

Clicker Scores now posted on TYCHO. Look in grades for “clicker”

Homework #8 due tonight (Nov 21) at midnight

Homework #9 due next Wed (Nov 28).

Hour exam Nov 30

More graded exams to be returned at end of class.

Exam grades were posted Monday afternoon

Average is 75 and std dev is 14. Expect average to increase by 1 or 2 when missing pages are found.

Correction:

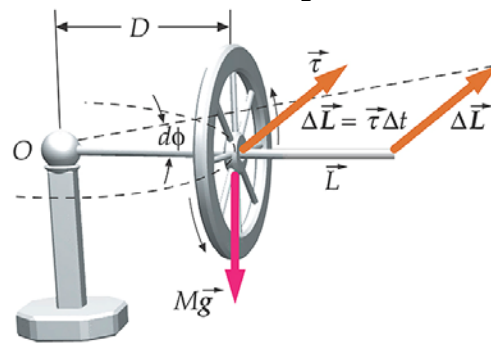
$\vec{\omega} = \vec{r} \times \vec{v} / r^2$ is correct, not $\vec{\omega} = \vec{r} \times \vec{v}$ which has the wrong units. The direction is given by $\vec{r} \times \vec{v}$

The other things, angular momentum and torque were correct.

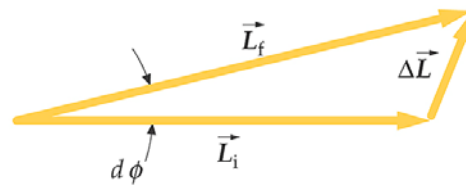
Gyroscope: (precession)

\vec{L}_{spin} is in horizontal plane. Gravity provides **torque**, also in horizontal plane, perpendicular to \vec{L}_{spin} .

Therefore $\Delta\vec{L}_{\text{spin}}$ changes **direction** of \vec{L}_{spin} , **not** its **magnitude** and it stays in horizontal plane.



(a)



(b)

$\Delta\vec{L} = \vec{\tau}\Delta t = MgD\Delta t$ with a direction perpendicular to the axel and to \vec{g}

angle that axis shifts is ϕ :

$$\Delta\phi = \frac{\Delta L}{L_{\text{spin}}} = \frac{MgD\Delta t}{I_g\omega_{\text{spin}}}$$

so precession angular speed is

$$\omega_p = \frac{d\phi}{dt} = \frac{MgD}{I_g \omega_{\text{spin}}}$$

clicker

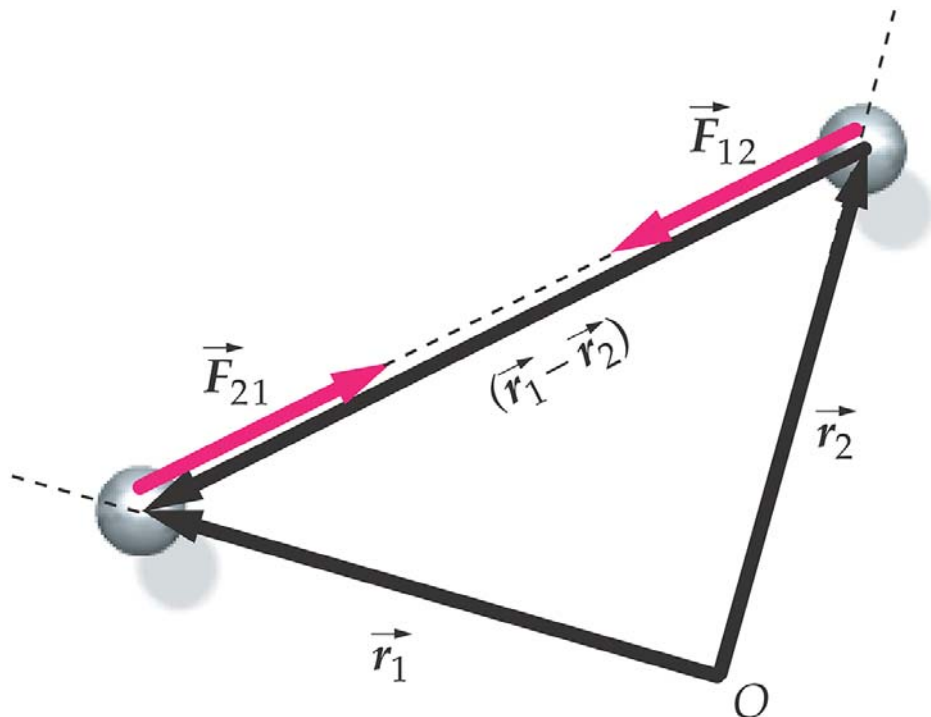
Section 10.3 Conservation of Angular Momentum

$$\vec{\tau}_{\text{net,external}} = \frac{d\vec{L}_{\text{sys}}}{dt} \text{ therefore if } \vec{\tau}_{\text{net,external}} = 0$$

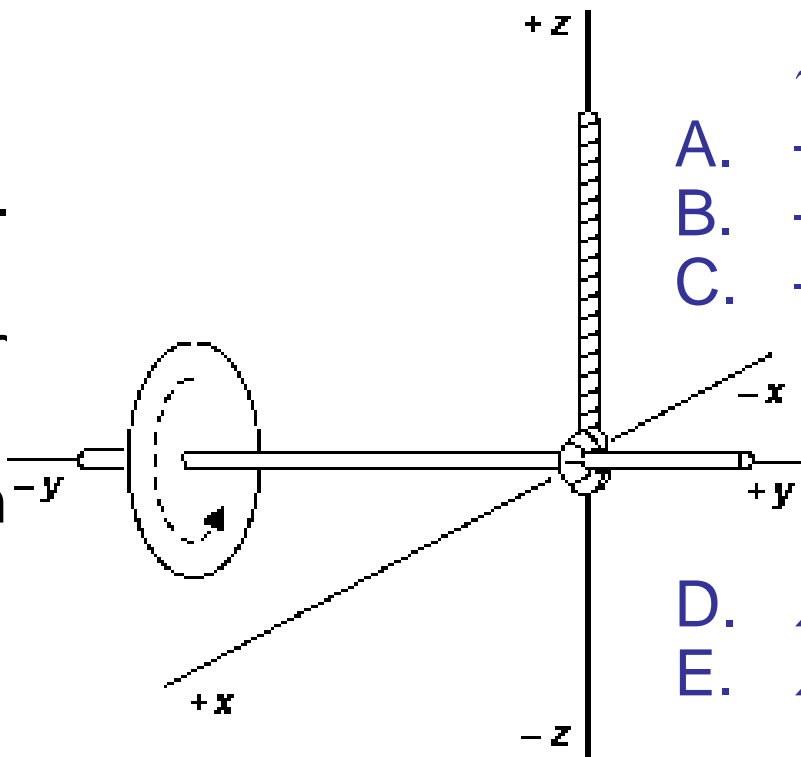
\vec{L}_{sys} is constant.

Angular momentum is conserved in the absence of external torque, or more generally, when the net external torque is 0.

This depends on internal torques canceling, which follows from Newton's 3rd: $\vec{F}_{12} = -\vec{F}_{21}$:



A gyroscopic toy is spinning as shown. The torque τ , angular momentum of the wheel, and angular precession velocity ω_p are in which directions?



- | | τ | L | ω_p |
|----|--------|------|------------|
| A. | $-z$ | y | x |
| B. | $-x$ | $-y$ | $-z$ |
| C. | $-x$ | $-y$ | z |
| D. | x | y | z |
| E. | x | y | $-z$ |

Examples: Diver, Ballet dancer, ice-skater, gymnast. All manipulate their moment of inertia to control ω in spite of L being conserved.

Physics example – **Bullet into stick.**

System: stick+“bullet”

initial: $\vec{L} = \vec{r} \times \vec{p} = m\vec{v}x$

(out of page)

final (m sticks to M):
and system rotates about pivot.

$$L = L_{\text{stick}} + L_{\text{bullet}}$$

$$L_{\text{bullet}} = mx^2\omega$$

$$L_{\text{stick}} = I_{\text{stick about pivot}}\omega$$

$$L_{\text{init}} = L_{\text{final}} \text{ so}$$

$$m\vec{v}x = (mx^2 + I_{\text{stick about pivot}})\omega$$

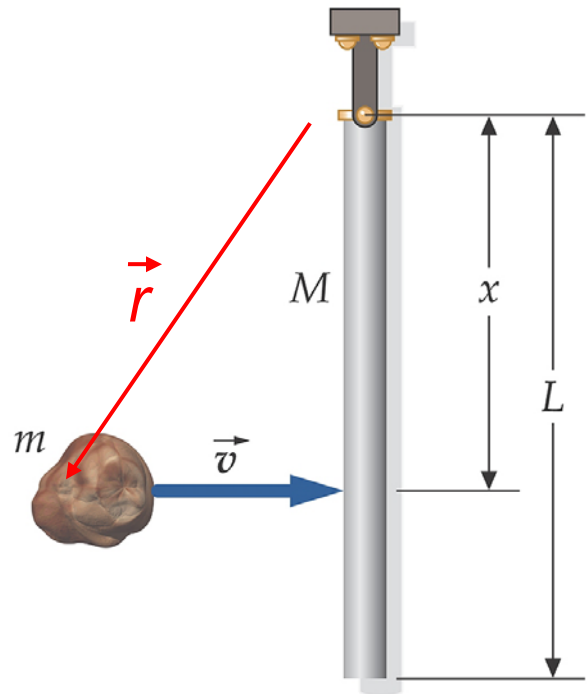
Note $I_{\text{system}} = mx^2 + I_{\text{stick about pivot}}$ when m sticks to M

Given ω we can get

$$K = \frac{1}{2} I_{\text{system}} \omega^2,$$

then considering gravity, find motion after collision --

L is conserved, because gravity does not produce any torque until the stick swings from vertical, and we are ignoring effect of gravity on the flying bullet.



Conservation of L with “Central Forces”

E.g. gravity of Sun on Earth, atoms, ...

No torque about center by a central force. Why?

Example of comet orbit.

Evolution of astronomical systems

many galaxies are disks

**the solar system is disk-like. (planetary orbits
in approximate plane.)**

Neutron stars – pulsars.