Rubinstein's and our modeling approaches. In particular, the Rubinstein model requires Pareto efficient outcomes, in that agreements are always reached and arrived at immediately. This means that the time to agreement is always the same and is essentially $t^* = 0$, unlike in our context, where the time to agreement is given for example by equation (4.4), which clearly varies as per the negotiation situation, the individuals involved, and their initial demands and offers. Additionally, in our model we do not predict that agreements between the parties will always be reached. In fact, we are able to predict inefficiencies in the bargaining games, in consonance with observed empirical evidence, as a function of the model's parameters.

A major point of similarity is that both models explicitly take into account the time pressures on the negotiators. The difference, however, lies in the manner in which they are modeled. Rubinstein employs the economic framework of discount rates to shrink the size of the pie each time period. We, on the other hand, along with Van Damme et al. (1990), believe that such small interest losses need not be explicitly considered; this seems to be corroborated, in fact, by available experimental and anecdotal evidence (Ochs and Roth 1989). Our procedure models time pressure as a psychological force that compels negotiators to make offers.

With regard to predictions, the Rubinstein model and its related variants essentially rely on process-oriented arguments to define the subgame perfect equilibrium. Consequently, they do not, unlike in our model, predict the dynamic sequence of offers and counteroffers that an outsider might expect to observe for two-party negotiations. In the case of our model, an outside observer (e.g., mediator) should be able to predict conditions under which an exponentially decaying and oscillatory sequence of offers and counteroffers is likely to evolve. However, our formulation assumes that the first two demands made by the Seller and the initial offer made by the Buyer are arbitrary (i.e., not specified endogenously). On the other hand, in the game-theoretic models the first offer, which is made strategically, should be accepted instantaneously. While the latter modeling approach may fail as a descriptive model, it has an advantage in prescribing the optimal first offer that should be made.

5. Empirical Issues

In this section, various empirical issues related to the model and its testing are discussed. We begin by presenting findings and insights obtained via a field survey.

5.1. Field Survey Results

In order to first assess whether the constructs considered in the development of the negotiation model are also perceived as crucial by industry experts, a mail survey was conducted. A total of 30 usable responses from experienced negotiators was obtained. Of the top eight factors (e.g., perceptions of power, time pressure, ideal settlement), from a much larger list, considered by these industry experts to be most important in affecting the course and outcome of business negotiations, seven of these are explicitly considered and form the basis of our analytical model.

Next, based upon the results implied by the model, a set of *if–then* type predictions (described below) was extracted and evaluated by the respondents. (Note that this investigation is directed toward testing the implications derived from the model, unlike the ones discussed previously, which were oriented toward the reasonableness of its assumptions and the face validity of the underlying constructs).

Discriminating Condition: Propositions 4.2.1 and 4.2.2 specify conditions necessary to obtain the two different offer sequences (Figures 1 and 2). To test their validity, the respondents were provided explicitly with the discriminating conditions expressed in verbal terms (i.e., the reciprocity types of the negotiating parties and the nature of the concessions) and asked to predict which of the two conditions would give rise to the corresponding offer pattern. This part of the survey indicates that 80% of the respondents agreed with the model-based pattern in predicting that a montonically decreasing offer pattern would be obtained when the two parties are of similar reciprocity type (Proposition 4.2.1). To rule out the possibility that the executives surveyed were merely choosing their responses randomly, they were also asked to predict the offers pattern that would arise when the reciprocity types of the two parties are different (Proposition 4.2.2). The percentage agreeing with the model-based prediction is also 80%.

Negotiation Agreements: Conditions under which negotiators will or will not reach agreements were presented in Propositions 4.3.1 through 4.3.3. The conditions which discriminate among three propositions were stated verbally and presented to the executives surveyed as three prevailing scenarios, labeled A, B, and C, respectively. They were then asked to identify under which of the scenarios an agreement between the two parties was most likely. The results show that 17% of the respondents indicated that an agreement was most likely under scenario A (described in Proposition 4.3.1, which predicts no agreement); 80% indicated that an agreement would be most likely under scenario B (described in Proposition 4.3.2, which predicts an agreement at some finite time); and situation C (described in Proposition 4.3.3, predicting no agreement) was indicated by 3% of those surveyed. Taken together, the above field-study-based findings provide some empirical support and external validity to the predictions obtained from the model.

5.2. Parameters Assessment

In order to operationalize the model it is necessary to assess the value of its various parameters. Here we report the results of an initial attempt at scale developments from the perspective of an outside observer (e.g., researcher or mediator). The reliability assessment of the scales reported here was conducted on 49 student subjects.

In order to measure the parameter values of the subjects' coefficient of reciprocity (K_i) a new scale was developed. The basis for the development of this scale was the Personality Attitude Schedule constructed for use in experimental bargaining studies by Shure and Meeker (1967) and the revised and adapted version of Harnett and Cummings (1980). Based upon these earlier published works, a battery of items were generated. These items were revised to make them more relevant to the context of two-party negotiations. The conceptual definition of reciprocation/exploitation provided by Bartos (1974, page 38) was used as the guiding principle in the generation of these items. An instrument consisting of five statements (i.e., items) is proposed (see Table 1). For each statement the subject had to circle the appropriate response, on a five-point scale (with 1 = Strongly Disagree and 5 = Strongly Agree). The Cronbach alpha reliability coefficient (Nunnally 1978) for the five items is over 0.65.

The perceived relative bargaining power scale is based on Eliashberg et al. (1986). Four five-point scale items

Table 1 **Scale Reliability Analysis**

Construct	Items Retained		Cronbach Alpha Coefficient
1. Coefficient of Reciprocity	1. If the other party hardens their previous position, we will act similarly.		0.651
	2. If the other party tries to exploit us, our party will respond similarly.		
	3. If the other party will not cooperate with us then it is a good idea for us to also not cooperate with them.		
	4. If the other party is uncooperative, the most effective way to deal with them is to be cooperative.*		
	5. If the other party makes a concession, we will respond with a concession.		
2. Perceived Power	1. powerless . . .	. . powerful	Self-Power = 0.821
	2. submissive . . .	. . dominant	Other Party's Power = 0.779
	3. weak . . .	. . strong	
	4. in control . . .	. . being controlled*	
3. Perceived Time Pressure of Self	1. I feel that the time pressure I am under is ______:		0.798
	2. The influence of time pressure on my general attitude toward the other party is ______:		
	3. The influence of time pressure on the offers that I made are ______:		
	4. The influence of time pressure on my concession making behavior is:		
4. Perceived Time Pressure on Other Party	1. I feel that the time pressure the other party is under is ______:		0.890
	2. In my view, the influence of time pressure on the other party's general attitude toward me is ______:		
	3. I feel that the influence of time pressure on the offers that the other party made are ______:		
	4. I think that the influence of time pressure on the other party's concession making behavior is ______:		

Note: * = Item was reverse scored.

(shown in Table 1) were used after purifying the measure. The subjects were asked to rate their perception of own as well as their negotiating partner's power. They were also asked to provide a measure of their perceived power using a 100-point constant-sum scale to be split between the two parties in such a manner that it reflected their perception of the relative powers. The reliability coefficients for the buyer's perception of the bargaining power of self and of the other party (i.e., Seller) are 0.82 and 0.78, respectively. The correlation between the relative perceived power calculated using the multi-item and the constant-sum scale measures was 0.78 ($p < 0.0001$).

Similar to the above, the perception of relative time pressure facing each individual was assessed. The Cronbach alpha reliability coefficients for the subject's perception of the time pressure on self is 0.80 and that for the subject's perception of the time pressure on the other party is 0.89. The correlation between the constant-sum measure scale and the ratio of the self to other four-item category scales was 0.71 ($p < 0.0001$).

Overall, this aspect of the study suggests that the parameters employed in our model are empirically assessable. Since the correlations between the constant-sum scale and the multi-item category scales are significant, we recommend that further analyses should employ the constant-sum scale because, in part, it has much better ratio scale properties.

5.3. A Proposed Laboratory Experiment

In order to test the validity of the analytically derived propositions, and to determine the usefulness of the negotiation process model as a forecasting tool, we sketch out a description of a possible experimental design. To this end a computer stooge can be designed to mimic the behavior of one negotiator. The stooge can be made to play for instance the role of the Seller based on equations (3.1), (3.2), (3.5), (3.6), and (3.9). Human subjects, all assigned to playing the Buyer's role, will be informed that they will be negotiating over a computer network with another subject located elsewhere in the computer laboratory.

The independent parameters of the model that could be manipulated are essentially the initial concession magnitude of the stooge ($X_0 - X_1$) and the negotiation-specific parameter (K). From equation (B.2) (see Appendix B), we note that this negotiation-specific constant (K) comprises the product of parameter values characterizing both the stooge (K_S) as well as the real subject (K_B). Therefore, if (1) the impact of the stooge's initial concession magnitude and (2) the stooge portion of the negotiation specific constant are manipulated at two levels each, say high and low, this would lead to a 2×2 experimental design. On the other hand, if we choose the same value for K_S for all the cells, this would provide the ability to test whether or not identical values of K_S, which conceivably may be generated by drastically different parameter values for the adjustment coefficient (θ_S) and the relative power to time pressure advantage (π_S / α_S), make any difference.

5.3.1. Predictive Validity—A Proposed Procedure. To employ this model as a forecasting tool such as in a laboratory experiment described in the previous subsection, we need to know the first two offers made by the Buyer to calibrate equation (B.9). An alternative benchmark model is needed to compare the accuracy of the forecasts made by our proposed process model (B.9). This is necessary since any measure of error that is employed cannot in a vacuum be said to be good or bad, especially given the innovative nature of our current model. For the purpose of fair comparison, the alternative process model should utilize the same amount and type of information as our model (i.e., only the first two offers).

The following linear model could, for instance, be employed as an alternative for making comparisons since it can also be calibrated on the first two Buyer's offers, and it generates monotonic patterns.

$$Y_t = Y_0 + ([Y_1 - Y_0] \cdot t). \tag{5.2}$$

The symbols here have the same meaning as those in equation (B.9). A nice feature of the linear model proposed for the comparison is that it belongs to a different class of models than the new analytical model; that is to say, there exists no value of K for which the two models are equivalent at all times (t).

To determine the rank ordering of the alternative models, the choice of the error loss function is not too critical (Granger and Newbold 1973). If the task, however, is the comparison of the relative accuracy of the two models, Steece (1982) notes that this choice could be crucial. One can, therefore, employ Theil's U_1 coefficient for the mean absolute deviation (MAD) and the root mean square (RMS) measures to determine the relative forecast accuracy. This comparison can be done at the individual negotiator level.

6. Discussion and Conclusions

In this paper we developed a framework for examining dynamically the process of negotiations by integrating a wide body of literature. The proposed new analytical process model explicitly incorporates constructs considered important by negotiations scholars and practitioners. Specifically, we have incorporated in the model factors such as the aspiration level and limit (i.e., reservation price) of each party, concession points, the relative power and time pressure advantage as perceived by each of the parties as well as the tendency to reciprocate in negotiations, characterizing each negotiator. Our modeling approach has emphasized descriptive and empirical grounding from the perspective of an outside (and objective) observer. It yields various types of characterizations and predictions; namely steady-state individual level insights, discriminating conditions of a negotiator's dynamic offer patterns, and certain dyadic level dynamic results.

The main benefit of this research, of course, is the development of a descriptive model of a negotiator. By changing various parameter values of the model it is possible to simulate many different types of negotiation experiences. One can then predict, from the vantage point of an outside observer, which dyads are more likely to reach an agreement.

We have also subjected the model to an initial empirical investigation, to validate the underlying constructs and also test some of its implications. The results of the field study are quite encouraging.[4] Additionally, published empirical findings from Siegel and Fouraker (1960) suggest, for instance, that the exponential shape of the offer plots as predicted by the model has descriptive validity. Also, recent empirical evidence from Murnighan et al. (1987) experiments supports the notion that bad faith bargaining typically results in disagreement, which is consistent with another implication of our model.

Future research could be directed profitably along empirical avenues. A number of further research questions have been provided in the previous section. They are designed specifically to test both the internal and predictive validity of the model. Also, further scale development and refinements are needed.[5]

[4] Further details on sample instructions for proposed data collection can be obtained either by writing to the first author or consulting Balakrishnan (1988).

[5] The authors gratefully acknowledge the research support provided by the Wharton Risk and Decision Processes Center and the SEI Center for Advanced Studies in Management. They also thank Colin Camerer, Irv LaValle, Gary Lilien, Rich Oliver, Arvind Rangaswamy, Rakesh Vohra, Jerry Wind, the anonymous referees, and the Associate Editor for their helpful comments and suggestions.

Appendix A

The Analytical Process Model: A Formal Derivation

We assume that the play of the game occurs at an infinite sequence of time periods (or innings). The set of time periods is indexed by the nonnegative integers with generic element $t \in N$. The time intervals are, however, so chosen that precisely two moves in the game occur at any one period. Each time period (or inning), the Seller makes the first move, and this is followed by a response from the Buyer. That is to say the two parties involved in the negotiation make their moves sequentially. This order of play repeats itself every inning.

Each party's problem of determining the appropriate offer level is as represented analytically in the text in equations (3.1) through (3.10). Making the appropriate substitutions of equations (3.5) and (3.6) in (3.1) and (3.2) leads to the following offer by Seller:

$$X_t^* = \frac{\pi_S}{\alpha_S + \pi_S}\tau_S + \frac{\alpha_S}{\alpha_S + \pi_S}\beta_{S,t} \quad \text{for} \quad t = 0, 1, 2, \cdots \tag{A.1}$$

That is, the Seller's offer is a convex combination of his or her aspiration level and his or her concession point at time t ($\beta_{S,t}$), which is modeled as follows:

$$\beta_{S,t} = \beta_{S,t-1} + \theta_S(Y_{t-1} - Y_{t-2}) \quad \text{for} \quad t = 2, 3, \cdots$$
$$\text{with} \quad -1 \le \theta_S \le 1, \quad \text{and for some } \beta_{S,0} \text{ and } \beta_{S,1}. \tag{A.2}$$

Similarly, for the Buyer, appropriate substitutions of equations (3.7) and (3.8) in (3.3) and (3.4) result in:

$$Y_t^* = \frac{\pi_B}{\pi_B + \alpha_B}\tau_B + \frac{\alpha_B}{\pi_B + \alpha_B}\beta_{B,t} \quad \text{for} \quad t = 0, 1, 2, \cdots \tag{A.3}$$

Similar to (A.2), we model the Buyer's concession point at time t as:

$$\beta_{B,t} = \beta_{B,t-1} + \theta_B(X_t - X_{t-1}) \quad \text{for} \quad t = 1, 2, \cdots$$
$$\text{with} \quad -1 < \theta_B < 1 \quad \text{and for some } \beta_{B,0}. \tag{A.4}$$

We can now write the analytical expression for the process of demands and offers for the two parties. Suppressing the parties' superscript and lagging (A.1) by one period we obtain

For Seller:

$$X_{t-1} = \frac{\pi_S}{\alpha_S + \pi_S} \tau_S + \frac{\alpha_S}{\alpha_S + \pi_S} \beta_{S,t-1}. \tag{A.5}$$

Subtracting (A.5) from (A.1) we have

$$X_t - X_{t-1} = \frac{\alpha_S}{\alpha_S + \pi_S} [\beta_{S,t} - \beta_{S,t-1}]. \tag{A.6}$$

Now, rewriting equation (A.2) we have

$$\beta_{S,t} - \beta_{S,t-1} = \theta_S(Y_{t-1} - Y_{t-2}). \tag{A.7}$$

Substituting (A.7) in (A.6), we obtain

$$X_t - X_{t-1} = \frac{\alpha_S}{\alpha_S + \pi_S} \theta_S[Y_{t-1} - Y_{t-2}] \quad \text{for} \quad t = 2, 3, \ldots. \tag{A.8}$$

For Buyer:
Based upon (A.3) we can write

$$Y_{t-1} = \frac{\pi_B}{\alpha_B + \pi_B} \tau_B + \frac{\alpha_B}{\alpha_B + \pi_B} \beta_{B,t-1}, \tag{A.9}$$

and

$$Y_{t-2} = \frac{\pi_B}{\alpha_B + \pi_B} \tau_B + \frac{\alpha_B}{\alpha_B + \pi_B} \beta_{B,t-2}. \tag{A.10}$$

Similarly to (A.8) we obtain:

$$Y_{t-1} - Y_{t-2} = \frac{\alpha_B}{\alpha_B + \pi_B} \theta_B[X_{t-1} - X_{t-2}] \quad \text{for} \quad t = 2, 3, \ldots. \tag{A.11}$$

We note that the LHS of (A.11) corresponds to the expression in parentheses of the RHS of (A.8). After appropriate substitution in equation (A.8) we obtain for the seller:

$$X_t - X_{t-1} = \frac{\alpha_S}{\alpha_S + \pi_S} \theta_S \frac{\alpha_B}{\alpha_B + \pi_B} \theta_B[X_{t-1} - X_{t-2}] \quad \text{for} \quad t = 2, 3, \ldots. \tag{A.12}$$

Rewriting (A.12) results in the following difference equation representation of the negotiation process

$$X_t - (K+1)X_{t-1} + KX_{t-2} = 0 \quad \text{for} \quad t = 2, 3, \ldots, \tag{A.13}$$

where

$$K = \frac{\alpha_S}{\pi_S + \alpha_S} \theta_S \frac{\alpha_B}{\pi_B + \alpha_B} \theta_B; \quad (-1 \le K \le 1). \qquad \text{(A.14), (4.2)}$$

Rewriting the above second-order difference equation to describe the process from time t to $t+2$, we have the following difference equation:

$$X_{t+2} - (K+1)X_{t+1} + KX_t = 0 \quad \text{for} \quad t = 0, 1, \ldots. \qquad \text{(A.15), (4.1)}$$

To obtain a solution to (A.15) we need to specify the (two) initial conditions:

$$X_0 = \text{Seller's initial demand}, \tag{A.16}$$

$$\text{and} \quad X_1 = \text{Seller's second demand}. \tag{A.17}$$

A similar procedure yields an identical second-order difference equation for the Buyer with

$$Y_0 = \text{Buyer's initial offer}, \tag{A.18}$$

$$\text{and} \quad Y_1 = Y_0 + \frac{\alpha_B}{\alpha_B + \pi_B} \theta_B[X_1 - X_0] = \text{Buyer's second offer}. \tag{A.19}$$

Obtaining an Analytical Dynamic Solution

Applying Z-transforms (see Gue and Thomas 1968) to obtain an analytical solution to the difference equation (4.1), we get

$$Z^{-2}[X(Z) - X_0 - X_1 Z] - Z^{-1}(K+1)[X(Z) - X_0] + KX(Z) = 0. \tag{A.20}$$

Collecting terms $X(Z)$, we obtain

$$X(Z)[Z^{-2} - Z^{-1}(K+1) + K] = X_0 Z^{-2} + X_1 Z^{-1} - (K+1)X_0 Z^{-1}. \tag{A.21}$$

Multiplying throughout by Z^2, we get

$$X(Z)[1 - (K+1)Z + KZ^2] = X_0 + Z[X_1 - (K+1)X_0]. \tag{A.22}$$

Solving for $X(Z)$ yields

$$X(Z) = \frac{X_0}{(1-KZ)(1-Z)} + \frac{[X_1 - (K+1)X_0]Z}{(1-KZ)(1-Z)}. \tag{A.23}$$

Using partial fraction expansion this becomes

$$X(Z) = \frac{A}{(1-KZ)} + \frac{B}{(1-Z)}. \tag{A.24}$$

Multiply $X(Z)$ by $(1-Z)$ and let $Z = 1$. We have

$$(1-Z)X(Z) = \frac{A(1-Z)}{(1-KZ)} + B. \tag{A.25}$$

We find

$$B = \frac{X_1 - KX_0}{(1-K)}. \tag{A.26}$$

The parameter A can be determined by multiplying $X(Z)$ by $(1-KZ)$ and evaluating this result similarly for $Z = 1/k$:

$$(1-KZ)X(Z) = A + \frac{B(1-KZ)}{(1-Z)} \tag{A.27}$$

$$(1-KZ)X(Z)\Big|_{Z=1/K} = \frac{X_0 + [X_1 - (K+1)X_0]Z}{(1-Z)}\Big|_{Z=1/K}. \tag{A.28}$$

Hence,

$$A = \frac{X_0 - X_1}{(1-K)}. \tag{A.29}$$

Thus, we have

$$X(Z) = \frac{(X_0 - X_1)(1 - K)}{(1 - KZ)} + \frac{(X_1 - KX_0)(1 - K)}{(1 - Z)}. \quad \text{(A.30)}$$

To find the solution, all that remains now to be done is to obtain the inverse transform of these terms. Standard tables can be used to determine these. The inverse transform of (A.30) is:

$$X_t = \frac{X_1 - KX_0}{(1 - K)} + (K)^t\left[\frac{X_0 - X_1}{(1 - K)}\right] \quad \text{for all} \quad t \in N \quad \text{(A.31), (4.3)}$$

Appendix B

Proofs of Propositions

Here we present proofs of some of the propositions presented in §4. First, those relating to the steady-state value of the bargaining process, followed by those for the dyadic level dynamic results.

We have from equation (4.3):

$$X_t = \frac{X_1 - K \cdot X_0}{1 - K} + (K)^t\left[\frac{X_0 - X_1}{1 - K}\right], \quad \text{(B.1)}$$

where $t \in N$, $K = K_S \times K_B$ and

$$K_i = \frac{\alpha_i}{\pi_i + \alpha_i} \cdot \theta_i, \text{ and } -1 < \theta_i < 1; \text{ for all } i = S, B. \quad \text{(B.2)}$$

From (B.1) it is obvious that for this process of offers to reach a steady state we require the value K to be between -1 and 1, i.e.,

$$K \in [-1, 1] \quad \text{for} \quad \lim_{t \to \alpha} X_t \to \text{constant}.$$

Formally, we require

$$|K| = \left|\frac{\alpha_S}{\pi_S + \alpha_S} \cdot \theta_S \cdot \frac{\alpha_B}{\pi_B + \alpha_B} \cdot \theta_B\right| < 1. \quad \text{(B.3)}$$

This condition, we state, always holds as

$$0 \leq \frac{\alpha_i}{\pi_i + \alpha_i} \leq 1 \quad \text{and} \quad |\theta_i| < 1. \quad \text{(B.4)}$$

Under condition (B.3), the steady-state solution exists and is given by

$$\lim_{t \to \alpha} X_t = \frac{X_1 - K \cdot X_0}{1 - K}. \quad \text{(B.5)}$$

For notational ease, we shall in the rest of the appendix refer to

$$\lim_{t \to \alpha} X_t \equiv X_\alpha. \quad \text{(B.6)}$$

Proof of Proposition 4.1.1. Differentiating (B.5) with respect to the second demand, we have

$$\frac{\partial X_\alpha}{\partial X_1} = \frac{1}{1 - K}. \quad \text{(B.7)}$$

The RHS of (B.7) will be positive iff

$$1 - K > 0$$

$$\Rightarrow \quad K < 1$$

The above inequality essentially implies that the RHS of (B.7) will be positive in the entire domain of $K \in [-1, 1]$. Q.E.D.

Proof of Proposition 4.1.2.

$$\frac{\partial X_\alpha}{\partial K_S} = \left(\frac{\partial X_\alpha}{\partial K}\right)\left(\frac{\partial K}{\partial K_S}\right) = \frac{(X_1 - X_0)}{(1 - K)^2} \cdot \left(\frac{\alpha_B}{(\alpha_B + \pi_B)}\right) \cdot \theta_B. \quad \text{(B.8)}$$

The denominator of (B.8) is always positive.

Hence, the RHS of (B.8) will be positive if

a) $X_1 < X_0$ and $\theta_B < 0$.

or

b) $X_1 > X_0$ and $\theta_B > 0$;

Note from §3.2 that as the Seller's coefficient of reciprocity (K_S) increases, the Seller becomes less reciprocative. Q.E.D.

Proofs of propositions 4.1.3 through 4.1.5 follow similarly (see Balakrishnan 1988 for details).

Proofs of Dyadic Level Dynamic Propositions

From equation B.1 and by symmetry we have the negotiation process representation for Buyer as follows:

$$Y_t = \frac{Y_1 - K \cdot Y_0}{1 - K} + (K)^t \frac{[Y_0 - Y_1]}{1 - K}, \quad \text{(B.9)}$$

Where Y_t is the offer made by the Buyer at time period t ($\in N$), and Y_0 and Y_1 represent the initial and second offers.

As before, the steady-state solution exists and is given by

$$\lim_{t \to \alpha} Y_t = \frac{Y_1 - K \cdot Y_0}{1 - K}. \quad \text{(B.10)}$$

Proof of Proposition 4.3.1. In the situation when both parties start with a concession and both are of a similar reciprocity type, it easy to see from equation (B.1) and (B.9) that the sequence of offers of the seller and buyer will be monotonically decreasing and increasing, respectively.

Now, given that there is no immediate agreement at the end of two offers and counteroffers (i.e., $Y_0 < X_0$ and $Y_1 < X_1$), the two parties will not be able to reach an agreement if their steady-state final offers are still not acceptable to the other party. In other words, under these conditions there will be no agreement if:

$$\lim_{t \to \alpha} X_t > \lim_{t \to \alpha} Y_t. \quad \text{(B.11)}$$

This implies that there exists no finite time at which the sequence of offers of the two parties intersect.

Substituting for the steady-state final offers in (B.11) we have

$$\frac{X_1 - K \cdot X_0}{1 - K} > \frac{Y_1 - K \cdot Y_0}{1 - K}. \quad \text{(B.12)}$$

Therefore, this implies that

$$K < \frac{X_1 - Y_1}{X_0 - Y_0}. \tag{B.13}$$

Proof of Proposition 4.3.2. Proof is by similar arguments, except now we require that at the least the final offers must be acceptable to the other party for an agreement to be reached. Therefore, with all other conditions the same as in 4.3.1, we require for an agreement to be reached that:

$$K > \frac{X_1 - Y_1}{X_0 - Y_0}. \tag{B.14}$$

Proof of Corrollary 4.3.2. From (A.31), (B.9) and the definition of when agreement is reached, that is, when $Y_t \geq X_t$ (or $X_{t+1} \leq Y_t$), we have t^*, the number of innings needed to reach an agreement:

$$\frac{Y_1 - K \cdot Y_0}{1 - K} + (K)^{t*}\left[\frac{Y_0 - Y_1}{1 - K}\right] \geq \frac{X_1 - K \cdot X_0}{1 - K} + (K)^{t*}\left[\frac{X_0 - X_1}{1 - K}\right]. \tag{B.15}$$

Rearranging (B.15) and taking logarithms on both sides we obtain

$$t^* \geq \log_K\left[\frac{K(X_0 - Y_0) - (X_1 - Y_1)}{(X_0 - Y_0) - (X_1 - Y_1)}\right]. \quad \text{(B.16), (4.4)}$$

Proof of Proposition 4.3.3. Under condition (4) stated in Proposition 4.3.3, we know that the steady-state final offers of the two parties are not acceptable to each other. In addition, if the Seller starts with a concession and the two parties are not of a similar reciprocity type (i.e., one is exploitative and the other reciprocative), we note from proposition 4.2.2 that the sequence of offer of both parties will resemble a damped sawtooth pattern. Consequently, that implies that the only opportunity for an agreement is at the time when the Buyer makes his or her second offer (Y_1). At this instance an agreement can occur only if the Buyer is reciprocative and his or her second offer (Y_1) is greater than or equal to that of the Seller's (X_1). Otherwise, there will be no agreement.

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