phys 225 Homework due Nov 6

5.6 Longest wavelengths → lowest energies

$E \propto \left(\frac{1}{n'^2} - \frac{1}{n^2} \right)$ so lowest 5 energies have $n = n'+1, n'+2, \ldots, n'+5$

a.) For Lyman series $n' = 1$ so longest λ are

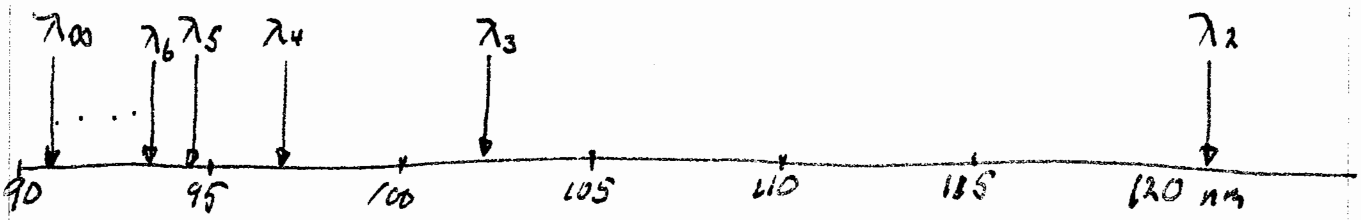
$$1/\lambda = R\left(1 - 1/n^2\right) \quad n = 2, 3, 4, 5, 6.$$

$$\lambda = \frac{1}{R}\left(\frac{n^2}{n^2-1}\right) = \frac{1}{2}\left(\frac{4}{3}, \frac{9}{8}, \frac{16}{15}, \frac{25}{24}, \frac{36}{35}\right)$$

$$= \frac{1}{R}\left(1.333,\ 1.125,\ 1.067,\ 1.042,\ 1.029\right)$$

since $R = 0.0110\ \text{nm}^{-1}$, $1/R = 90.91$ nm

$\lambda = 121.2,\ 102.3,\ 97.0,\ 94.7,\ 93.5$ nm



b.) for $n = \infty$, $\lambda^{-1} = R(1-0) \Rightarrow \lambda = \frac{1}{R} = 90.9$ nm

$$\Delta_n \lambda \equiv \lambda_n - \lambda_{n+1} = \frac{1}{R}\left[\frac{n^2}{n^2-1} - \frac{(n+1)^2}{(n+1)^2-1}\right]$$

$$= \frac{1}{R}\left[\frac{1}{1-1/n^2} - \frac{1}{1-1/(n+1)^2}\right]$$

$$\doteq \frac{1}{R}\left[\left(1 + 1/n^2\right) - \left(1 + \frac{1}{(n+1)^2}\right)\right] \quad \text{for large } n$$

since $\frac{1}{(n+1)^2} = \frac{1}{n^2} \cdot \frac{1}{\left(1+\frac{1}{n}\right)^2} \doteq \frac{1}{n^2}\left(1 - 2/n\right)$

$\Delta_n \lambda = \frac{1}{R}\left[2/n^3\right]$ plus correction terms of higher power in $1/n$

as $n \to \infty$, $\Delta_n \lambda \to 0$ and the lines are all near $\lambda_\infty = 1/R$.

5.7 a) $a_B = \hbar^2 / k_e e^2 m$

$\hbar = (1.05 \times 10^{-34})$ J·s

$k_e = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ S.I. units

$e = 1.6 \times 10^{-19}$ C

$m = 9.11 \times 10^{-31}$ kg.

$$a_B = \frac{(1.05)^2 \times 10^{-68}}{(9\times10^9)(1.6)^2 10^{-38} (9.11) 10^{-31}}$$

$$= \frac{(1.05/1.6)^2}{(9)(9.11)} 10^{-30+22} = 5.3 \times 10^{-11}\ m$$

b.) $\hbar c = 197$ eV nm

$ke^2 = 1.44$ eV nm

$mc^2 = 511 \times 10^3$ eV

$$a_B = \frac{(\hbar c)^2}{k_e e^2 m c^2} = \frac{197^2}{(1.44)(511\times10^3)} \frac{(eV \cdot nm)^2}{(eV \cdot nm)\,eV}$$

$$= 5.27 \times 10^{-2}\ nm$$

5.8 $$E_R = \frac{ke^2}{2a_B} = \frac{ke^2}{2\hbar^2/ke^2 m} = \frac{(ke^2)^2 m}{2\hbar^2} \cdot \frac{c^2}{c^2}$$

$$= \frac{1.44}{(2)(5.29\times10^{-2})} \frac{eV \cdot nm}{nm} \qquad = \frac{(1.44)^2\ 511\times10^3}{2\,(197)^2} \frac{eV \cdot nm \cdot eV}{eV \cdot nm}$$

$$= 13.6\,eV \qquad = 13.7\,eV$$

also $$E_R = \frac{1}{2}\left(\frac{ke^2}{\hbar c}\right)^2 mc^2 = \frac{1}{2}\alpha^2 mc^2$$

$$= \frac{1}{2}\left(\frac{1}{137}\right)^2 511\times10^3$$

$$= 13.6$$

5.10 $L_n = p_n \cdot r_n = n\hbar$ Bohr requirement

a.)

$$\frac{k_e e^2}{r_n^2} = \frac{m v_n^2}{r_n}$$ — Coulomb force = centrip. accel.

so $ke^2 = p_n v_n r_n = n\hbar v_n$

so $v_n = \frac{ke^2}{n\hbar}$ $\quad \frac{eV\cdot nm}{eV\cdot sec}$ ← units check

b.) $v_n \propto 1/n$ so $n=1$ is fastest.

$$v_1 = \frac{ke^2}{\hbar}\frac{c}{c} = \alpha c = \frac{1}{137}c \approx 0.7\%\, c$$

pretty fast, but remember

$$\gamma = \frac{1}{\left(1-v^2/c^2\right)^{\frac{1}{2}}} \doteq 1 + \frac{1}{2}\frac{v^2}{c^2} = 1 + 2.5\times10^{-5}$$

so γ pretty near 1.0 still.

thus non-relativistic treatment is ok.

5.13 a) from 5.7, $a_B \propto 1/m$

so for μ, $a_{B\mu} = \frac{m_e}{m_\mu} a_B$

$$= \frac{0.053}{207} = 2.6\times10^{-4}\,nm$$

$$= 0.26\times10^{-12}\,m = 0.26\,pm$$

~~$\lambda \propto 1/E$~~ from 5.8 $E_R \propto m$

so $E_{1\mu} = (207)(E_1) = 207(13.6) = 2.84\,keV$

b.) $\lambda \propto 1/E$

Lyman-α for H is when $n'=1$, $n=2$

this is the lowest E Lyman line, found to be 121 nm in Prob 5.6

so Lyman-$\alpha \Big)_\mu = \frac{121}{207} = 0.59\,nm$

an x-ray

5.18 Since e^+ and e^- have the same mass, their center of mass (the center for the orbit) is half way between them. All the orbit properties, are the same with $m \to m_r = \frac{1}{\frac{1}{m_1}+\frac{1}{m_2}} = \frac{m_e}{2}$

Since E_R for H is $\propto m$ (see prob 5.8)

the groundstate E for positronium is ½ that of H, namely $\frac{13.6}{2} = 6.8$ eV

520 $a_B \propto 1/e^2$ one e from the electron and the other from the nucleus.

So in general $a_B \propto 1/Ze^2$

and for an orbiting particle of different mass from the electron

$$a_{Bx} \propto m_e/m_x \quad \text{(see prob 5.13)}$$

thus $a_{B\pi,C} = a_B \frac{m_e}{m_\pi}\frac{1}{Z} = \frac{a_B}{(6)(273)}$

$= 3.2 \times 10^{-5}$ nm ≈ 32 fm $= 32 \times 10^{-15}$ m

which is bigger than the nucleus.

but $a_{B\pi,Pb} = \frac{a_B}{(82)(273)} = 2.2$ pm

which is smaller than the nucleus.