ESS 460 – ESS 560 Cosmogenic Nuclides

Homework Week 2

Shielding depths: Absorption of radiation is governed by the amount of mass traversed. Dense media like Pb absorb radiation more efficiently *per unit path length* than low-density media like water or air.

Therefore it makes sense to measure length in a way that avoids having to specify the density of the material we're working with. We define *'shielding thickness'* in units of g/cm^2 – think of the cumulative mass of a column of material with a cross-sectional area 1 cm^2.

Given a shielding thickness Z, the equivalent length l (in centimeters) in a material of density ρ (g/cm^3) is given by:

$$Z = \rho\, l$$
$$l = Z / \rho$$

(1) Calculate the shielding depth at the bottom of a 4 m deep swimming pool.

(2) Calculate the shielding depth of a sample collected from the base of a 4 m deep road-cut, in stratified sedimentary rocks:

	0
Sandstone (ρ = 2.38 g cm^{-3})	
	1.6 m
Shale (ρ = 2.14 g cm^{-3})	
	2.7 m
Sandstone (ρ = 2.44 g cm^{-3})	
	4.0 m

Atmospheric pressure is closely related to atmospheric 'depth'. Pressure P is measured in Pa (1 Pa = 1 kg m^{-1} s^{-2}; kg wt per unit area), so shielding depth below the top of the atmosphere is given by P/g, where g is the acceleration due to gravity (9.807 m s^{-2}). The US Standard Atmosphere defines pressure as a function of altitude by:

$$P(z) = P_{sl}\, Exp\left[-\frac{gM}{R\xi}\left(\ln T_{sl} - \ln\left(T_{sl} - \xi z\right)\right)\right]$$

where P_{sl} is the sea-level pressure (101,325 Pa), M is the molar weight of air (0.0289644 kg/mol), R is the gas constant (8.314 J K^{-1} mol^{-1}), T_{sl} is sea-level temperature (288.15 K), and ξ is the adiabatic lapse rate (i.e. the decrease in temperature with altitude; 0.0065 K m^{-1}). Numerically, this becomes:

$$P(z) = P_{sl}\ Exp\left[-\frac{0.03417}{\xi}\left(\ln T_{sl} - \ln\left(T_{sl} - \xi z\right)\right)\right]$$

(3) Calculate the atmospheric pressure at 1000 m, 3000 m and 6000 m altitude. Calculate the shielding depth (below the top of the atmosphere) at each altitude in g cm^{-2}. Note – the shielding depth at sea level is 1033.2 g cm^{-2}. Take care with units, and make sure you can get the correct value at sea-level before calculating the high-altitude values.

(4) Based on the formula given in class for the geometric cross-section of a nucleus, show that the total nuclear cross-section of a material of density ρ and atomic weight A is equal to:

$$0.034\ \rho A^{-1/3}\quad cm^2\ per\ cm^3$$

This is usually referred to as the 'macroscopic' cross-section, with units cm^{-1}. Its inverse has dimensions of length, and is a measure of the mean distance a particle will travel between nuclear collisions. Show that this 'mean-free-path' L is equal to:

$$29\ A^{1/3}\quad g\ cm^{-2}$$

Calculate L for air, given A = 14.548 g/mol.

Based on this value, comment on the probability of a primary cosmic ray proton reaching ground level.

(5) Calculate the threshold energy for fast-neutron induced spallation of ^{16}O to produce ^{10}Be:

$$^{16}O(n, 4p3n)^{10}Be$$

Data you will need:

Mass ^{16}O = 15.994915 amu

Mass p = 1.007825 amu

Mass n = 1.008665 amu

Mass ^{10}Be = 10.013535 amu

Suppose instead that the fragments produced by the reaction contain an α-particle:

$$^{16}O(n, \alpha 2pn)^{10}Be$$

Calculate the threshold neutron energy for this reaction. M_α = 4.002603 amu.

Account for the difference between the threshold energies of the two possible reactions.

Compositional data for air:

	Fraction by mass	Molar fraction	Molecular species	Partial pressure (atm)
Carbon	0.000124	0.00015	CO_2	0.000383 (in 2007)
Nitrogen	0.755267	0.7844	N_2	0.78084
Oxygen	0.231781	0.2108	O_2	0.20946
Argon	0.012827	0.00467	Ar	0.9340