phys 225

Homework due Oct 16

2.7 $E = 2mc^2 = \gamma mc^2$

so $\gamma = (1-\beta^2)^{-1/2} = 2$

square $1-\beta^2 = 1/4 \rightarrow \beta^2 = 3/4$ so $\beta = \frac{\sqrt{3}}{2}$

$u = \beta c = \frac{\sqrt{3}}{2} c = 2.6 \times 10^8 \, m/s$

this answer is ok too

2.12 ① $\xrightarrow{u_1}$ ② $u=0$ then ① $\xrightarrow{u_3}$ ② $\xrightarrow{u_4}$

eq 2.19 says $u_3 = \dfrac{m_1^2 - m_2^2}{m_1^2 + m_2^2 + 2m_1m_2\sqrt{1-u_1^2/c^2}}\, u_1$

write $m_1/m_2 = \delta \ll 1$

factor out m_2^2

$$u_3 = \frac{m_2^2(\delta^2 - 1)}{m_2^2(\delta^2 + 1 - 2\delta\sqrt{1-u_1^2/c^2})}\, u_1$$

can take $\delta \rightarrow 0$ without getting 0 in numerator or denominator

$u_3 \approx -u_1$

2.20 $50\, GeV = 50 \times 10^3\, MeV$ $= K$ or $= E$ - so little difference here

classical $K = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\frac{u^2}{c^2}$

$mc^2 \approx 0.5\, MeV$ for electron

thus $50 \times 10^3 = (\frac{1}{2})(0.5)\, u^2/c^2$

$u^2/c^2 = 2 \times 10^5 \longrightarrow u = 447c$

correct: $50\, GeV = \gamma mc^2$

$\gamma = \dfrac{50 \times 10^3}{0.5} = 10^5$ (wow!)

for $u \approx c$ write $\beta = 1-\delta$

$\beta^2 \doteq 1-2\delta$ (drop δ^2 - tiny)

$$\gamma \doteq \frac{1}{(1-\beta^2)^{\frac{1}{2}}} = \frac{1}{\sqrt{2\delta}}$$

$$\delta = 1/2\gamma^2 = \frac{1}{2\times 10^{10}} = 0.5\times 10^{-10}$$

so $\beta = (1 - 0.5\times 10^{-10}) \rightarrow u = (1-0.5\times 10^{-10})c$

thus $u \approx c$ for any purpose except when you need δ explicitly

2.21 $mc^2 = 5\,\text{GeV}$, $K = (\gamma-1)mc^2 = 8\,\text{GeV}$, $E = K + mc^2 = 13\,\text{GeV}$

$E^2 = (pc)^2 + (mc^2)^2$ so $(pc)^2 = 13^2 - 5^2$

$pc = 12\,\text{GeV} \rightarrow p = 12\,\frac{\text{GeV}}{c}$

$$\frac{pc}{E} = \frac{\gamma\beta mc^2}{\gamma mc^2} = \beta = \frac{12}{13} = 0.92.$$

speed $u = \beta c = 0.92c = 2.77\times 10^8\ m/s$

2.22 eq 2.22: $\beta = pc/E$ eq 2.23 $E^2 = (pc)^2 + (mc^2)^2$

divide 2.23 by $(pc)^2$

$$\left(\frac{E}{pc}\right)^2 = 1 + \left(\frac{mc^2}{pc}\right)^2$$

so $\frac{E}{pc} > 1$ if $m > 0$, $\frac{E}{pc} = 1$ if $m = 0$

plug into 2.22

$\beta < 1$ if $m > 0$, $\beta = 1$ if $m = 0$

2.27 $\Lambda^0 \rightarrow p + \pi^-$

moving — at rest — moving

$m_\Lambda = 1116$ MeV/c^2
$m_p = 938$ "
$m_\pi = 140$ "

moment is conserved, p has none, so Λ^0 and π^- move along the same line

initial Λ^0 has u_Λ giving us γ_Λ and β_Λ
final π^- has u_π, γ_π, β_π

conserve E: $\gamma_\Lambda m_\Lambda = m_p + \gamma_\pi m_\pi$ (1.)

conserve p: $\gamma_\Lambda \beta_\Lambda m_\Lambda = 0 + \gamma_\pi \beta_\pi m_\pi$ (2.a)

It would be nice to have 2 equations with just γ's and no extra β's.

use $(pc)^2 = E^2 - (mc^2)^2 = (\gamma^2 - 1)(mc^2)^2$

then conserve momentum

$$(\gamma_\Lambda^2 - 1) m_\Lambda^2 = (\gamma_\pi^2 - 1) m_\pi^2 \quad (2b)$$

square (1) $\quad \gamma_\Lambda^2 m_\Lambda^2 = m_p^2 + 2\gamma_\pi m_\pi m_p + \gamma_\pi^2 m_\pi^2$

subtract (2.b) $\quad m_\Lambda^2 = m_p^2 + 2\gamma_\pi m_\pi m_p + m_\pi^2$

$$\text{so } \gamma_\pi = \frac{m_\Lambda^2 - m_\pi^2 - m_p^2}{2 m_\pi m_p}$$

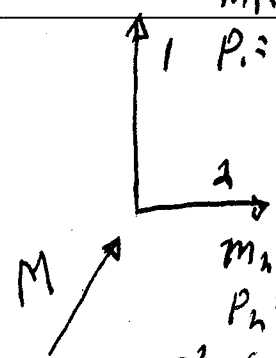
$$E_\pi = \gamma_\pi m_\pi c^2 = \frac{m_\Lambda^2 - m_\pi^2 - m_p^2}{2 m_p} c^2$$

$$= \frac{(m_\Lambda c^2)^2 - (m_\pi c^2)^2 - (m_p c^2)^2}{2 m_p c^2} = 184.4 \text{ MeV}$$

$E_\Lambda = m_p c^2 + E_\pi = 1122$ MeV

these are total E's.

2.31



$m_1 = 0.5$ GeV/c^2
$p_1 = 2$ GeV/c

$m_2 = 1.0$ GeV/c^2
$p_2 = 1.5$ GeV/c

$E^2 = (pc)^2 + (mc^2)^2$ can get the mass

momentum is conserved so

$$\vec{P}_M = \vec{P}_1 + \vec{P}_2 = (1.5, 2) \rightarrow P_m^2 c^2 = 6.25 \ (\text{GeV})^2$$

$$E_M = E_1 + E_2 = \left[(p_1 c)^2 + (m_1 c^2)^2\right]^{1/2} + \left[(p_2 c)^2 + (m_2 c^2)^2\right]^{1/2}$$

$$E_M = (4 + .25)^{1/2} + (1.5^2 + 1)^{1/2} = 3.864 \text{ GeV}$$

$$E_m^2 = 14.93 \ (\text{GeV})^2$$

$$(Mc^2)^2 = E_m^2 - (P_m c)^2 = 14.93 - 6.25 \ (\text{GeV})^2$$

so $M = 2.95$ GeV/c^2

$$\frac{P_m c}{E_m} = \beta_m = \frac{\sqrt{6.25}}{3.864} = 0.65 \rightarrow u_m = 0.65c$$

2.44 we are given $R = P/eB$

an orbit is $2\pi R$ around

the protons go $\dfrac{2\pi P}{eB}$ which takes $\dfrac{2\pi P}{eB}\dfrac{1}{\beta c} = T$

a) $P = \gamma\beta mc$ so $T = \dfrac{2\pi\gamma\beta mc}{eB\beta c} = \dfrac{2\pi\gamma m}{eB}$

b) we are allowed $\gamma = 1.02$

so the protons get $K = (\gamma - 1)mc^2$

$\approx (0.02)(938)$ MeV

$K \approx 19$ MeV