

**Hour Exam III answers posted on the website
Earlier Hour Exam answers also posted.**

**Exam grades posted on Tycho. Average was only
39 ! We will go over the exam Friday.**

**Office hrs NOT WED, but Thurs and Fri (normal)
from 4pm to 5:30 or so.**

Final homework assignment due Friday Dec 7

Final exam Monday Dec 10 at 0830 here.

**Exam has M.C and work out problem by me
M.C and workout problem by Tutorial people
M.C problem from lab. Lab problem based on labs
you did and *don't forget uncertainty analysis.***

Simple Harmonic Motion summary

Had $\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} x(t)$ for mass on spring

Most general solution is

$$x(t) = A \sin(\omega t) + B \cos(\omega t)$$

$$\omega = +\sqrt{k/m}$$

with A and B chosen to give the right $x(0)$ and $v(0)$.

Other ways to write it:

$$A_s \sin(\omega t + \phi_s) \text{ or } A_c \cos(\omega t + \phi_c)$$

now the two adjustable parameters are A and ϕ

Textbook uses $A \cos(\omega t + \phi_c)$

Amplitude: $|A_s|$ or $|A_c|$ or $\sqrt{A^2 + B^2}$

Phase: ϕ

Angular frequency: ω

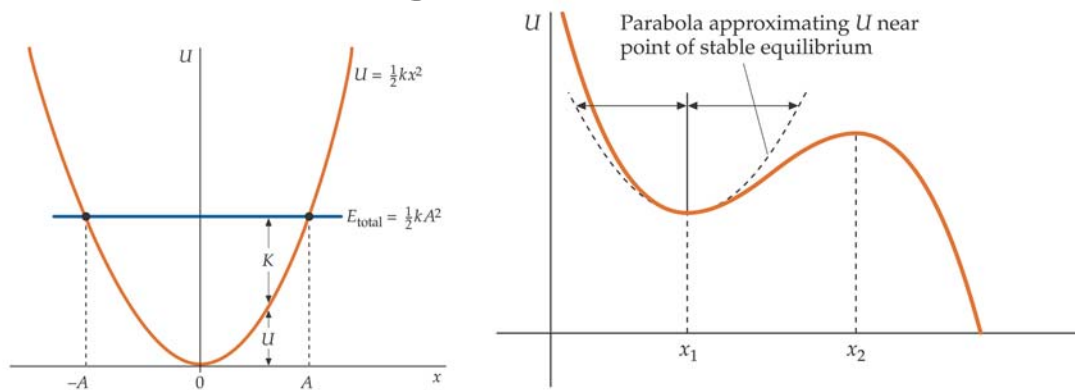
Period $T = 2\pi / \omega$

Frequency $f = 1/T$ (Note error in text eqn 14-11.)

$$\omega = 2\pi \frac{1}{T} = 2\pi f \text{ is correct.}$$

Energy in S.H.M. $E = \frac{1}{2} kx^2 + \frac{1}{2} mv^2 = \frac{A^2 k}{2}$

Use of S.H.O. for general physical problem:



Equilibrium point in $U(x)$ is at a minimum at x_1 .

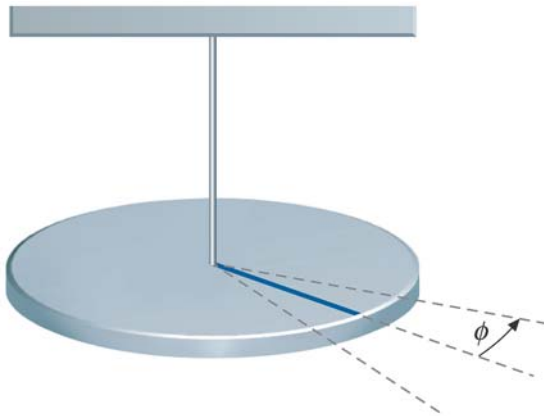
Expand $U(x)$ about x_1 with Taylor expansion.

First derivative of $U(x)$ is 0 at a minimum so get $U(x_1) + \Delta x^2 [d^2U/dx^2](1/2) + \dots$ for small amplitudes.

This is a spring with $k = [d^2U/dx^2]$

Examples – molecules, nuclei, anything that vibrates.

Other examples of S.H.O



Torsion pendulum (like in Cavendish expt.)

Object with I hanging on a wire with κ
so $\tau = -\kappa\phi = I\alpha$
(like mass on spring)

Thus motion is given by

$$\phi(t) = A \cos(\omega t + \phi_0) \text{ with } \omega = \sqrt{\kappa / I}$$

This is how you get κ which you need to get G with the Cavendish experiment.

Physical Pendulum:

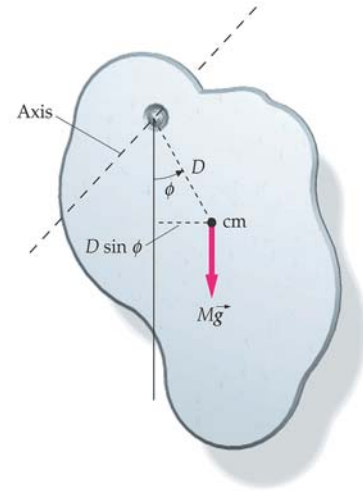
Torque about axis

$$\tau = -MgD \sin(\phi)$$

Need to know I

$$\tau = I\alpha \text{ and } \alpha = d^2\phi/dt^2$$

$$-MgD \sin(\phi) = I \frac{d^2\phi}{dt^2}$$



Not the same equation. Have **sin(phi)**, not ϕ

Expand about 0: $\sin(x) = x - x^3/6 + \dots$

replace $\sin(\phi)$ with ϕ to get $\frac{d^2\phi}{dt^2} = -\frac{MgD}{I}\phi$

good for small amplitudes only

$$\text{So } \omega = \sqrt{\frac{MgD}{I}} \text{ Note } M/I$$

does not depend on M .

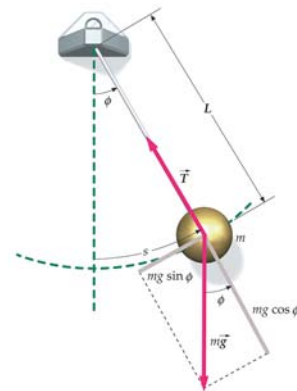
Simple Pendulum: Small sized mass on string.

D becomes L

$$I \text{ is } ML^2 \text{ so } \omega = \sqrt{\frac{g}{L}} \text{ This}$$

is used to compare g at different locations.

Clicker



Both a mass–spring system and a simple pendulum have a period of 1 s. Both are taken to the moon in a lunar landing module. While they are inside the module on the surface of the moon,

- A. the pendulum has a period longer than 1 s.
- B. the mass–spring system has a period longer than 1 s.
- C. both A and B are true.
- D. the periods of both are unchanged.
- E. one of them has a period shorter than 1 s

If the length of a simple pendulum is increased by 4% and the mass is decreased by 4%, the period is

- A. not changed.
- B. increased by 2%.
- C. decreased by 4%.
- D. increased by 4%.
- E. decreased by 2%.

Damping

If a **force** is **proportional to $-v$** it will **slow** the oscillator. This also makes a solvable differential

equation.
$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} x(t) - \frac{b}{m} \frac{dx}{dt}$$

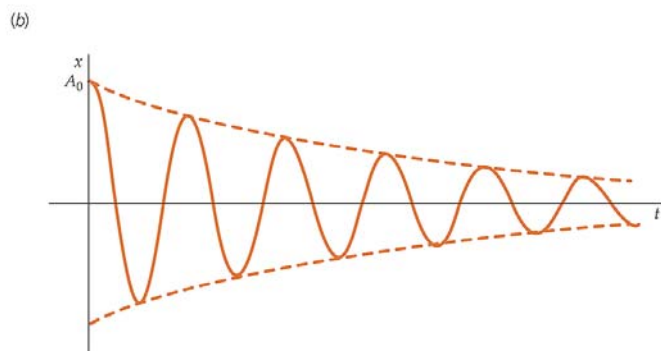
Note that **b/m** has dimensions **$1/\text{time}$** . The time **$\tau = m/b$** is the “**damping time**” and the system **energy** drops by **$1/e$** in that time. (this τ is time, not torque – sorry.) Note **small b** is **long τ**

Solution, if **$\tau > T$** (actually $\tau > 1/(2\omega_0) = T/4\pi$)

$$x(t) = A_0 e^{-t/2\tau} \cos(\omega' t + \phi) \quad \text{where}$$

$$\omega' = \omega_0 \left(1 - \frac{1}{(2\omega_0 \tau)^2}\right) \quad \text{and} \quad \omega_0 = \sqrt{k/m}$$

for large τ frequency is not shifted much.



If $2\omega_0\tau = 1$, then $\omega' = 0$ and the system does not oscillate. **Critically damped.**

$2\omega_0\tau = 1 \rightarrow \mathbf{b_c = 2\omega_0 m}$ gives critical, and bigger b gives “over damping”

