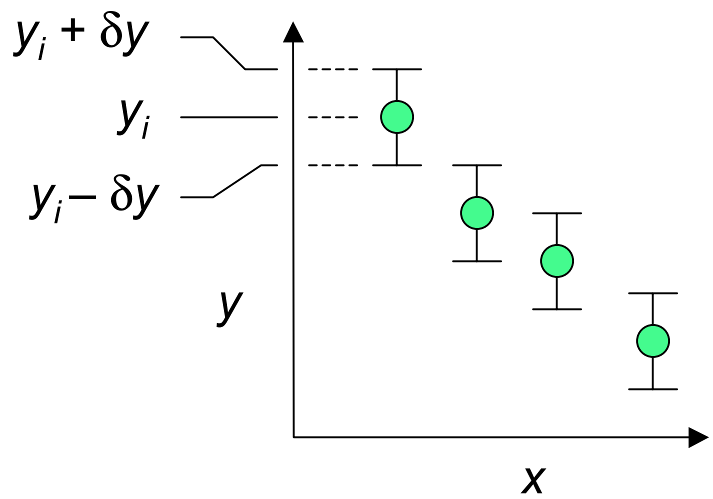
Logarithmic Error Bars

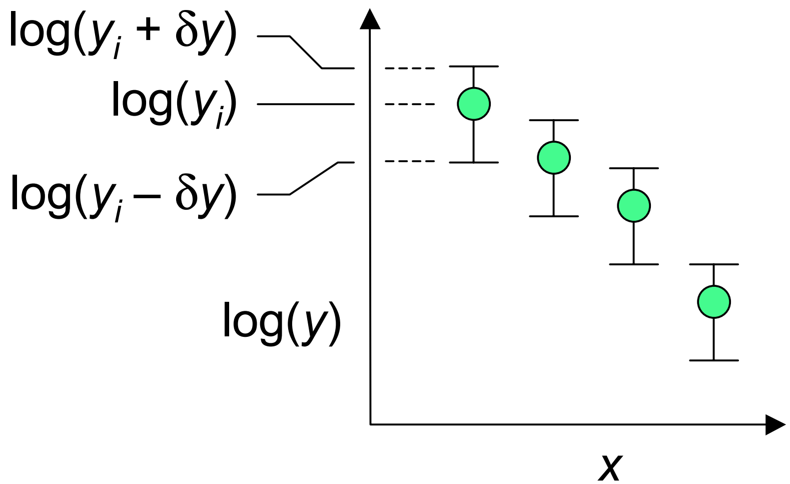
- Suppose that one has a sufficient number of measurements to make an estimate of a measured quantity *y* and report its error, $\pm\ \delta y$.

- The error, $\pm\ \delta y$, is represented on a Cartesian plot by extending lines of the appropriate size above and below the point *y.*



log Error Bars (cont.)

- If plotted on a logarithmic plot, however, this practice leads to asymmetric error bars.



- This gives a misleading view of measurement precision, especially when measured quantities vary by several orders of magnitude.

log Error Bars (cont.)

- To represent error bars correctly on a log plot, one must recognize that the quantity *z* being plotted is different than the measured quantity *y*.

$$z = \log(y)$$

- The error δz of is

$$\delta z = \delta\left[\log(y)\right]$$

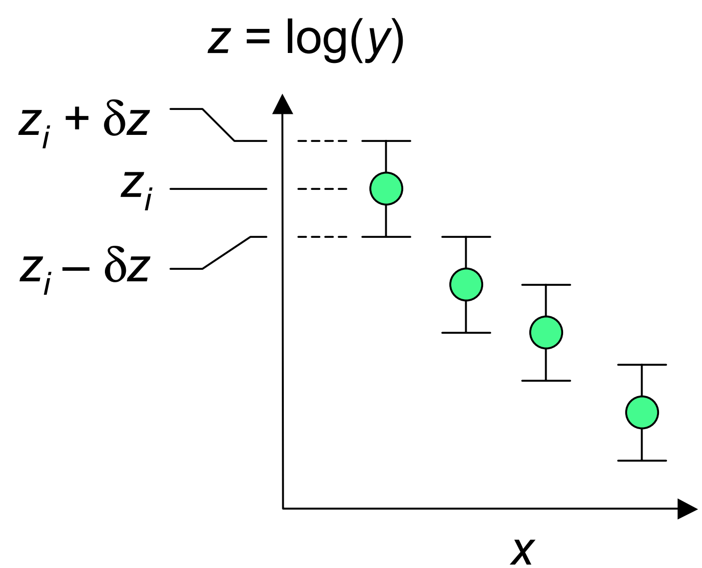
log Error Bars (cont.)

- On the assumption of small errors, a differential analysis can be used

$$\delta z \approx dz = d\left[\log(y)\right] = \frac{1}{2.303}\frac{dy}{y} \approx 0.434\frac{\delta y}{y}$$

- The error δz is thus given by the *relative error* in y

$$\delta z \approx 0.434\frac{\delta y}{y}$$



- The error bars now display correctly on a logarithmic plot.

Example: log Error Bars

- Plot the following data with error bars on a log-log plot

x	y	δy
0.03	0.011	0.003
0.1	0.042	0.006
0.2	0.093	0.018
0.5	0.21	0.02
1	0.28	0.05
2	0.53	0.12
5	0.77	0.12
10	0.92	0.10
20	1.88	0.30
50	3.56	0.40

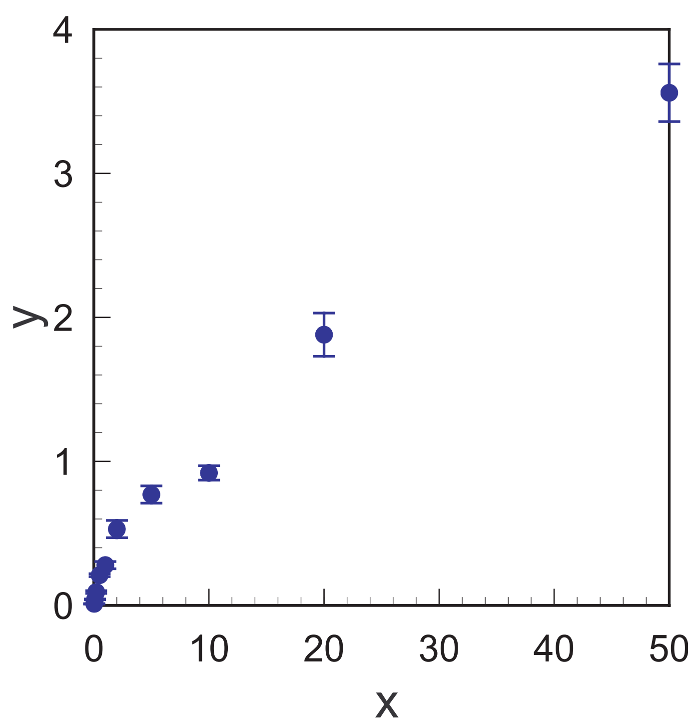
Example: log Error Bars

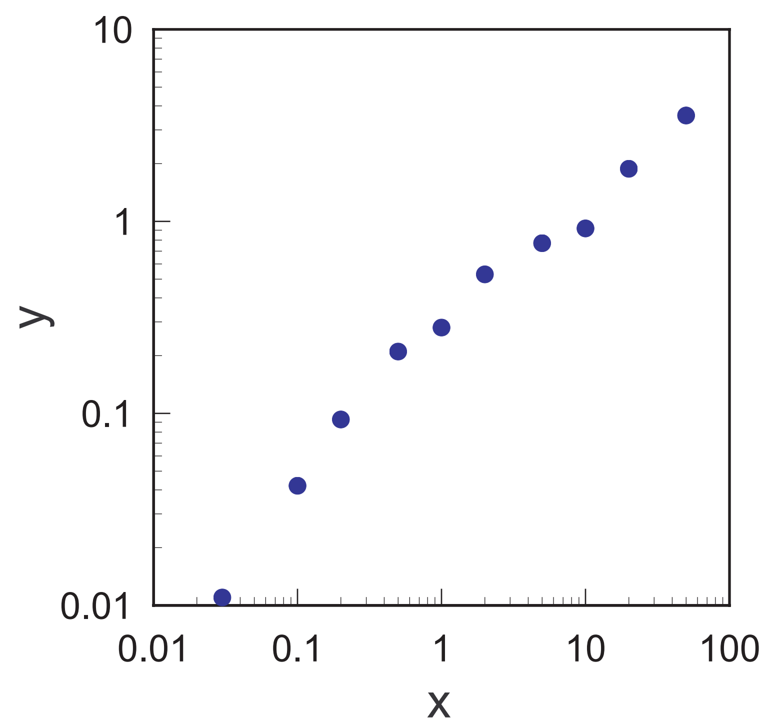
- First we calculate the quantities we need to plot:

x	y	δy	log(x)	log(y)	log(y - δy)	log(y + δy)	δy/y	0.434 δy/y
0.03	0.011	0.003	-1.523	-1.959	-2.097	-1.854	0.273	0.118
0.1	0.042	0.006	-1.000	-1.377	-1.444	-1.319	0.143	0.062
0.2	0.093	0.018	-0.699	-1.032	-1.125	-0.955	0.194	0.084
0.5	0.21	0.02	-0.301	-0.678	-0.721	-0.638	0.095	0.041
1	0.28	0.05	0.000	-0.553	-0.638	-0.481	0.179	0.078
2	0.53	0.12	0.301	-0.276	-0.387	-0.187	0.226	0.098
5	0.77	0.12	0.699	-0.114	-0.187	-0.051	0.156	0.068
10	0.92	0.10	1.000	-0.036	-0.086	0.009	0.109	0.047
20	1.88	0.30	1.301	0.274	0.199	0.338	0.160	0.069
50	3.56	0.40	1.699	0.551	0.500	0.598	0.112	0.049

Example (cont.)

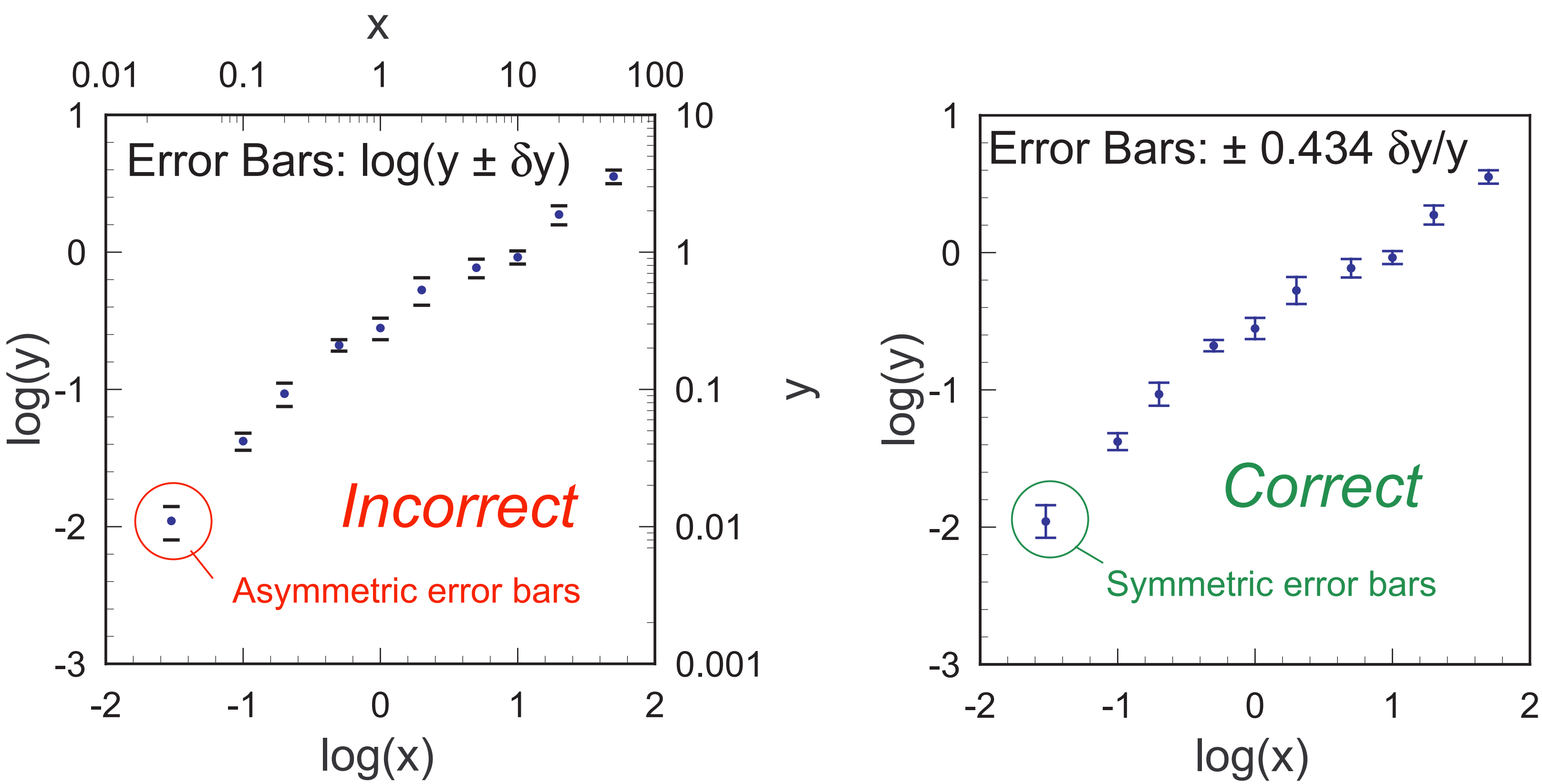
- It is interesting to compare the Cartesian and logarithmic *y* vs. *x* plots. Note that the logarithmic plot displays the data better.





Example (cont.)

- Plot on left shows error bars plotted incorrectly.
- Plot on right shows error bars plotted correctly.



Comments on Example

- The column $\delta y/y$ is the *relative error*. It varies from 10-27% in this example. The relative error is used for the error bars on a logarithmic plot.

- The asymmetric error bars are best seen for the points with large errors, like the first point.

- The logarithmic error bars are plotted on the *log*(y) *scale*. That means on the scale that reads –3, –2, –1, 0, 1; *not* on the *y*-scale, which reads 0.001, 0.01, 0.1, 1, 10.