Phys 225 - homework for last day of class

7.52 $\psi_2 = A_2(1 - 2x^2/b^2)e^{-x^2/2b^2}$

first get ψ_2'': $\psi_2' = A_2\left[-\frac{4x}{b^2} + (1 - 2x^2/b^2)\left(-\frac{2x}{2b^2}\right)\right]e^{-x^2/2b^2}$

$$\psi_2' = A_2\left(-5x/b^2 + 2x^3/b^4\right)e^{-x^2/2b^2}$$

so $\psi_2'' = A_2\left[-5/b^2 + 6x^2/b^4 + \left(-\frac{5x}{b^2} + 2\frac{x^3}{b^4}\right)\left(-\frac{2x}{2b^2}\right)\right]e^{-x^2/2b^2}$

$$= \frac{A_2}{b^2}\left(-5 + 11x^2/b^2 - 2x^4/b^4\right)e^{-x^2/2b^2}$$

if this is proportional to ψ_2, we can factor $(1 - 2x^2/b^2)$ out of the polynomial

$$\psi_2'' = \frac{A_2}{b^2}(1 - 2x^2/b^2)(x^2/b^2 - 5)e^{-x^2/2b^2}$$

$$= \frac{1}{b^2}(x^2/b^2 - 5)\psi_2 \quad \text{and } b^2 = \hbar/m\omega_0$$

$$\psi_2'' = \frac{m\omega_0}{\hbar}(x^2 m\omega_0/\hbar - 5)\psi_2$$

$$= \frac{2m}{\hbar^2}\left(\frac{x^2 m\omega_0^2}{2} - \frac{5}{2}\hbar\omega_0\right)\psi_2$$

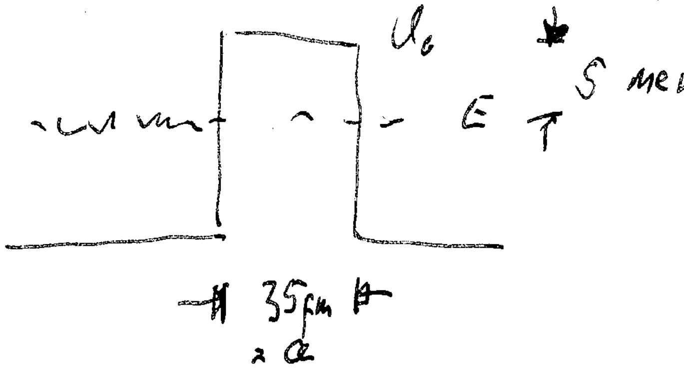
since $\omega_0^2 = K_H/m$ this becomes

$$\psi_2'' = \frac{2m}{\hbar^2}\left(\underbrace{\tfrac{1}{2}K_H x^2}_{U(x)} - \underbrace{\tfrac{5}{2}\hbar\omega_0}_{E_2}\right)\psi_2 \Rightarrow E_2 = \frac{5}{2}\hbar\omega_0$$

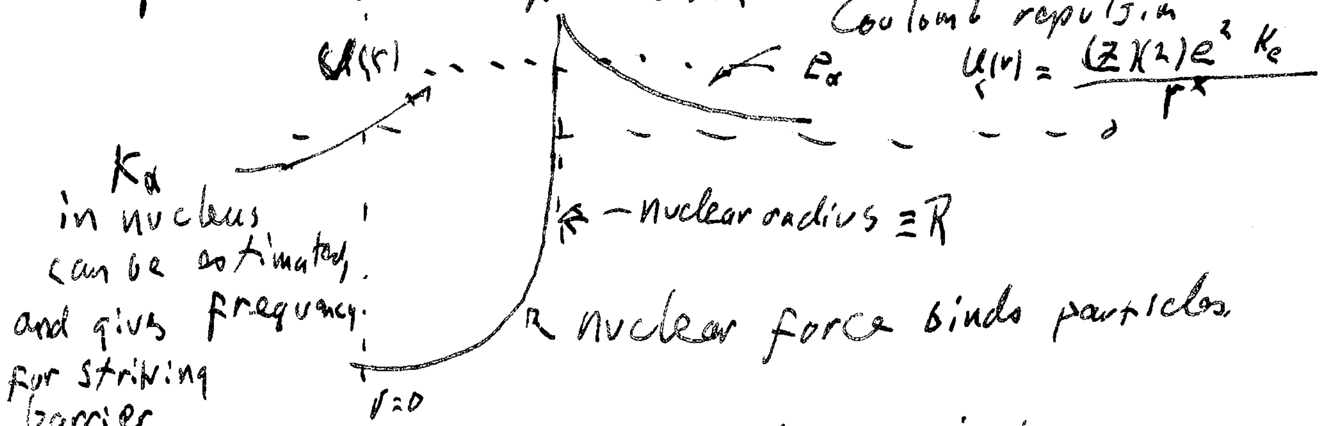
7.56 $e^{i\varphi} = \cos\varphi + i\sin\varphi \Rightarrow e^{i\pi} = \cos\pi + i\sin\pi$
$= -1 + 0i = -1$

7.55 - next page

7.55



by the way, this is a simplification of the real problem:



E_α is measured after decay, $U_c(R)$ is known

The barrier is more like a triangle than a rectangle.

barrier penetration: $P = e^{-2\alpha a}$ is probability of penetration per hit

$$\alpha = \left(\frac{2m(U_0 - E)}{\hbar^2}\right)^{1/2} = \frac{(2mc^2(U_0-E))^{1/2}}{\hbar c}$$

$$\alpha = \frac{((2)(4\times10^3)(5))^{1/2}}{197}$$ using $m_\alpha = 4 m_{nucleon}$

so $\alpha = 1$ if $197 = 200$, which is more precise than anything else in this problem.

[note, using "correct" m_α and 197 gives $\alpha = 0.98$]

$P = e^{-2\alpha a} = e^{-70}$ prob. of penetration/hit

$= 4 \times 10^{-31}$ (note using "exact" α gives answer in back of book, which differs by a factor of $e^{(2)(.02)(35)} = e^{1.4} = 4$)

Since there are 10^5 sec/day, if 5×10^{21} hits/sec we find $P = 5.4 \times 10^{-31} \times 10^{26} = 20 \times 10^{-5} = 2 \times 10^{-4}$ escapes/day

7.58 eq 7.114: $\Psi = \frac{1}{\sqrt{2}}[\Psi_1 + \Psi_2]$

$$|\Psi|^2 = \frac{|\Psi_1|^2}{2} + \frac{|\Psi_2|^2}{2} + \frac{1}{2}[\Psi_1^* \Psi_2 + \Psi_1 \Psi_2^*]$$

$$= \frac{|\Psi_1|^2}{2} + \frac{|\Psi_2|^2}{2} + \frac{1}{2} \underbrace{\psi_1 \psi_2}_{\text{real}} [e^{+i\omega_1 t} e^{-i\omega_2 t} + e^{-i\omega_1 t} e^{+i\omega_2 t}]$$

$\Delta\omega \equiv \omega_2 - \omega_1$ so $[\,] = 2\cos\Delta\omega t$

$$\int_{-\infty}^{\infty} |\Psi|^2 = \int_{-\infty}^{\infty} \frac{|\Psi_1|^2}{2} + \int_{-\infty}^{\infty} \frac{|\Psi_2|^2}{2} + \cos\Delta\omega t \int_{-\infty}^{\infty} \psi_1 \psi_2 \, dx$$

$\psi_1 \psi_2$ is an odd function, since ψ_1 is even and ψ_2 odd or vice-versa.

$$\int_{-a}^{a} g(x)\,dx = 0 \quad \text{if } g(x) \text{ is odd.}$$

i.e. $\int_{-a}^{a} g(x)\,dx = \int_{-a}^{0} g(x)\,dx + \int_{0}^{a} g(x)\,dx$

and $\int_{-a}^{0} g(x)\,dx = \int_{a}^{0} -g(-y)\,dy$ if $y = -x$ is substituted

$= \int_{a}^{0} g(y)\,dy = -\int_{0}^{a} g(y)\,dy$

So $\int_{-\infty}^{\infty} |\Psi|^2 = \frac{1}{2} + \frac{1}{2} + 0 = 1$ and so Ψ is normalized

$\uparrow \Psi_2$ normalized

$\nearrow$ if Ψ_1 normalized