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Power law and evolutionary trends in stock markets

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Abstract

We model the distribution of daily stock trading volume using the power law and document a new phenomenon. The power law exponent systematically increases with time suggesting that trading is becoming increasingly concentrated in a subset of stocks.

Keywords: Zipf's Law; Hirschman–Herfindahl Index; Stock trading volume

JEL classification: G10; H39; D39

1. Introduction

Trading in stocks has grown dramatically over the last five decades but it is an open question as to whether this growth has been uniform across all stocks or is disproportionately concentrated in a subset of stocks. To measure the unevenness of trading across stocks, we model the distribution of daily trading volume across all stocks in the U.S. market and in each of the three U.S. exchanges as a power law function. We discover a new phenomenon in which the power law exponent systematically increases with time and we find exponents evolve for the market and for each of the exchanges over the years 1962 to 2005. This non-static, evolutionary trend is striking because physical and social science studies that employ the power law consistently report that exponents are stable over time. Prior studies that employ the power law to model financial variables such as firm size (Axtell, 2001), stock returns (Gabaix et al.,

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2003) and net income (Okuyama et al., 1999) report that the exponents are similar across different types of markets and tend to be stable over time. Changes in the power law exponent over time allow us to select from competing hypotheses to describe the evolution of concentration in trading across stocks and we conclude that daily trading is becoming more concentrated in a subset of stocks.

Even casual observation of the financial press shows an explosive growth in stock trading volume in recent decades. New savings opportunities such as Individual Retirement Accounts and 401(K) programs and the explosive growth of financial information have drawn new investors to the market. Stock exchanges, in turn, have responded by expanding the stock offerings available to investors. The effects of these changes on the market may be examined through the lens of Zipf's Principle of Least Effort. This principle holds that there are two opposing forces at play, those of unification and diversification. The influence of these opposing forces suggests three alternative hypotheses. The first hypothesis, consistent with the force of diversification, is that an increase in the number of investors and investment opportunities would increase competitiveness in the stock market with investors spreading their money over a larger set of competitive choices. This implies that investors devote attention and analysis to all stocks and trading is uniformly distributed over all stocks. An alternative hypothesis, consistent with the force of unification, is that, investors, overwhelmed by the plethora of stock investments offered, limit their analyses and choices to a small subset of highly regarded stocks, and in the limiting case the daily choice reduces to a single stock, consistent with herding behaviour. A third hypothesis is that increases in trading volume and investment opportunities are unrelated to market competitiveness leading to purely random fluctuations in market concentration. In this case, forces of unification and diversification offset, leading to a static level of trading concentration. To determine if and how the stock market is evolving, we examine if there are changes over time in a measure of market concentration and discern between these three alternatives.

The power law is widely recognized as a universal law in describing many physical phenomena. Zipf (1949) extended the use of the power law beyond the physical sciences and showed that the law was applicable to the social sciences. A generalization of Zipf's finding, applied to any item that can be ranked by size, can be stated as:

$$(\mathrm{Size})_i \times (\mathrm{Rank}_i)^q = \mathrm{constant} \tag{1}$$

The exponent 'q' is specific to the item examined and is known as Zipf's parameter or the power law exponent. Zipf's Law is a special case of the power law in which the exponent 'q' equals 1. Power law functions and, in some cases, Zipf's Law, have been found to accurately model a remarkably wide range of physical science, social science, and economic phenomena. For example, Gopikrishnan et al. (2000) study the trading of individual stocks over a 2-year (1994–95) period and show that the distribution of trading volume for a stock obeys a power law function with an exponent of approximately 1.5 and that this value seems to hold for each of the 1000 largest stocks. Naldi (2003) shows that the power law exponent can also be used as a general measure of market concentration, and that the larger the exponent the greater the level of concentration.

2. Analysis

In this paper, we model the distribution of the trading volumes for each day across all stocks in the US stock market as a power law function and estimate the value of the daily power law exponent both at the market and at the exchange level. We estimate the power law model for each trading day, from 1962 to 2005, with three objectives in mind. First, we determine if the power law accurately models daily trading volume and if it holds across the exchanges and over time. Second, we examine differences in the exponents, if any, across stock exchanges. Third, we investigate if the power law exponent fluctuates randomly or varies systematically over time within each exchange, and if so, the implications of such changes on trading concentration.

The trading volume data for our analysis are obtained from the Center for Research in Security Prices (CRSP) database. We model the distribution of daily trading volume for all stocks traded on the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX) from July 1962 through December 2005 and on NASDAQ from November 1982 to December 2005. Table 1 provides descriptive statistics of the trading volume data, rank ordered from the most heavily traded to the least heavily traded stock on that day on a specific exchange.

To estimate the power law exponent, studies have typically used the ordinary least squares regression approach (Axtell, 2001; Gabaix et al., 2003). We first employ the OLS regression and find that the daily adjusted R-squared for the U.S. market ranges from 0.68 to 0.88, suggesting that the power law appears to be a reasonably good model of daily trading volume distribution. However, as Doray and Luong (1995), Naldi and Salaris (2006) and Goldstein et al. (2004) point out, the maximum likelihood estimate produces a more accurate and robust estimate of the power law exponent and that this has the least variance in

Table 1
Descriptive data of daily trading volume (in number of shares traded) for the U.S. market, the New York Stock Exchange, and the American Stock Exchange for the period July 2, 1962 to December 31, 2005 (10,951 trading days) and for the Nasdaq exchange from November 1, 1982 to December 31, 2005 (5848 trading days)

	Mean	SD	Maximum	Minimum
Panel A: U.S. market (all 3 exchanges)				
Number of Stocks	4545	2308	8776	1570
Daily total trading volume	766,548,230	1,250,642,013	6,153,538,108	2,525,360
Most heavily traded stock volume	20,405,692	46,723,220	1,514,053,570	39,500
Least heavily traded stock volume	53	43	100	1
Panel B: New York Stock Exchange				
Number of Stocks	2023	562	3073	1113
Daily total trading volume	381,119,426	576,846,755	3,246,044,677	1,960,045
Most heavily traded stock volume	11,207,026	23,634,574	399,208,700	36,200
Least heavily traded stock volume	79	35	200	1
Panel C: American Stock Exchange				
Number of Stocks	649	71	874	386
Daily total trading volume	26,275,275	53,058,726	509,784,520	446,375
Most heavily traded stock volume	5,686,096	14,407,630	176,088,500	13,300
Least heavily traded stock volume	69	37	100	1
Panel D: Nasdaq Exchange				
Number of Stocks	3427	680	4972	1956
Daily total trading volume	672,156,709	739,612,529	3,301,948,719	23,253,111
Most heavily traded stock volume	34,896,225	56,965,131	1,514,053,570	537,600
Least heavily traded stock volume	55	49	100	1

Table 2
Maximum likelihood estimate of the power law exponent is computed for every trading day for the U.S. market and for each of the three exchanges [New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and Nasdaq]

Exchange	Mean	Median	Max	Min
Power law exponents (q_t)				
U.S. market (all three exchanges)	0.81	0.81	1.15	0.66
New York Stock Exchange	0.79	0.78	1.12	0.65
American Stock Exchange	0.98	0.94	2.11	0.64
Nasdaq Exchange	0.92	0.90	1.32	0.76
Standard errors of the power law exponents				
U.S. market (all three exchanges)	0.00008	0.00004	0.00038	0.00000
New York Stock Exchange	0.00010	0.00005	0.00049	0.00001
American Stock Exchange	0.00027	0.00020	0.00095	0.00002
Nasdaq Exchange	0.00003	0.00003	0.00011	0.00001

The summary measures are reported in the top panel. The standard error of the power law exponent is computed daily for each exchange and the summary measures of these standard errors are in the bottom panel.

comparison to the other methods. We, therefore, estimate the power law exponent, q_t, for each day (t), for the U.S. market and for each of the three major exchanges, using the maximum likelihood estimation (MLE) procedure as detailed by Naldi and Salaris (2006). Our measure of daily trading volume for each firm's shares is vol_{it}, the daily volume for firm i on day t. A comparison of the MLE and OLS estimates of the power law exponent finds that though the numerical values of the exponent are different, our substantive conclusions are similar; consequently, only the MLE exponents are reported in this paper.

The summary statistics for the MLE power law exponent estimates from 1962 to 2005 are reported in Table 2. This table shows that the average of the daily power law exponents across the trading periods examined is 0.81 for the U.S. market as a whole and 0.79, 0.98, and 0.95 for the NYSE, AMEX and Nasdaq exchanges, respectively. The maximum likelihood estimates of the exponents are remarkably robust given the relatively small standard errors associated with them. These exponent values indicate that daily trading volume does not quite follow the special case of Zipf's Law (i.e., an exponent value of 1). To examine the presence or absence of trend, we plot the daily values of the power law exponents for the US

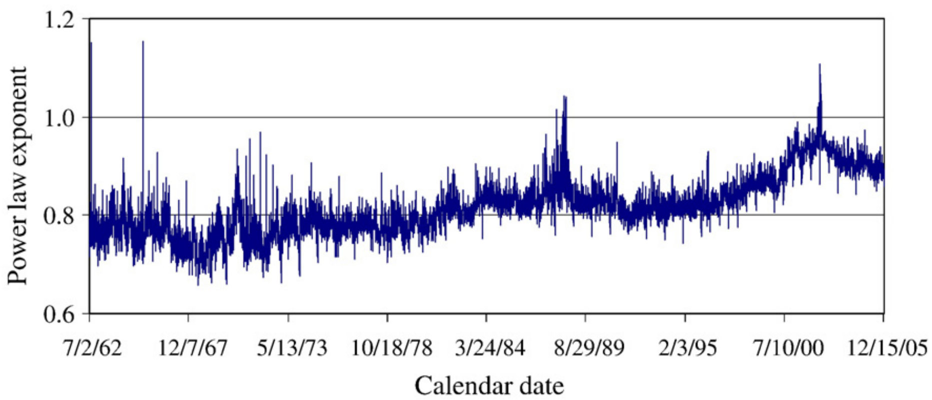

Fig. 1. ML estimates of power law exponents for US stock markets (July 1962–December 2005).

market (Fig. 1) and for each of the three exchanges (Fig. 2). The time-series plots of the power law exponents as depicted in these plots are revealing. There is a clear upward trend in the daily power law exponent both for the market as a whole and for each of the three exchanges over the four decades studied. The upward trend in the daily exponents is confirmed with a time-series regression of q_t, suggesting very similar values in the slope for each of the exchanges, as reported in Table 3. The similarities of the consistent upward trend and fluctuations across exchanges are confirmed by the high correlations of the exponent values that range from 0.59 to 0.77 between the exchanges (see Table 3). This upward trend in the values of the power law exponents suggests that stock trading has become increasingly concentrated in the period under study.

Interestingly, we find that the power law exponent appears to reflect the changes in stock market concentration better than a more commonly used measure of industry concentration in economics, the Hirschman–Herfindahl Index (HHI). Naldi (2003) has shown analytically that the HHI is a more elastic measure of concentration in those situations where a power law applies. This leads to situations, as in this study, where we find the daily measures of the HHI to be extremely volatile and the huge fluctuations in

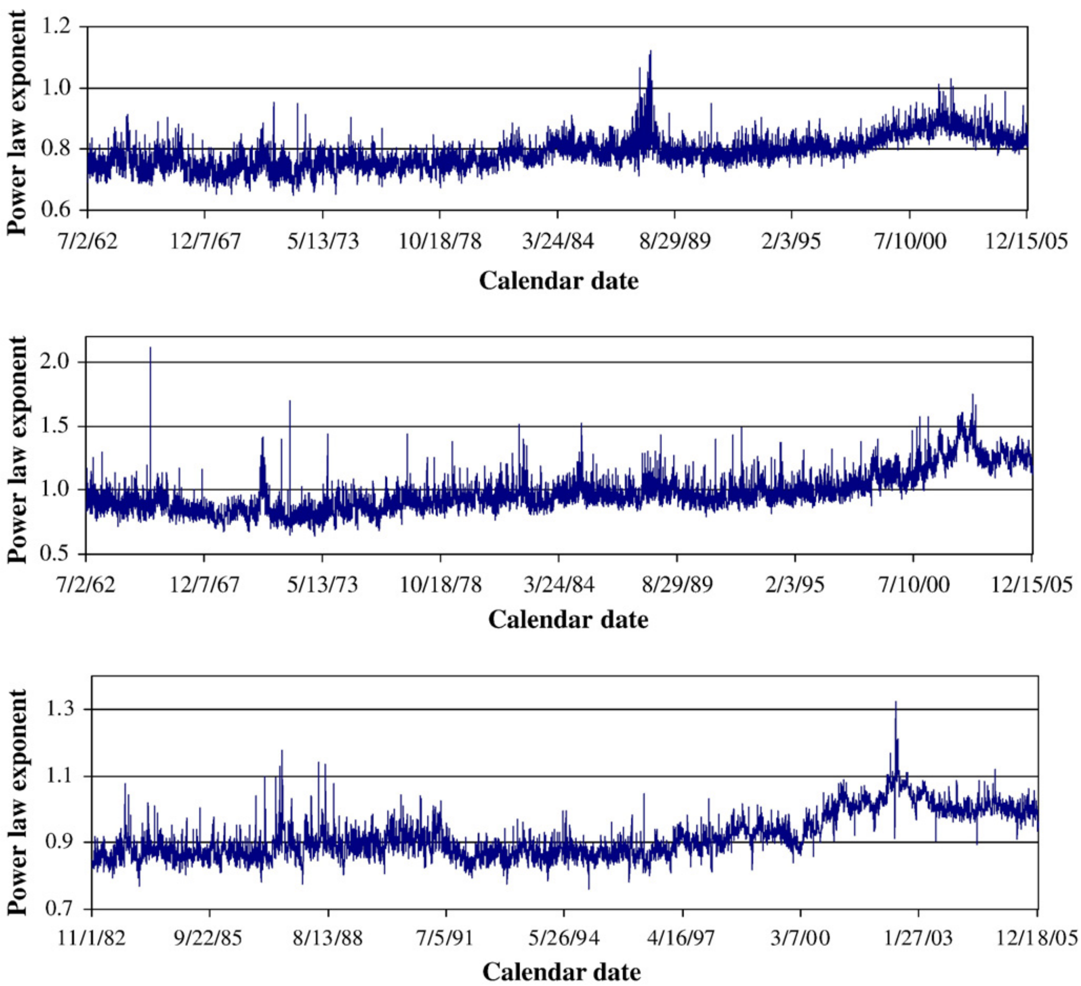

Fig. 2. ML estimates of power law exponents for NYSE stocks (July 1962–December 2005), AMEX stocks (July 1962–December 2005), and Nasdaq stocks (November 1982–December 2005).

Table 3
Trend analysis of power law exponents

Exchange	Slope of regression line	F-statistic	R-square for time-series regression
Panel A: Goodness of fit measures for the time-series regression of the daily power law exponent (q_t) vs. calendar time. The time-series regressions are based on 10,951 daily observations for the U.S. market, the New York Stock Exchange (NYSE), and the American Stock Exchange (AMEX) from 1962 to 2005 and 5,848 daily observations for the Nasdaq exchange from 1982 to 2005.			
U.S. market	9.76×10^{-6}	17111 ($p<0.0001$)	0.61
NYSE	7.47×10^{-6}	10060 ($p<0.0001$)	0.48
AMEX	24.3×10^{-6}	11606 ($p<0.0001$)	0.51
Nasdaq	18.7×10^{-6}	4945 ($p<0.0001$)	0.46

Panel B: Correlation coefficients of daily power law exponents (q_t) between the U.S. market, NYSE, AMEX, and Nasdaq exchanges across 5,848 daily observations from 1982 to 2005.

	U.S. market	NYSE	AMEX	Nasdaq
U.S. market	1			
NYSE	0.84	1		
AMEX	0.78	0.58	1	
Nasdaq	0.91	0.59	0.77	1

the daily HHI obscure any noticeable trend in the concentration of daily trading. We note that the time-series regressions of HHI measures have R-square coefficients of 0.00 for the U.S. market and 0.00, 0.24 and 0.21 for the NYSE, AMEX and Nasdaq, respectively. However, we do find mildly significant positive slopes in these regressions, providing additional evidence that market concentration is increasing with time. We also find that the correlation of the daily HHI measures with the contemporaneous MLE power law exponents is relatively high, with correlation coefficients of 0.50, 0.88, and 0.85 for NYSE, AMEX, and Nasdaq exchanges respectively.

3. Conclusions

Viewing the power law exponent as a measure of concentration, we see that daily trading is consistently becoming increasingly concentrated over time. Within the framework of Zipf's Principle of Least Effort, our empirical results suggest that the force of unification seems to be steadily outweighing the force of diversification; that is, investors, perhaps overwhelmed by the number of investments available, appear to be restricting their choices to a small subset of highly regarded stocks. These actions appear to be consistent with the herding behaviour premise. It would be interesting to investigate if such an evolutionary pattern characterizes other international stock exchanges as well.

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