

ESS 460 – ESS 560 Cosmogenic Nuclides
Homework Week 2

Shielding depths: Absorption of radiation is governed by the amount of mass traversed. Dense media like Pb absorb radiation more efficiently *per unit path length* than low-density media like water or air.

Therefore it makes sense to measure length in a way that avoids having to specify the density of the material we're working with. We define '*shielding thickness*' in units of g/cm^2 – think of the cumulative mass of a column of material with a cross-sectional area 1 cm^2 .

Given a shielding thickness Z , the equivalent length l (in centimeters) in a material of density ρ (g/cm^3) is given by:

$$Z = \rho l$$

$$l = Z / \rho$$

- (1) Calculate the shielding depth at the bottom of a 4 m deep swimming pool.
- (2) Calculate the shielding depth of a sample collected from the base of a 4 m deep road-cut, in stratified sedimentary rocks:

Sandstone ($\rho = 2.38 \text{ g cm}^{-3}$)	0
Shale ($\rho = 2.14 \text{ g cm}^{-3}$)	1.6 m
Sandstone ($\rho = 2.44 \text{ g cm}^{-3}$)	2.7 m
	4.0 m

Atmospheric pressure is closely related to atmospheric 'depth'. Pressure P is measured in Pa ($1 \text{ Pa} = 1 \text{ kg m}^{-1} \text{ s}^{-2}$; kg wt per unit area), so shielding depth below the top of the atmosphere is given by P/g , where g is the acceleration due to gravity (9.807 m s^{-2}). The US Standard Atmosphere defines pressure as a function of altitude by:

$$P(z) = P_{sl} \text{Exp} \left[-\frac{gM}{R\xi} (\ln T_{sl} - \ln(T_{sl} - \xi z)) \right]$$

where P_{sl} is the sea-level pressure (101,325 Pa), M is the molar weight of air (0.0289644 kg/mol), R is the gas constant (8.314 J K⁻¹ mol⁻¹), T_{sl} is sea-level temperature (288.15 K), and ξ is the adiabatic lapse rate (i.e. the decrease in temperature with altitude; 0.0065 K m⁻¹). Numerically, this becomes:

$$P(z) = P_{sl} \text{Exp} \left[-\frac{0.03417}{\xi} (\ln T_{sl} - \ln(T_{sl} - \xi z)) \right]$$

(3) Calculate the atmospheric pressure at 1000 m, 3000 m and 6000 m altitude. Calculate the shielding depth (below the top of the atmosphere) at each altitude in g cm⁻². Note – the shielding depth at sea level is 1033.2 g cm⁻². Take care with units, and make sure you can get the correct value at sea-level before calculating the high-altitude values.

(4) Based on the formula given in class for the geometric cross-section of a nucleus, show that the total nuclear cross-section of a material of density ρ and atomic weight A is equal to:

$$0.034 \rho A^{-1/3} \text{ cm}^2 \text{ per cm}^3$$

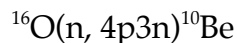
This is usually referred to as the ‘macroscopic’ cross-section, with units cm⁻¹. Its inverse has dimensions of length, and is a measure of the mean distance a particle will travel between nuclear collisions. Show that this ‘mean-free-path’ L is equal to:

$$29 A^{1/3} \text{ g cm}^{-2}$$

Calculate L for air, given $A = 14.548$ g/mol.

Based on this value, comment on the probability of a primary cosmic ray proton reaching ground level.

(5) Calculate the threshold energy for fast-neutron induced spallation of ¹⁶O to produce ¹⁰Be:



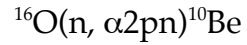
Data you will need: Mass ¹⁶O = 15.994915 amu

Mass p = 1.007825 amu

Mass n = 1.008665 amu

Mass ¹⁰Be = 10.013535 amu

Suppose instead that the fragments produced by the reaction contain an α -particle:



Calculate the threshold neutron energy for this reaction. $M_n = 4.002603$ amu.

Account for the difference between the threshold energies of the two possible reactions.

Compositional data for air:

	Fraction by mass	Molar fraction	Molecular species	Partial pressure (atm)
Carbon	0.000124	0.00015	CO ₂	0.000383 (in 2007)
Nitrogen	0.755267	0.7844	N ₂	0.78084
Oxygen	0.231781	0.2108	O ₂	0.20946
Argon	0.012827	0.00467	Ar	0.9340