physics 225 Homework #2
Due Oct 9

1.41 Event #1 $t=0$, origin
#2 $t=0$, $x = 4c\,yr$, $y=z=0$

a.) S' $v=0.6c$ so $\gamma = \frac{1}{\sqrt{1-.36}} = \frac{1}{\sqrt{.64}} = \frac{10}{8} = 5/4$

Event #1 $t'=0$, $x'=y'=z'=0$

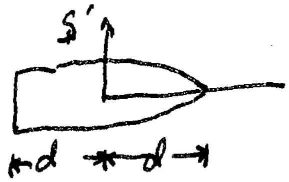
#2 $x' = \gamma(x - vt) = \frac{5}{4}(4-0) = 5\,c\,yr$

$y'=z'=0$, $t' = \gamma(t - vx/c^2) = -\frac{5}{4}(0.6)4 = -3\,yr$

b.) $x_2' - x_1' = 5\,c\,yr$

c.) No - they are 3yr apart. #2 came first

1.42



rocket at rest in S'
flash moves at c so

$$\left.\begin{array}{l} x'_F = d, \quad t'_F = d/c \\ x'_B = -d, \quad t'_B = d/c \end{array}\right\} \text{in } S'$$

in S $\quad x_i = \gamma(x_i' + \beta c t_i')$ $\quad i = F$ or B

$ct_i = \gamma(ct_i' + \beta x_i')$ $\quad \beta \equiv v/c$

So $x_F = \gamma(d + \beta d) = \gamma(1+\beta)d$

$ct_F = \gamma(d + \beta d) = \gamma(1+\beta)d \Rightarrow t_F = \gamma(1+\beta)d/c$

same algebra gives

$x_B = -\gamma(1-\beta)d$, $t_B = \gamma(1-\beta)d/c$

times unequal in S because rocket is moving while light propogates from flash. light goes c in S, reaches back first (in S) because rocket is moving in S

1.43 In example 1.6, boy chops so back hatchet just shaves the snake's tail. Snake's proper length is contracted in S

$\gamma = 5/4$ (as above), $L_0 = 100$ cm becomes 80cm

Thus the snake smashes into the front hatchet but is not chopped

1.43 - continued

Using Lorentz transformation

back hatchet falls at $x = x' = 0$ at $t = t' = 0$

front one falls at $x = 80\text{cm}$, $t = 0$ in S

in S' front hatchet falls at

$$x' = \gamma(x - vt) = \frac{5}{4}(80) = 100\text{cm}$$

$$ct' = \gamma(ct - \beta x) = \frac{5}{4}(0 - (0.6)(80)) = -60\text{cm}$$

so $t' = -60\text{cm}/c = -60/3\times10^{10} = -2\times10^{-9}$ s.

So hatchet falls in front of snakes nose - the time doesn't matter since the snake is at rest in S'.

1.45

$u_x' = 0.6c$ (shot forward)

$$u_x = \frac{u_x' + V}{1 + Vu_x'/c^2} = \frac{0.6 + 0.5}{1 + (0.6)(0.5)}c = 0.85c$$

1.46

$\xrightarrow[0.9c]{A}$ $\xleftarrow[0.9c]{B}$ speeds in S

rocket A in S', rocket B has $u_x = -0.9c$ in S

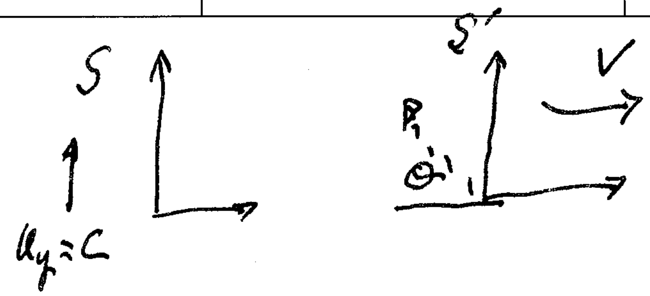
in S' rocket B has $u_x' = \dfrac{u_x - V}{1 - u_x V/c^2}$

$V = 0.9c$

so $u_x' = -\dfrac{2(0.9c)}{1 + 0.9^2} = -0.994c$

A sees B going toward $-x$, as in the diagram

1.48



a.) $u_z' = 0$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{V u_x}{c^2}\right)} = c/\gamma \qquad \text{since } u_x = 0,\ u_y = c$$

$$u_x' = \frac{-V}{1 - V u_x/c^2} = -V$$

b.) $$\tan\theta = u_y'/(-u_x') = \frac{c}{V\gamma}$$

c.) $$u'^2 = u_x'^2 + u_y'^2 + u_z'^2 = V^2 + c^2/\gamma^2 + 0$$

$$= c^2\left(\beta^2 + \frac{1}{\gamma^2}\right)$$

$$= c^2\left(\frac{\gamma^2\beta^2 + 1}{\gamma^2}\right)$$

$$= c^2(1-\beta^2)\left(\frac{\beta^2}{1-\beta^2} + 1\right)$$

$$= c^2\left(\frac{1-\beta^2}{1-\beta^2}(\beta^2 + 1 - \beta^2)\right)$$

$$= c^2$$

Showing again that complicated combinations of β, γ simplify

$u' = c$ — Speed of light stays c in relatively moving frames.

1.52 – eq 1.51: $\frac{f_{obs}}{f_{src}} = \frac{1}{(1-\beta)\gamma}$ all for doppler shift

eq 1.53: $\frac{f_{obs}}{f_{src}} = \sqrt{\frac{1+\beta}{1-\beta}}$ for approaching source

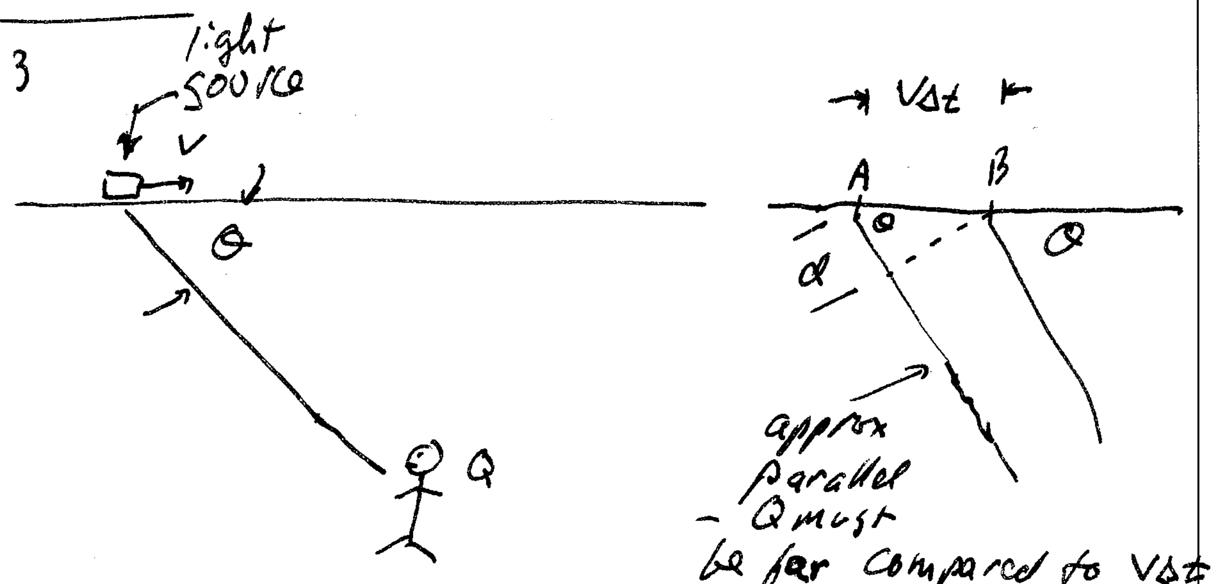
eq 1.57: $\frac{f_{obs}}{f_{src}} = \gamma(1+\beta)$

use $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{(1-\beta)(1+\beta)}}$

eq 1.51: so $\frac{1}{(1-\beta)\gamma} = \frac{[(1-\beta)(1+\beta)]^{1/2}}{(1-\beta)} = \frac{(1+\beta)^{1/2}}{(1-\beta)^{1/2}}$ eq 1.53

eq 1.57 $\gamma(1+\beta) = \frac{1+\beta}{\sqrt{(1-\beta)(1+\beta)}} = \sqrt{\frac{1+\beta}{1-\beta}}$ again eq 1.53.

1.53



Δt is period in frame of observer
light from B goes $d = v\Delta t\cos\theta$ less distance than from A, so second crest takes $d/c = \frac{v\Delta t\cos\theta}{c}$ less time to get to Q than does first crest

so Q sees the crests with period $\Delta t - \frac{v}{c}\Delta t \cos\theta$

ie $\Delta t_Q = \Delta t(1 - \beta\cos\theta)$

this is the "ordinary" doppler shift.

Δt is not a proper time

T_0 is the period in the rest frame of the source, so $\Delta t = \gamma T_0$

and frequency = 1/period

so $f_{obs} = \frac{1}{\Delta t_Q} = \frac{1}{\gamma T_0(1-\beta\cos\theta)} = \frac{f_{source}}{\gamma(1-\beta\cos\theta)}$

if $\theta = 0$, $\cos\theta = 1$ and we get $f_{obs} = \frac{f_{src}}{\gamma(1-\beta)}$ which is eq 1.51

Note if $\theta = 90°$ there is no "ordinary" doppler shift, but for light a factor $\frac{1}{\gamma}$ remains.

This is the purely relativistic "transverse" doppler shift.