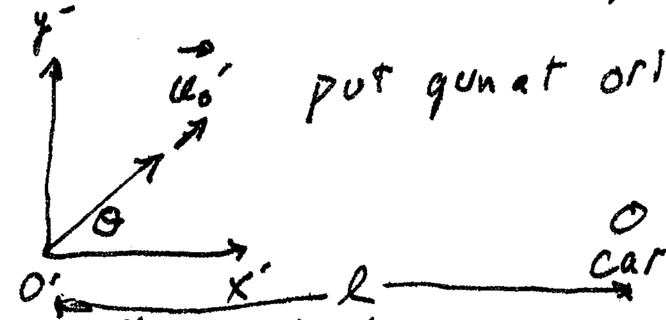
physics 225 Homework for Oct 2

1.8 put gun at origin

assume point car!!



frame S' in which gun + car are at rest.

note $\vec{u}'$ is velocity of bullett in S', and initially $\vec{u}_0' = u_0 \cos\theta\,\hat{x} + u_0 \sin\theta\,\hat{y}$

u_0 is the "muzzle velocity" of the bullett - i.e. the speed it leaves the gun

the horizontal component of $\vec{u}'$ is constant, and the x' position is

$$x' = u_x' t = u_0(\cos\theta)\,t$$

so it hits the car after $t = l/u_0\cos\theta$

the vertical motion is subject to gravity so

$$u_y' = u_0 \sin\theta - gt$$

and

$$y' = \int_0^t u_y' dt = u_0 \sin\theta\, t - gt^2/2$$

to hit the car $y' = 0$ at $t = l/u_0\cos\theta$

so

$$u_0 \sin\theta = \frac{g}{2} \frac{l}{u_0 \cos\theta}$$

or

$$u_0^2 \sin\theta \cos\theta = gl/2$$

$$\tfrac{1}{2} u_0^2 \sin 2\theta = gl/2$$

so

$$\sin 2\theta = gl/u_0^2$$

in S, bullett has

gun angle

$$\vec{u}_0 = (u_0 \cos\theta + v)\hat{x} + (u_0 \sin\theta - gt)\hat{y}$$

$\tan\theta_{bullett} = u_x/u_y$ at $t = 0$.

θ_{bullet} in S different from value in S'
θ for gun is same ($v \ll c$ for car)

to do problem in S, again calculate time to go from gun to (moving) car. Vertical motion is the same, muzzle velocity has $v_x \hat{x}$ added ...

1.10 $U_s = 330$ m/s in air (w.r.t the air)

$$\vec{u} = U_s \hat{y} \quad \text{from } B \text{ to } D^*$$
$$(U_s + v)\hat{x} \quad \text{" } B \text{ to } C$$
$$(U_s - v)\hat{x} \quad \text{" } B \text{ to } A$$

* - note, because of wind, sound doesn't go straight to D but must go at a small angle and go $\frac{1}{(1-v^2/U_s^2)^{\frac{1}{2}}}$ farther.

but this factor is $\doteq 1 + \frac{1}{2}\left(\frac{30}{330}\right)^2$ which is a $\frac{1}{2}$% correction, small compared to the accuracy of the stated wind speed.

a house is about 10 m. Imagine everybody is 10 m from B

$t = 10/u \quad t_{BA} = 30$ms, $t_{BC} = 27$ms, $t_{BA} = 33$ms

these differences easily measured with microphones and oscilloscopes.

1.12 $(1-x)^n \doteq 1-nx$ for case $n = -\frac{1}{2}$

x	$\frac{1}{(1-x)^{1/2}}$	$1+x/2$	% deviation
0.5	1.414	1.25	-11.6%
0.1	1.054	1.05	-0.4%
0.01	1.005	1.005	$-4\times10^{-3}\%$
0.001	1.0005	1.0005	$-4\times10^{-5}\%$

1.21 $\Delta t' = \Delta t/\gamma$ $\quad \Delta t = 1$ day $= 10^5$ seconds

$\Delta t - \Delta t' = 1s \Rightarrow 10^5(1-\frac{1}{\gamma}) = 1$

$\gamma - 1 = 10^{-5}\gamma$

$\gamma \doteq 1 + \frac{1}{2}\beta^2 \Rightarrow \beta^2 = 2\times10^{-5}$

$\beta = \sqrt{20} \times 10^{-3} = v/c$

so $v = \beta c = \sqrt{20}\times10^{-3} \times 3\times10^8$ m/s

$= \sqrt{180}\times10^5$

$= \sqrt{1.8}\times10^6 \doteq 1.4\times10^6$ m/s

to loose 60 sec β^2 is 60 times bigger

so $\beta = 11\times10^6$ m/s

1.22

Earth bound folks OBSERVE traveler ages $\frac{1}{\gamma}(T_{flight})$ each leg of trip

he goes same speed to and from star, so each leg takes half total time

so he ages $80/\gamma$ years

$\gamma = \frac{1}{(1-.95^2)^{1/2}}$ $\quad$ so increase in age is $(80)(1-.95^2)^{1/2} = 25$ yrs

1.23

$$1 = 3/\gamma \Rightarrow \gamma = 3 = (1-\beta^2)^{-1/2}$$

$$9 = \frac{1}{1-\beta^2}$$

$$9 - 9\beta^2 = 1 \Rightarrow \beta^2 = 8/9 \Rightarrow \beta = 2\sqrt{2}/3 = 0.94$$

$$v = \beta c = 2.82 \times 10^8 \, m/s$$

1.26

a.) $\gamma = \frac{1}{(1-\beta^2)^{\frac{1}{2}}} = \frac{1}{(1-.64)^{\frac{1}{2}}} = \frac{1}{\sqrt{.36}} = \frac{1}{.6} = 1.7$

b.) $t_{\frac{1}{2} \text{ in lab}} = \gamma t_{0\frac{1}{2}} = (1.7)(1.8 \times 10^{-8}) = 3.1 \times 10^{-8}$ sec.

c.) going 36m at 0.8c

$$\Delta t_{lab} = \frac{d}{v} = \frac{d}{\beta c} = \frac{36}{(.8)(3 \times 10^8)} = 15 \times 10^{-8} s.$$

$$\Delta t / t_{lab\frac{1}{2}} = \frac{15}{3} = 5 \text{ halflives} \quad N = N_0/2^5 = N_0/32 = 1000$$

d.) without time dilation

$$\Delta t / t_{0\frac{1}{2}} = \frac{15}{1.8} = 8.33 \text{ halflives}$$

$$N = N_0 / 2^{8.33} = \frac{32\,000}{322} = 100$$

1/10 as many.

1.33 0.8c again, $\gamma = 1.7$ so

a.) $l = \frac{1^m}{\gamma} = 0.6 \, m$

b.) $l = 1 \, m$

1.33 - continued

c.) $l'_x = (1\,m)\cos 60° = \frac{1}{2}\,m$

$l'_y = (1m)\sin 60° = \frac{\sqrt{3}}{2}\,m = l_y$

$l_x = 0.6\, l'_x = 0.3\,m$

$l^2 = l_x^2 + l_y^2 = 0.09 + \frac{3}{4} = 0.84$

so $l = 0.92\,m$

d.) in S stick is at 60° to v

$l_x/l = 1/2 \qquad l_y/l = \sqrt{3}/2$

$l_0 = 1\,m$ = length in S′ where stick is at rest

$l'_x/\gamma = l_x \quad l'_y = l_y$

$l'^2_x + l'^2_y = l_0^2 = 1\,m^2$

so $\gamma^2 l_x^2 + l_y^2 = 1\,m^2$

divide by l^2, use l_x/l, l_y/l above

$\gamma^2 \frac{1}{4} + \frac{3}{4} = 1/l^2$

$\left(\frac{1}{.36} + 3\right)/4 = 1/l^2$

$l = 0.83\,m$