Sign here to give permission for me to return your exam during class:____________________
Otherwise you can come to me and I will return your exam in private.

You may use one side of a single sheet of notes you personally prepared.

Please put your name on each page. There are 4 problems with a total of 8 main parts. (Some of the main parts have a couple of questions.) All main parts count equally.

Please make your logic clear. If you just write down numbers it is impossible to tell what you are doing. If you need to do scratch work, use the back of the pages. If you mess up the space for the answer, put it *on the back of the page with the question on it,* in a box and clearly labeled, and make a note in the space where the answer should go.

1. Light of wavelength 400 nm at a certain intensity is incident on a sodium photo-cathode. The work function of sodium is 2.3 eV.

   (a) What is the maximum kinetic energy of the elections that are emitted.
   For the photoelectric effect, we found $K_{max} = hf - \phi$
   since $f = c/\lambda$ this is $K_{max} = hc/\lambda - \phi$
   You are expected to know $hc = 1240$ nm-eV, so

   $$K_{max} = 1240/400 - 2.3 = 0.80 \text{ eV}$$

   (b) The intensity of the light is doubled. What does this do to

      i. the number of electrons emitted per second?

         The number of electrons is doubled.

      ii. the maximum kinetic energy of an electron?

         The energy of the electrons is unchanged.
         Remember each electron comes from a single photon, so doubling the number of photons does not effect the energy distribution of the electrons, but just doubles the number that are ejected.

2. A beam of X-rays of 17.0 keV is incident on a thin foil. Compton-scattered x-rays are observed at 90° with respect to the direction of the incident x-rays.

   (a) What energy does one of these Compton-scattered x-rays have?

   For Compton scattering, we found

   $$\lambda_1 - \lambda_0 = \frac{hc}{mc^2}(1 - \cos\theta)$$

   which in this case ($\theta = 90°$) is

   $$\lambda_1 - \lambda_0 = hc/mc^2$$

   Since $E = hf = hc/\lambda$ this becomes

   $$1/E_1 - 1/E_0 = 1/(mc^2)$$

   $$\text{so} \quad E_1 = (1/511 + 1/17.0)^{-1} = 16.5 \text{ keV}$$

   using the value for mc^2 that you are supposed to know.

   (b) What is the kinetic energy of the corresponding electron?

   Energy is conserved, and we started with 17.0 keV, giving 16.5 keV to the final x-ray, so the electron got the difference, 0.5 keV.

   (c) What is the component of the electron's momentum along the direction of the incident x-rays?

   The incident x-ray has all its momentum in the direction it is going. After the scattering, the final x-ray is going perpindicular to the initial one, so it has no momentum along the direction of the incident one. Thus the electron's momentum in that direction must be equal to that of the incident x-ray. For a photon (x-ray) $p = E/c$, so this momentum is 17.0 keV/c.

   Although the problem didn't request it, the electron's momentum perpindicular to this direction must be -1 times that of the final x-ray, which is 16.5 keV/c, and then the total momentum of the electron is $\sqrt{17^2 + 16.5^2} = 23.7$ keV/c. This gives a kinetic energy of $p^2/2m = 0.55$ keV, which is consistent with the previous answer.

3. Gold has atomic weight $A = 197.$, atomic number $Z = 79$, and, when solid, density 19. grams/cm^3.

   (a) Sizes:

      i. Using (some of) those figures and Avogadro's number, find the approximate size of a gold atom. (i.e. find the volume an atom takes up, and give the edge of a cube of that volume.)

         A mole is A grams and contains N_A atoms (or molecules), so a mole of gold weighs 197 grams, and occupies a volume of 197/19 cm^3. Thus a single atom occupies $(197/19)/N_A$ cm^3. There are 10^6 cm^3 in a m^3, so a single atom occupies $(197)/[19(6\times 10^{23})(10^6)] = 1.7 \times 10^{-29}$ m^3 $= 1.7 \times 10^{-2}$ nm^3
         The side of a cube of this volume is the cube root of that, which is 0.26 nm, which is the approximate diameter of a gold atom.

      ii. Compare this figure with the diameter of the lowest Bohr orbit for hydrogen, which is a good measure of the size of the hydrogen atom.

         Another number you should know is $a_B = 0.05$ nm. If you forgot that, a formula for it is

$$a_b = \frac{\hbar c}{\alpha}\frac{1}{mc^2} = \frac{197 \times 137}{(511 \times 10^3} = 0.053 \text{ nm}$$

         and we see the diameter of this orbit is 0.1 nm, which is 2.5 times smaller than the estimate for the gold atom. Thus their sizes are comparable.

      iii. Do you expect these two numbers to be of similar magnitude, or not, and why?

         They should be similar sizes because although heavy atoms have big nuclear charges, the last electrons to be added see the nuclear charge screened by the inner electrons, and so the very last electron sees about a charge of +1, just like the only electron in hydrogen. Thus we expect all atoms to be roughly the same size.
         Incidentally, if you do the earlier calculation for liquid hydrogen, for which the density is 0.07 g/cm^3 you will find a "size" a little bigger than 0.2 nm, just as we did for gold.

4. The Balmer series of lines in the hydrogen spectrum involve transitions to the $n = 2$ state.

   (a) Wavelengths:

      i. What is the wavelength of the lowest energy Balmer photon?
      The energy is the difference between the n=3 and n=2 levels, using $E_n = -E_R/n^2$ this is

      $$E_{\text{min}} = -E_R(1/3^2 - 1/2^2) = 13.6(1.4 - 1/9) = 1.89 \text{ eV}$$

      and the wavelength is

      $$\lambda = hc/E_{\text{min}} = 1240/1.89 = 656 \text{ nm}.$$

      ii. What is the wavelength of the highest energy Balmer photon? This is the difference between 0 (or E_∞) and $E_2 = -E_R/4$
      and the wavelength is

      $$\lambda = hc/E_{\text{min}} = \frac{1240}{13.6/4} = 365 \text{ nm}.$$

      If you remembered the value for $R = 0.0110 \text{ nm}^{-1}$ you could use the actual Balmer formula, which says

      $$1/\lambda = R(1/4 - 1/n^2)$$

      but since I don't remember that value, I can't expect you to.

   (b) Another photon is usually emitted after one of the Balmer photons.

      i. What two energy levels are involved for this second photon? (Give the n's.)

      The atom is left in the $n = 2$ state by the emission of any Balmer photon. What happens next? It goes to the $n = 1$ state. So the next photon after any Balmer photon is the one from the $n = 2$ to $n = 1$ transition.

      ii. What is the energy of this photon?

      This is the Lyman-α photon, and its energy is $E_R(1/1^2 - 1/2^2) = 3E_R/4 = 10.2$ eV.