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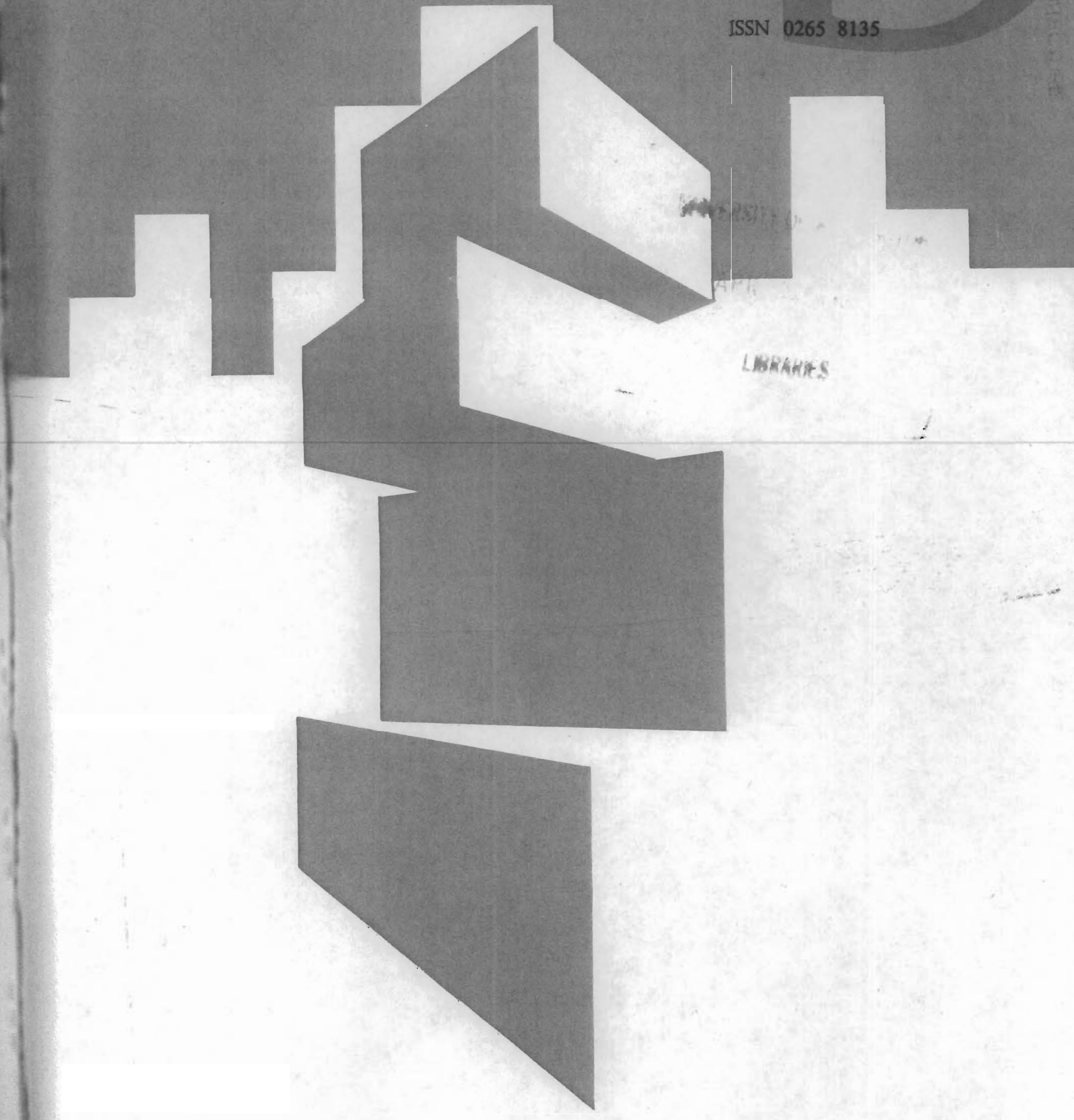

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Efficiency evaluation of retail outlet networks

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Abstract. The authors generate alternative location-covering scenarios for retail outlet networks by using a programming model which guarantees the spatial market 'adequacy' of individual facility sites. The notion of relative spatial efficiency is then used to evaluate these scenarios and to develop information constructs which support many of the managerial decisions necessary for the appropriate structuring of such networks.

1 Introduction

Recognition of the import of geography and spatial structures for store location strategy has resulted in a fruitful marriage between location analysis and operations research techniques. A variety of spatial models have resulted in the development of formal approaches to determining optimal retail sites. Early theoretical developments were focused on the location of a single store (Applebaum, 1965; Huff, 1964); more-recent studies have led to the development of sophisticated models for siting store networks (Achabal et al, 1982; Ghosh and McLafferty, 1987).

A major contributing factor to the feasibility and success of a network of retail outlets is the existence of *adequate* markets, within a given *range*, to support the stores. This concept of the minimum amount of demand necessary to support the supply of a good at a specified location has been operationalized in the literature on location analysis by incorporation of *threshold* and *range* constraints (see Balakrishnan and Storbeck, 1991). However, a feature common to these models is that they seek an *optimal* solution based upon the optimization of a set of predefined objectives. In so doing, these optimization models typically ignore nonoptimal sites and thereby lose information that may be useful, in the context of actual decisionmaking, when changes in the markets require contraction or expansion of the facilities.

To accommodate the dynamic aspects of locational planning, Fisher and Rushton (1979) proposed the concept of spatial efficiency, which assigns to each locational configuration a value commensurate with its utility as measured on a variety of spatial criteria. Furthermore, in operationalizing the concept of *relative spatial efficiency* (RSE), it has been argued that, with the infrastructure pretty much given, the emphasis in location analysis is gradually moving away from the determination of entire network systems to the examination of issues related to more limited locational decisionmaking (Desai and Storbeck, 1991). That is, within the realm of decisionmakers, analytical techniques which are sensitive to incremental changes within locational systems are increasingly valued.

In this paper, we continue our work on the preservation of choices in such planning exercises by evaluating not only those locations deemed *optimal* in terms common to location-allocation modeling, but also those sites which are *not* optimal within algorithmic frameworks, but may prove 'interesting' and valuable to the decisionmaker for other reasons (Desai and Storbeck, 1991). Moreover, we study how the imposition of threshold requirements alters the spatial efficiency of a network of retail outlets and discuss how further concerns about other managerially important (aspatial) factors such as market equity can be incorporated in the analysis.

We begin in the next section by introducing Balakrishnan and Storbeck's approach to modeling maximum coverage with threshold constraints. In section 3 we introduce the basic ideas of RSE, and in section 4 we introduce market inequity as an aspatial evaluation criterion. In section 5, we use numerical examples to illustrate how the efficiency of networks depends upon the spatial characteristics of the market and further discuss the implications, in terms of spatial efficiency, of incorporating aspatial criteria in this type of analysis. We conclude the paper in section 6 with remarks on the utility of *efficiency* (as opposed to *optimal*) perspectives in locational planning.

2 Network configurations with threshold

Balakrishnan and Storbeck (1991) developed the McTHRESH model to capture the spatial interplay between the range of a good and the threshold of a supplier at particular points in a region. Specifically, this model addresses the need to site facilities, such that the overall spatial market coverage of demand within some range is maximized. Although this model allows the overlapping of the spatial markets of different facilities, it requires that each facility must nonetheless receive a stipulated threshold level of demand. Thus, the McTHRESH analytical model corresponds to the following verbal statement:

> *Find the number and locations of facility sites which (1) maximize the spatial extent of market coverage within some predetermined range, and (2) maintain required threshold levels of demand for all sites.*

The working of the model is best illustrated in reference to figure 1. In this hypothetical example, facilities are located at sites 1 and 2. Facility 1 covers all the demand in areas A and B, and facility 2 covers that in B and C. According to the McTHRESH model, these two sites are optimal if no other two sites offer better coverage of the market, and if the demand in area A (area C) plus one half of the demand in area B is at or above the threshold requirements of facility 1 (facility 2).

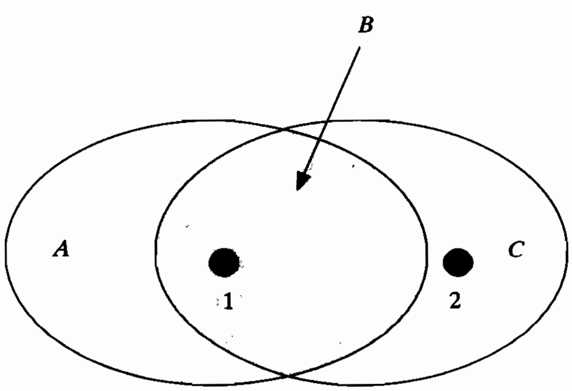

Figure 1. A hypothetical example of facility location.

Though the siting process within this model is driven primarily by the goal of maximizing the overall market coverage, the eventual choice of sites is greatly influenced by several parameters: the number of facilities to be sited, the required threshold level of demand at candidate sites, the allowed extent of the overlap in the spatial market, and the range of the good under consideration. Consequently, the McTHRESH model allows the decisionmaker to stipulate values for each of these parameters in structuring appropriate planning scenarios (Balakrishnan and Storbeck, 1991). See the appendix for a mathematical programming formulation of the McTHRESH model.

The McTHRESH program, in its flexibility, allows for the generation of numerous solutions, based on the variation of the parameters, to the problem of siting a network of facilities. However, it does not provide for the evaluation of these different scenarios. Indeed, McTHRESH, like all linear programs, merely suggests 'what's best' under very specific parametric conditions; it does not assist the decisionmaker in evaluating the appropriateness of given conditions. Consequently, the focus of most single-objective (and multiobjective) location-allocation programs is on the generation of the best set of sites, given stated criteria and known constraints. Our purpose, however, is to determine the appropriateness of different sets of sites—however generated—along accepted dimensions, as specified by the decisionmakers. Thus, in this paper, we use McTHRESH as a site-generation technique, developing network configurations where each configuration is optimal under different parametric conditions.

It ought to be emphasized that these McTHRESH solutions are optimal in a rather restricted sense; that is, only when one is maximizing coverage while maintaining a required threshold for all sites. Additional criteria, not readily implementable in these spatial models, must be used to select the 'best' or most efficient configurations. Thus, models such as McTHRESH are used to generate a reduced choice set while maintaining requisite variety, RSE measures are then constructed to select the most efficient among these.

Hence, our unit of analysis is the whole system or configuration of sites. These configurations differ from each other incrementally, usually with one site replaced by another or having an additional site. Thus, in comparing these configurations, we are evaluating incremental changes in the system.

3 Models for measuring efficiency

In proposing that consumers' demand for a good is more effectively understood in terms of its attributes, Lancaster (1971) modified the concept of a good and consequently the analysis of consumer demand (Balakrishnan and Desai, 1991; Hauser and Simmie, 1981; Ladd and Zober, 1977). Following a similar logic, techniques based on *data envelopment analysis* (DEA) have been used in the analysis of the spatial and aspatial attributes of locational configurations to obtain measures of RSE (Desai and Storbeck, 1991; Desai et al, 1994). Such modeling efforts suggest that decisionmakers are, most often, more interested in the attributes of a siting configuration than in the sites themselves. Using the McTHRESH model as a basis, we extend the analysis of spatial efficiency to study and evaluate retail outlets. In particular, the focus of this paper is on the 'cost' of constraints, such as those caused by *adequacy* and *equity* of the market area, that decisionmakers (franchisors, for instance) must consider in making decisions about the siting of retail outlets.

However, before discussing the concepts of market adequacy and equity, we first establish links between the efficiency models introduced by Debreu (1951) and Farrell (1957), and their estimation and the basic notions underlying RSE. Debreu and Farrell developed measures to evaluate the performance of a production process in terms of its efficiency in converting product inputs into outputs. Such measures are based on identifying a production function that embodies the optimal relationship between the inputs and outputs (Fare et al, 1985). Production units which exhibit this optimal rate of conversion of inputs into outputs are said to be efficient and those which use more inputs to produce the outputs or produce less outputs with the same amount of inputs are said to be inefficient. DEA (Charnes et al, 1978) is a technique which is used to obtain such measures of *relative* efficiency.

In implementing a DEA-based measure of RSE, however, we depart from the context of the typical production process; instead, we follow Lancaster (1971) in obtaining these measures of efficiency in terms of the attributes of a configuration of sites. Therefore, the unit of analysis is a configuration of sites (or a network of retail stores) which is a solution obtained from the McTHRESH model. Associated with each configuration are attributes, some of which are 'bad' and some 'good'. In other words, one would want to assign a high-performance or efficiency score to configurations which have minimal levels of the bad attributes and maximal levels of the good attributes. Thus, without making the procedural link of a production technology wherein inputs are converted into outputs, we treat the undesirable attributes, which are to be minimized, as 'inputs' and the desirable attributes, which are to be maximized, as 'outputs' (Adolphson et al, 1991). Hence, there is a conceptual link between the Debreu-Farrell technical efficiency measurement and our analysis of RSE in attribute space. A variety of techniques based on econometric and mathematical programming methods have been developed to obtain these measures. DEA is one such technique in which linear programming is used to construct these measures. To help provide an understanding of the computations involved, we next introduce the underlying linear programs.

In order to present the linear programming models underlying the measurement of RSE, we must first introduce some notation. Let e_{na} denote the observed level of desirable attribute n of the ath network. Let f_{ua} denote the level of undesirable attribute u of the ath network. Assuming that there are L networks with N desirable and U undesirable attributes, we denote by $\mathbf{E}$ the matrix of desirable attributes, where $\mathbf{E}$ is of order $N \times L$. Similarly, we let $\mathbf{F}$ be the matrix of undesirable attributes, where $\mathbf{F}$ is of order $U \times L$.

Assume that in attribute space, these networks can be represented by combinations of these $\boldsymbol{f}$ and $\boldsymbol{e}$ vectors, then the set of all feasible networks, such that a proportionality is maintained between $\boldsymbol{f}$ and $\boldsymbol{e}$, is

$$C = \{(\boldsymbol{f}, \boldsymbol{e}): \boldsymbol{e} \leqslant \mathbf{E}\boldsymbol{z}, \mathbf{F}\boldsymbol{z} \leqslant \boldsymbol{f}, \boldsymbol{z} \in \mathbb{R}^L_+\} ,$$

where $\boldsymbol{z}$, $\boldsymbol{z} = (z_1, z_2, ..., z_L)$, denotes the vector of parameters which determine the combination of the attributes $(\boldsymbol{f}^a, \boldsymbol{e}^a)$ of the ath network. Let $(\boldsymbol{f}^0, \boldsymbol{e}^0)$ be a given observation of attributes of some network. Then the relative spatial efficiency of that network is:

$$\mathrm{R}(\boldsymbol{f}^0, \boldsymbol{e}^0) = \min\{\theta: (\theta\boldsymbol{f}^0, \boldsymbol{e}^0) \in C\} ,$$

which is the optimal solution of the following linear program:

$$\text{minimize } \theta ,$$

subject to

$$\mathbf{E}\boldsymbol{z} \geqslant \boldsymbol{e}^0 ,$$

$$\mathbf{F}\boldsymbol{z} \leqslant \theta\boldsymbol{f}^0 ,$$

and

$$z \geqslant \mathbf{0}.$$

Thus, in this instance, the RSE measure, θ, denotes the proportional reduction in the undesirable attributes that can be achieved while maintaining the current level of the desirable attributes.

4 Measuring the effect of threshold and range

To lend some substance to the attributes, consider a network of retail locations and the distance that a person has to travel to reach the store. We can associate, with each potential network, measures of *distance*, such as the average distance or the maximum distance, a person within the market area has to travel. Similarly, another attribute of the network could be the number of people not within the range of any of the stores in the network. In terms of location analysis, these people are left *uncovered* and ideally one would want this number to be zero. Very often, one of these attributes must be traded off against the other.

The unit of analysis in our evaluation is an entire network of sites, so we are presented with solutions from the McTHRESH model, each of which represents a viable system configuration or network, maximizing overall coverage of its market demand. However, this optimization process does little to guide the allocation of spatial markets to individual stores, which can vary considerably. Considering unevenness of demand to be a negative attribute, then, we use the variation in allocated demand as a measure of *market inequity*. In so doing, we include in this analysis certain aspects of locational decisions which have both spatial and aspatial dimensions, reflecting concerns with regional variations in the market, as well as 'fairness' within the organizational structures of an operating environment.

Specifically, the *inequity* of spatial market allocations at different sites in a network represents a particularly troublesome source of potential conflict in a marketing system. As management now spends considerable time dealing with conflict between members in a distribution channel (Thomas, 1976), it makes sense for decisionmakers to attempt the 'design' of *minimum conflict* decisions. Market inequity can be caused by a number of factors, but it most commonly manifests itself in terms of uneven demand. Thus, as a measure of this inequity, we use the root mean square (RMS) deviation of market demand, defined simply as the sum of the squared deviations of demand within the spatial market of a site from that of the network mean, divided by the number of sites in the network. This simple measure captures the regional variability in demand across markets and can, therefore, be construed to be an indicator of inequities in the market. Such inequities in outcomes often lead to interpersonal dissatisfaction and in turn to organizational conflict (Bazerman et al, 1992). Therefore, this measure essentially penalizes those network configurations in which certain sites have disproportionately larger or smaller demand than the network average.

To maintain the links with the results of the McTHRESH model (Balakrishnan and Storbeck, 1991) and previous measurements of RSE, we use geographical data from Swain (1971) to measure the effect of threshold requirements on the efficiency of retail networks. The average distance and demand uncovered are two (spatial) measures of performance used in this analysis, and we introduce the RMS realization of spatial market equity as a third (aspatial) measure. Consequently, this study is focused on these three attributes of siting configurations, each of which is to be minimized.

5 RSE measurement

The investigation of RSE begins, in this context, by generation of twenty-five locational siting scenarios based on the solution of the McTHRESH program for different parameter values of the model. Although *any* set of sites can be generated for this analysis, we limit our consideration of candidates to McTHRESH solutions for the purposes of direct comparison of *optimal* versus *efficient* solutions. Specifically, we generate maximum coverage networks for various values of the following parameters: number of facilities (4–6); extent of overlap (2, 3); and threshold (0–120). Each network is defined by its particular combination of these three characteristics. The left-most column of table 1 gives the names of the networks under consideration; S6250, for example, signifies the McTHRESH model solution for six facilities, with a maximum allowed market overlap value of two and a threshold of fifty. Where overlap has no effect on a solution (that is, sites for overlap values of 2 or 3 are identical), the second digit in the network identifier is replaced with a 'B'; so, S6B0 is the network identifier for the six-facility solution, with maximum overlap set at 2 *or* 3, and a threshold of 0.

Table 1. Input data.

Network	Distance	Uncoverage	Variation
S6B0	57.57	15.00	135.1
S6B20	57.46	18.00	109.3
S6B40	67.14	28.00	120.3
S6250	79.79	45.00	41.1
S6350	56.86	41.00	69.6
S6360	67.26	55.00	61.6
S6370	92.09	72.00	31.9
S5B0	65.91	31.00	106.9
S5B40	60.07	33.00	139.6
S5250	71.90	47.00	84.4
S5260	69.34	61.00	68.6
S5270	71.29	76.00	56.3
S5350	67.42	46.00	90.8
S5360	70.25	61.00	68.8
S5370	85.83	73.00	37.2
S5380	94.49	90.00	21.8
S5390	92.48	96.00	14.0
S4B0	74.06	59.00	155.8
S4B50	75.80	60.00	114.6
S4B60	83.84	68.00	96.6
S4B70	96.84	75.00	51.3
S4B80	96.65	96.00	23.6
S42110	95.84	104.00	10.4
S4B120	91.16	116.00	12.0
S43110	95.85	101.00	20.3

Our investigation of the above parameters is intended to examine primarily those aspects of the siting problem which are under the direct control of the decisionmaker. Determining the number of facilities, for example, has a direct effect on system performance (that is, demand coverage), as well as implications for system cost. Obviously, the more facilities one sites, the better coverage one achieves, and the greater costs one incurs. Setting the maximum extent of spatial market overlap carries with it implications for both producer and consumer. The more one allows one's markets to overlap, the more choices one gives to consumers as to the number of spatial markets in which they can participate. Increased overlap, for the producer, however, can also *cannibalize* the market area of individual sites in a system. Determining the threshold level of individual outlets in a network has an effect on the siting of a store vis-à-vis other stores in the system. It represents the 'guaranteed' market area allocated by the system to the individual store site. Consequently, it has profound implications for the spacing of facilities in a region and is intimately connected to the above determinations of the number of facilities and the extent of overlap. Although the range of a good can be manipulated somewhat by decisionmakers (via certain strategic actions available to them, such as advertising campaigns), we chose to fix this parameter's value at 10 miles for the purposes of this illustration, because it represents an aspect of the problem which is still largely determined by consumer behaviors (Storbeck, 1988).

Table 1 also shows the three (undesirable) attributes of each network, which are pertinent to the RSE analysis: average distance, number 'uncovered', and the RMS deviation measure of the demand variation or inequity in spatially allocated markets. In order to orient the reader with graphical depictions, we first examine the results of the RSE model based on just two dimensions: average distance and number uncovered.

Table 2 lists the RSE scores for this two-dimensional problem, and figure 2 displays the solutions in attribute space. As can be seen, the networks generally conform to our expectations with respect to spatial efficiency. That is, the greater the number of facilities and extent of overlap, and the less threshold required, the more spatially efficient the network. In other words, we would expect those siting configurations with the largest number of facilities, allowed to seek the greatest

Table 2. RSE (relative spatial efficiency) scores.

Network	Basic model (two dimensions)	Market equity model (three dimensions)
S6B0	1.000	1.000
S6B20	1.000	1.000
S6B40	0.853	0.853
S6250	0.715	1.000
S6350	1.000	1.000
S6360	0.845	0.950
S6370	0.617	0.927
S5B0	0.868	0.873
S5B40	0.951	0.951
S5250	0.792	0.832
S5260	0.820	0.892
S5270	0.796	0.938
S5350	0.844	0.844
S5360	0.809	0.888
S5370	0.662	0.937
S5380	0.602	0.943
S5390	0.615	1.000
S4B0	0.768	0.768
S4B50	0.750	0.750
S4B60	0.678	0.695
S4B70	0.587	0.797
S4B80	0.588	0.910
S42110	0.593	1.000
S4B120	0.624	1.000
S43110	0.593	0.928

amount of overlap under minimal threshold constraints, to have the most desirable attributes. Such is the case in our example. The RSE frontier is defined by the six-facility solutions for thresholds of 0, 20, and 50 (S6B0, S6B20, S6350). The first two of these networks represent solutions obtained when overlap was set to either 2 or 3 (that is, overlap had no effect on the selection of sites); the third network represents a solution where overlap was set at 3. One should note, however, that not all six-facility solutions are found on this frontier. In general, as threshold requirements are increased, even these solutions become less efficient. At an overlap value of 2, for example, six-facility solutions become inefficient when threshold rises to levels of 40 and 50. Not until the overlap maximum is moved to a value of 3 does the six-facility solution at a threshold of 50 (S6350) return to perfect efficiency. When the threshold value is moved still further to 60, however, even the six-facility network with an overlap value of 3 becomes inefficient, as S6360 has an RSE score of 0.845.

Figure 2 also displays the consequences of the possible removal of selected networks. For example, consider a situation wherein a threshold minimum is established for six-facility networks, such that solutions below a threshold value of 30 are not acceptable. Subsequently, networks S6B0 and S6B20 would be removed and a second frontier, defined by S6B40, S5B40, and S6350, would emerge. As in any DEA, each of these frontier networks is perfectly efficient in its own right. In this case, S6B40 represents the network with the least number uncovered; S6350 represents the network with the smallest average distance. The five-facility solution, S5B40, is relatively strong in both dimensions, and represents an interesting tradeoff between the other two networks. That is, there are six-facility solutions which are 'better' than the five-facility solution in each of the dimensions, but at different threshold levels. Such an analysis, then, would offer the decisionmaker a number of different choices of locational configurations, each evaluated not in terms of the characteristics of the network, but in the attributes of the desired outcome.

When the aspatial consideration of market equity is coupled with the two previous dimensions, even more interesting RSE patterns emerge. Naturally, those networks which were efficient in the above analysis remain efficient. But, a number of solutions with relatively higher threshold requirements experience dramatic increases in the RSE score. Indeed, four additional networks (S6250, S5390, S42110, S4B120) become perfectly efficient. Obviously, as threshold requirements rise, the location problem becomes increasingly constrained and the number of acceptable sites becomes more limited, more distant from the core. The net effect of increasing

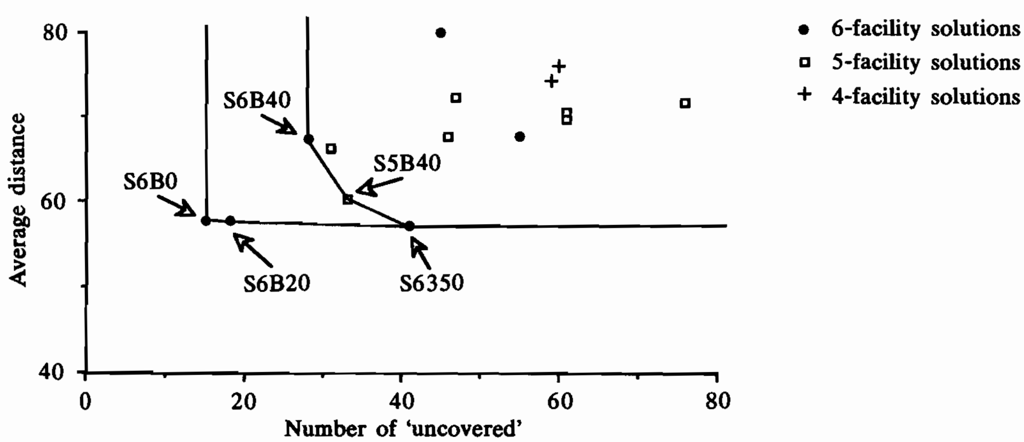

Figure 2. Efficient frontiers for selected McTHRESH solutions.

threshold values, then, is to 'even out' the differences of allocated markets, and to lower the overall spatial coverage of demand (see Balakrishnan and Storbeck, 1991). In the present analysis, such solutions are found to be desirable along the market-equity dimension and are thus evaluated favorably in terms of RSE.

6 Conclusion

Our primary goal in this research has been to extend the model of relative spatial efficiency, such that the resultant analytical tools can be used in the evaluation of retail outlet networks. An ancillary goal has been to divert the stare of location analysts from algorithmic notions of optimal facility sites toward those aspects of locational decisions which appear to be of greater interest and value to the decision-maker. To these ends, we have proposed a DEA-based approach to RSE, which includes both spatial and aspatial measures of siting performance.

The particular situation examined in this analysis is that of the decision to site a network of retail outlets, which is based on the assurance of spatial market adequacy for each individual facility in the system. To that framework we have added the concern of spatial market equity, as an important consideration in the organization of contemporary distribution channels in the marketing of goods and services. In this context, DEA has been used to examine the relationship between market adequacy and equity in various siting scenarios.

We have also attempted to demonstrate the superiority of DEA-based models over other multiobjective programming techniques in the support of locational decisionmaking. Specifically, this paper shows our RSE model to be a flexible multiobjective framework for evaluating planning decisions—generated by whatever means—which is able to incorporate very different measures of performance. Our study, for example, has combined common measures of system-wide accessibility (average distance) and individual accessibility (demand uncovered) with an aspatial measure of system equity (demand variation). However, although the concept of efficiency is general, the analysis is restricted to the types of data available. Thus, if such techniques are to realize their full potential, we will have to capture many more aspects of the location decision in a form which lends itself to analysis. As we can now accommodate a variety of criteria (in particular, aspatial criteria) in the analysis, we are in a position to fully capture the various criteria which often enter into location decisionmaking.

In coupling threshold issues with the measurement of efficiency we can now address the practical considerations of the *cannibalization* of existing facilities. We are achieving this capability at a cost which is mainly computational as, in theory, there is a large, combinatorial number of potential configurations of sites that must be evaluated. However, in practice, the choice set is highly constrained and the options are relatively few. So, although we are offering solution options which are more realistic, we also appear to be placing much greater computational requirements in obtaining these solutions. However, as the number of feasible options are, in practice, fairly limited, we do not believe the price to be high when compared with the benefits.

References

Achabal D D, Gorr W L, Mahajan V, 1982, "MULTILOC: a multiple store location decision model" *Journal of Retailing* **58**(2) 5-25

Adolphson D, Cornia G, Walters L C, 1991, "A unified framework for classifying DEA models", in *Operational Research '90* Ed. H Bradley (Pergamon Press, Oxford) pp 647-657

Applebaum W, 1965, "Can store location research be a science?" *Economic Geography* **41** 234–237

Balakrishnan P V, Desai A, 1991, "On the identification of superior products", working paper, College of Business, The Ohio State University, Columbus, OH

Balakrishnan P V, Storbeck J E, 1991, "McTHRESH: modeling maximum coverage with threshold constraints" *Environment and Planning B: Planning and Design* **18** 459–472

Bazerman M H, Lowenstein G F, White S B, 1992, "Reversals of preference in allocation decisions: judging an alternative versus choosing among alternatives" *Administrative Science Quarterly* **37** 220–240

Charnes A, Cooper W W, Rhodes E, 1978, "Measuring the efficiency of decision making units" *European Journal of Operational Research* **2** 429–444

Debreu G, 1951 *Theory of Value* (John Wiley, New York)

Desai A, Storbeck J, 1991, "A data envelopment analysis framework for measuring spatial efficiency" *Computers, Environment and Urban Systems* **14** 145–156

Desai A, Haynes K, Storbeck J, 1994, "A spatial efficiency framework for the support of locational decisions", in *Data Envelopment Analysis: Theory, Methodology and Applications* Eds W W Cooper, A Lewin, L Seiford (Kluwer-Nijhoff, Boston, MA) in press

Fare R, Grosskopf S, Lovell C A K, 1985 *The Measurement of Efficiency of Production* (Kluwer-Nijhoff, Boston, MA)

Farrell M J, 1957, "The measurement of productive efficiency" *Journal of the Royal Statistical Society Series A* **120** 253–281

Fisher H, Rushton G, 1979, "Spatial efficiency of service locations and the regional development process" *Papers of the Regional Science Association* **42** 83–97

Ghosh A, McLafferty S, 1987 *Location Strategies for Retail and Service Firms* (Lexington Books, Lexington, MA)

Hauser J R, Simmie P, 1981, "Profit-maximizing perceptual positions: an integrated theory for the selection of product features and price" *Management Science* **27** (January) 33–56

Huff D, 1964, "Defining and estimating a trading area" *Journal of Marketing* **28** 34–38

Ladd G W, Zober M, 1977, "Model of consumer reaction to product characteristics" *Journal of Consumer Research* **4** 89–101

Lancaster K J, 1971 *Consumer Demand: A New Approach* (Columbia University Press, New York)

Storbeck J, 1988, "The spatial structuring of central places" *Geographical Analysis* **20** 93–110

Swain R, 1971 *A Decomposition Algorithm for a Class of Facility Location Problems* PhD dissertation, Department of Civil Engineering, Cornell University, Ithaca, NY

Thomas K W, 1976, "Conflict and conflict management", in *Handbook of Industrial and Organizational Psychology* (Rand McNally, Chicago, IL) pp 889–935

APPENDIX

Following Balakrishnan and Storbeck's (1991) notation, the mathematical expression of the *M*aximum *c*over with *THRESH*hold (McTHRESH) problem is

$$\text{minimize } Z = \sum_i d_i y_i^- , \tag{1}$$

subject to

$$\sum_j b_{ij} x_j - \sum_{k=2}^{K} (k-1) y_i^{(k)} + y_i^- = 1 , \qquad \forall i, \tag{2}$$

$$\sum_{k=2}^{K} y_i^{(k)} + y_i^- \leqslant 1 , \qquad \forall i, \tag{3}$$

$$t x_j + \sum_{k=2}^{K} \sum_i b_{ij} d_i \left(\frac{k-1}{k} \right) y_i^{(k)} \leqslant M_j , \qquad \forall j, \tag{4}$$

$$\sum_j x_j = P, \tag{5}$$

$$x_j, y_i^{(k)}, y_i^- = (0, 1) , \qquad \forall i, j, k, \tag{6}$$

where

I is the number of demand nodes i,

J is the number of facility site 'candidates' j,

K is the maximum number of (spatially defined) markets in which demand nodes can participate,

t is the predetermined threshold value for market entry,

d_i is the amount of demand at node i,

$b_{ij} = \begin{cases} 1, \text{if node } i \text{ is within the spatial market } \textit{range} \text{ of site } j, \\ 0, \text{otherwise,} \end{cases}$

M_j is a constant which equals the amount of demand 'available' to site j in its trading area, $M_j = \sum_j b_{ij} d_i$,

P is the number of facilities to be sited,

$x_j = \begin{cases} 1, \text{if a facility is sited at node } j, \\ 0, \text{otherwise,} \end{cases}$

$y_i^- = \begin{cases} 1, \text{if node } i \text{ is } \textit{not} \text{ in the spatial market of any facility,} \\ 0, \text{otherwise,} \end{cases}$

$y_i^{(k)} = \begin{cases} 1, \text{if node } i \text{ participates in } \textit{exactly } k \text{ spatial markets,} \\ 0, \text{otherwise.} \end{cases}$

Equation (1) maximizes the coverage of demand, in that it is there to minimize the number of people *not* participating in any spatial markets. Equation (2) defines participation in these markets. The first term ($\sum_j b_{ij} x_j$) represents the collection of all possible spatial markets in which node i can participate. The participation of i in at least one market forces y_i^- to be 0 because the first term equals at least 1. Should i participate in k markets (that is, $\sum_j b_{ij} x_j = k$), then the corresponding $y_i^{(k)}$ variable (which is multiplied by $k-1$) will equal 1 and the identity will be maintained.

Constraint (3) states that additional participation will not be sought until node i is in the first market (that is, $\sum_k y_i^{(k)} \leqslant 0$, if $y_i^- = 1$).

Constraint (4) defines the threshold requirement for each candidate site. The right-hand-side (RHS) term, M_j, is equal to the amount of demand available to candidate site j in its trading area. The first term on the left-hand-side (LHS) is an expression of the threshold value for site j. The second LHS term is the amount of demand (covered by site j) multiplied by $(k-1)/k$ and the overcoverage variable, $y_i^{(k)}$; it represents the amount of demand in site j's trading area that is 'lost' to neighboring facilities which also cover this population. More generally, then, when some j is selected as a site, that site's available (trading area) demand—minus the demand accessed by other sites—must be greater than or equal to the threshold value. For those nodes j which are not sites (that is, $x_j = 0$), this relationship simply states that demand in trading areas 'lost' to neighboring facilities must be less than or equal to that which is available in that trading area. In this, the constraint set ensures that all sites selected by the program will have simultaneous access to their needed threshold populations.

Finally, constraint (5) simply states that p facilities must be sited. For another version of McTHRESH, wherein the number of facilities is determined endogenously, see the more complete development in Balakrishnan and Storbeck (1991).