Sign here to give permission for me to return your exam during class:____________________
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You may use one side of a single sheet of notes you personally prepared.

Please put your name on each page. There are 3 problems with a total of 8 parts. All parts count equally, even though some are really easy and others are harder.

Please make your logic clear. If you just write down numbers it is impossible to tell what you are doing. If you need to do scratch work, use the back of the pages. If you mess up the space for the answer, put it *on the back of the page with the question on it,* in a box and clearly labeled, and make a note in the space where the answer should go.

1. A supernova explosion was seen in Feb 1987, located 160×10^3 light-years away from Earth. A much nearer supernova (6×10^3 light-years away) was observed in the year 1054.

   (a) When did each of these occur in the Earth's frame of reference? (Give a date. Or give time before the present. Do not imply more accuracy than you have.)

   The one seen in 1987 was 160k light-yrs away, so it happened 160k years before 1987. Likewise, the one seen in 1054 happened 6k years before 1054. Since we are dealing in integral thousand years, we should round 1987 off to 2k and 1054 off to 1k. Then the nearer supernova happened in −5k (i.e. 5k BC) and the further one happened in −158k (i.e. 158k BC). Or, the nearer one happened 7k years before the present, and the further one happened 160k years before the present. These would be called −7k and −160k yrs in a system where t=0 now.

   (b) Show that if both of these were observed in the same direction (e.g. the positive x direction), there is a frame of reference in which they occur simultaneously.

   If there is such a frame, we can Lorentz transform to it. In the frame of the earth, S, the two supernovae are at $x_1 = 6$k light-yrs, $t_1 = -5$k yrs for the near one and $x_2 = 160$k light-yrs, $t_2 = -158$k yrs for the far one.
   The Lorentz transforms are

   $$t'_1 = \gamma(t_1 + vx_1/c^2); \qquad t'_2 = \gamma(t_2 + vx_2/c^2)$$

   so, since we want

   $$t'_1 = t'_2$$
   $$t_1 + vx_1/c^2 = t_2 + vx_2/c^2$$
   $$t_1 - t_2 = (v/c^2)(x_1 - x_2)$$

   so

   $$v/c^2 = (-5 + 158)/(6 - 160) = -153/154.$$

   Since $c = 1$ light-year/year, $v = -153/154$ light-years/year, and the desired frame moves to the left (v is minus) with a speed near but somewhat less than c. Note that as $t_1 - t_2$ is what matters, if we set $t = 0$ at the present, then we get the same result.

   Note also that if $t_1 - t_2$ were much bigger, then the required v would be bigger than c, and so there would be no such frame.

2. A π^- meson ($m_\pi c^2 = 140$ MeV) is moving at $0.8c$ in the lab.

(a) What is its momentum? (in MeV/c)

$$p = \gamma m u$$
$$pc = \gamma \beta m c^2$$
$$\gamma = (1-\beta^2)^{-1/2} = 1/0.6$$
$$pc = (0.8/0.6)140 = 187 \text{ MeV}$$
$$p = 187 \text{ MeV/c}$$

(b) What is its kinetic energy? (in MeV)

Since we already know γ, we can use $K = (\gamma - 1)mc^2 = 93$ MeV
Or, since we know pc and mc^2, we can use $K = E - mc^2$ with $E^2 = (pc)^2 + (mc^2)^2$

(c) An average π^- lives for 2.6×10^{-8} seconds in its rest frame. If our π^- is average, how far does it travel in the lab?

The lifetime given is a proper time (in the π^- rest frame) so it is dilated into the lab time. The distance a thing goes at a speed is

$$d = u\Delta t_{lab} = u\gamma\Delta t_0$$
$$d = (0.8c)(1/0.6)(2.6 \times 10^{-8} = (0.8)(3\ 10^8)(1/0.6)(2.6 \times 10^{-8}) \text{ m}$$

The powers of 10 cancel and we get

$$d = (0.8)(3)(2.6)/0.6 = 10 \text{ m}$$

(d) This π^- is brought to rest in some liquid deuterium. It combines with a deuterium nucleus (a proton and a neutron, with 2 MeV binding energy) and two neutrons come out. (The π^- disappears.) What is the combined kinetic energy of the two neutrons? (proton mass: 938 MeV/c^2, neutron mass: 939 MeV/c^2.)

We start with a π^- and a deuterium nucleus, and wind up with 2 neutrons and some kinetic energy.
Conservation of energy requires

$$m_\pi c^2 + M_d c^2 = 2m_n c^2 + K$$

and the deuteron is a bound proton and neutron

$$M_d c^2 = m_p c^2 + m_n c^2 - B$$

putting these together

$$K = m_\pi c^2 + m_p c^2 + m_n c^2 - B - 2m_n c^2 = m_\pi c^2 + (m_p c^2 - m_n c^2) - B = 140 - 1 - 2 = 137 \text{ MeV}$$

This is, incidentally, shared equally between the two neutrons, which have zero net momentum, and thus go in opposite directions with the same speed.

3. Two space ships going 0.6c in the Earth's frame are approaching each other, traveling along the x axis. (i.e. one goes in the $+x$ direction, and one in the $-x$ direction.)

   (a) What relative speed does the pilot of one ship observe for the other ship?

   Consider the ship going in the $+x$ direction, and call its rest frame S'. This frame has a velocity with respect to Earth of 0.6c. So we can ask what is u'_x for the other ship?

   $$u'_x = (u_x - v)/(1 - u_x v/c^2)$$

   with

   $$u_x = -0.6c$$

   for the ship going in the $-x$ direction. Being careful with the signs, we find

   $$u'_x = (0.6 + 0.6)c/(1 + 0.6^2) = 0.88c$$

   (b) If the radios on the ships transmit at 2 GHz, what frequency should the receivers be tuned to for the pilots to communicate? The ships are approaching each other, so we will get a "blue shift." The doppler shift requires the relative velocity of the two ships (note Earth is not involved in the communication), which we got in part (a) and we write

   $$f_{\text{obs}} = f_{\text{src}} \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$$

   where I chose the signs in the numerator and denominator to give me a blue shift. Putting in the value for β correspoinding to the speed in part (a), we find

   $$f_{\text{obs}} = 4 f_{\text{src}} = 8 \text{ GHz}$$