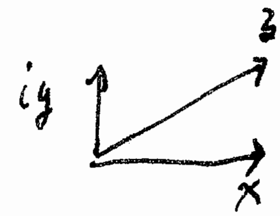
7.6 at max displacement, $K = 0$

so $E_{TOT} = K + U = U_0$

and E_{TOT} is conserved, so when there is no displacement, $U = 0$ and so $K = E_{TOT} = U_0$.

7.11

a.)



$z = x + iy$

$|z|^2 = x^2 + y^2$

$zz^* = (x+iy)(x-iy)$

$= x^2 + iyx - ixy - i^2y^2$

$= x^2 + y^2 = |z|^2$

b.) $|zw|^2 = (zw)(zw)^* = z\,w\,z^*w^* = zz^*\,ww^*$

$= |z|^2|w|^2$

Thus $|zw| = |z||w|$ — have to take positive square root because $|zw|$ is positive

c.) if $\Psi(x,t) = \psi(x)e^{-i\omega t}$

then $|\Psi(x,t)| = |\psi(x)||e^{-i\omega t}| = |\psi(x)|$

because $|e^{-i\omega t}|^2 = (e^{-i\omega t})(e^{+i\omega t}) = 1$

7.13 a.) $A\sin(\omega t + \phi) = A(\sin\omega t)(\cos\phi) + A(\sin\phi)\cos\omega t$

$= b\sin\omega t + a\cos\omega t$

with $b = A\cos\phi$ and $a = A\sin\phi$

b.) ~~if $\omega t' = \omega t + \phi$~~ the vice versa:

$a^2 + b^2 = A^2(\sin^2\phi + \cos^2\phi) = A^2 \Rightarrow A = \pm\sqrt{a^2+b^2}$

and $a/b = \frac{A\sin\phi}{A\cos\phi} = \tan\phi \Rightarrow \phi = \tan^{-1}(a/b)$

7.26

$\psi'' = -k^2\psi$: $\frac{d^2}{dx^2}\sin kx = \frac{d}{dx}(\cos kx)k = -k^2 \sin kx$

a.) and $\frac{d^2}{dx^2}\cos kx = -k^2 \cos kx$

and $\frac{d^2}{dx^2} e^{ikx} = -k^2 e^{ikx}$

so they are all solutions, as is e^{-ikx}

b.) $\sin kx : -ie^{ikx} + a\cos kx$

$= -i\cos kx + a\cos kx - i^2 \sin kx$

$= \sin kx$ if $a = +i$

so $\sin kx = i\left[\cos kx - e^{ikx}\right]$

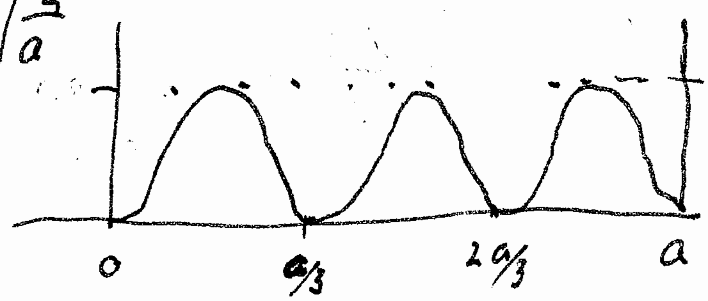
$\cos kx = e^{ikx} - i\sin kx$

and $e^{ikx} = \cos kx + i\sin kx$

7.30 for $n=3$ $|\psi_3(x)|^2 = \left|A\sin\frac{3\pi x}{a}\right|^2$ for box from 0 to a

$A = \sqrt{\frac{2}{a}}$

a.)



b.) most probable are where $\sin\frac{3\pi x}{a} = \pm 1$

so $3\pi x/a = \pi/2,\ 3\pi/2,\ 5\pi/2$

$x = a\pi/6,\ a\pi/2,\ 5a\pi/6$

both these intervals are 0.01a long. Lets do this approximately first, then exactly

$$\int_A^{A+\delta} |\psi_3(x)|^2 dx \doteq \left|\psi_3\left(A+\frac{\delta}{2}\right)\right|^2 \delta$$

for $A = 0.50a$, $\delta = 0.01a$

$|\psi_{(x)}|^2 = \frac{2}{a}\sin^2\left(\frac{3\pi x}{a}\right) \Rightarrow \frac{2}{a}\sin^2\left(0.505a \cdot \frac{3\pi}{a}\right)(0.01a)$

$= (0.02)(0.998) = 0.0200$ = Probability

and for $A = 0.75a$, $\delta = 0.01a$

we get $(0.02)(\sin^2(0.755 \times 3\pi))$

$= (0.02)(0.547) = 0.011$ = probability

---

to do it exactly we need

$\int_A^B \frac{2}{a}\sin^2(kx)\, dx$ with $k = 3\pi/a$

note $\cos 2\theta = \cos^2\theta - \sin^2\theta$

and $\cos^2\theta + \sin^2\theta = 1$

so $\cos 2\theta = 1 - 2\sin^2\theta$

or $\sin^2\theta = (1-\cos 2\theta)/2$

So the integral is $\frac{1}{a}\int_A^B [1 - \cos(2kx)]\, dx$

$= \frac{1}{a}(B-A) - \frac{1}{a}\frac{1}{2k}\left(\sin(2kB) - \sin(2kA)\right)$

$= \frac{1}{a}(0.01a) - \frac{1}{6\pi}\left(\sin 6\pi\begin{bmatrix} .51 \\ .76 \end{bmatrix} - \sin 6\pi\begin{bmatrix} .50 \\ .75 \end{bmatrix}\right)$

$= 0.01 + \frac{1}{6\pi}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.187 \\ 0.982 \end{bmatrix}\right)$

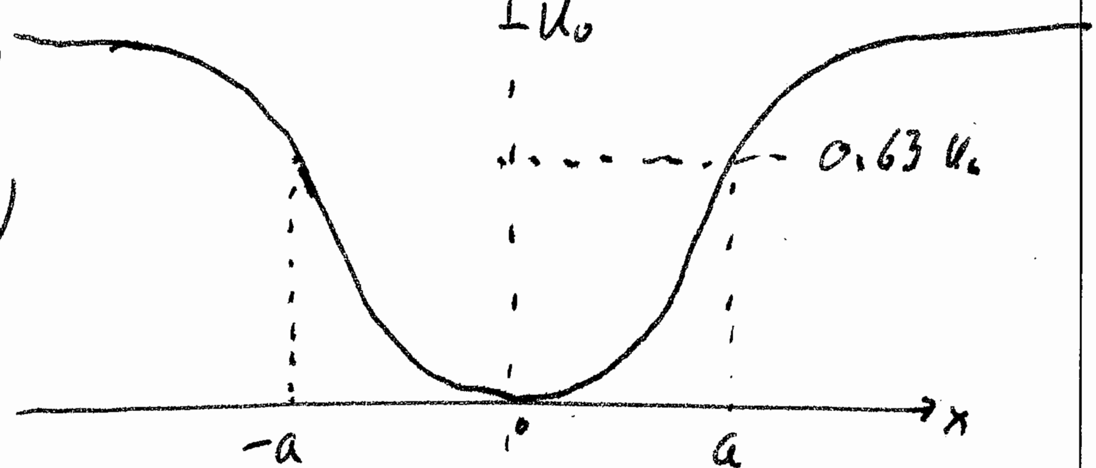
$= 0.01 + 0.0099 = 0.0199$ for first interval

$\quad\quad\quad - 0.0010 = 0.0110$ for second "

the exact result for this small interval is nearly the same as the approximate one.

7.37

a.)



b.) classical turning happens where

$$E = U(x) = U_0 - U_0 e^{-x^2/a^2}$$

so $\quad U_0 - E = U_0 e^{-x^2/a^2}$

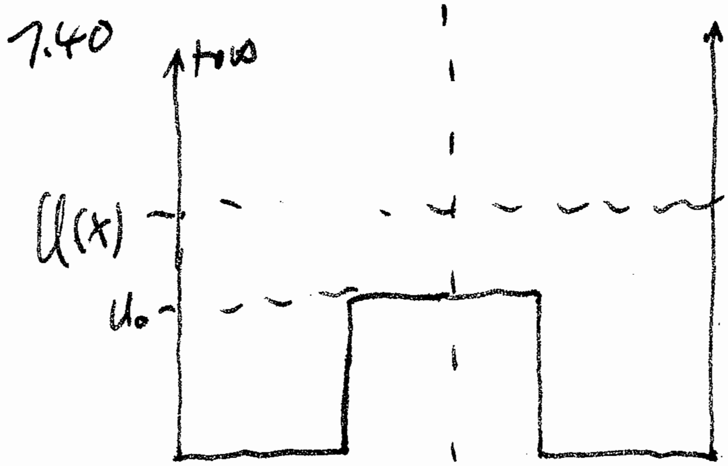
take ln: $\ln(U_0 - E) = \ln U_0 + \ln e^{-x^2/a^2}$

$$\ln U_0 - \ln(U_0 - E) = + x^2/a^2 \quad \text{at turning}$$

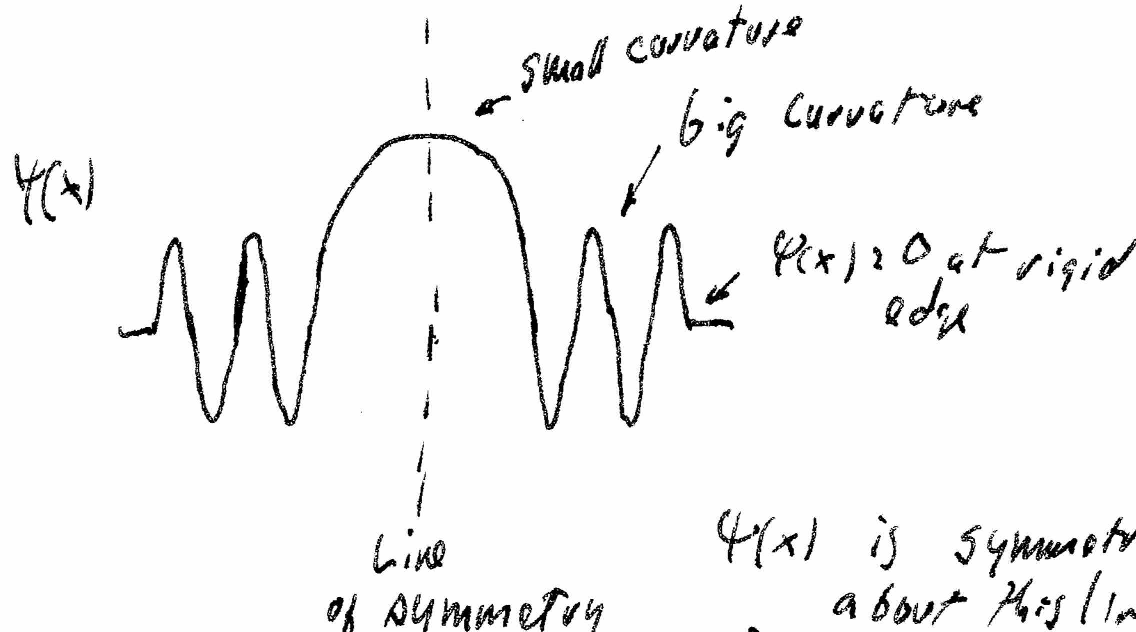
$$x = \pm a \left(\ln\left[\frac{U_0}{U_0 - E}\right]\right)^{1/2} \quad \text{at turning}$$

7.40 this wave function should be symmetric about the middle of the well, and the k value is bigger (λ shorter) at the edges where $U(x) = 0$. Whether or not any nodes are in the region where $U(x) = U_0$ depends on E, U_0, etc

7.40



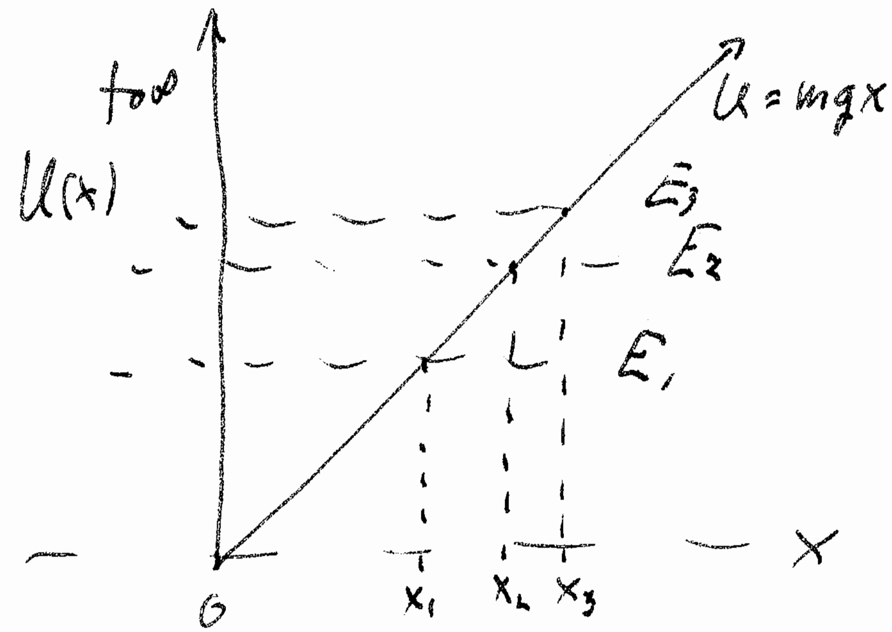
$E > U_0$ where E would be depends on mass and well dimensions.



ψ(x) is symmetric about this line if number of nodes was odd, it would be antisymmetric

the maximum at $x = 0$ may be bigger or ~~smaller~~ smaller than the others, depending on details.

for uniform grav. field $U(h) = mgh$ and we can call the height x instead of h. For the "hard surface" we can assume it is rigid, and have $U(0) \to \infty$.



classical turning at $x_i = E_i/mg$