

1 (i) $\delta^{18}O_{\text{snow}}$ of Pyroxene = 7‰

(1)

$$\delta^{18}O_{\text{snow}} = \left(\frac{(^{18}O/^{16}O)_{\text{pyroxene}}}{(^{18}O/^{16}O)_{\text{snow}}} - 1 \right) \times 1000 = 7$$

$$\text{So } \frac{(^{18}O/^{16}O)_{\text{px}}}{(^{18}O/^{16}O)_{\text{snow}}} = 1.007$$

$$\begin{aligned} \text{So } (^{18}O/^{16}O)_{\text{px}} &= 1.007 \times (^{18}O/^{16}O)_{\text{snow}} \\ &= 1.007 \times 0.0020052 \\ &= 0.00201924 \end{aligned}$$

$$\begin{aligned} \text{Likewise } (^{18}O/^{16}O)_{\text{plag}} &= 1.0077 \times (^{18}O/^{16}O)_{\text{snow}} \\ &= 1.0077 \times 0.0020052 \\ &= 0.00202064 \end{aligned}$$

difference is 1 in the 6th place

(actually 1.4 ppm)

$$\begin{aligned} 2(ii) \delta^{18}O_{\text{snow} - \text{snow}} &= \left(\frac{0.001925}{0.0020052} - 1 \right) \times 1000 \\ &= -40.0 \text{‰ relative to snow} \end{aligned}$$

$$\begin{aligned} \delta D_{\text{snow} - \text{snow}} &= \left(\frac{.00010747}{.00015576} - 1 \right) \times 1000 \\ &= -310 \text{‰ relative to snow} \end{aligned}$$

$$1 \text{ (iii)} \quad \delta^{18}\text{O}_{\text{Coral-SMOW}} = 26\%$$

$$\Rightarrow \left(\frac{(^{18}\text{O}/^{16}\text{O})_{\text{Coral}}}{(^{18}\text{O}/^{16}\text{O})_{\text{SMOW}}} - 1 \right) \times 1000 = 26\%$$

$$\Rightarrow \frac{(^{18}\text{O}/^{16}\text{O})_{\text{Coral}}}{(^{18}\text{O}/^{16}\text{O})_{\text{SMOW}}} = 1.026$$

$$\begin{aligned} \Rightarrow (^{18}\text{O}/^{16}\text{O})_{\text{Coral}} &= 1.026 \times 0.0020052 \\ &= 0.0020573 \end{aligned}$$

1 (iv) As above,

$$\begin{aligned} (^{18}\text{O}/^{16}\text{O})_{\text{PDB}} &= 1.0309 \times 0.0020052 \\ &= 0.0020672 \end{aligned}$$

$$\begin{aligned} \text{So } \delta^{18}\text{O}_{\text{Coral-PDB}} &= \left(\frac{(^{18}\text{O}/^{16}\text{O})_{\text{Coral}}}{(^{18}\text{O}/^{16}\text{O})_{\text{PDB}}} - 1 \right) \times 1000 \\ &= \left(\frac{0.0020573}{0.0020672} - 1 \right) \times 1000 \\ &= (0.99525 - 1) \times 1000 \\ &= (-0.0048) \times 1000 \\ &= -4.8\% \end{aligned}$$

$$\begin{aligned} \delta^{18}\text{O}_{\text{PDB-SMOW}} + \delta^{18}\text{O}_{\text{Coral-PDB}} &= 30.9\% - 4.8\% \\ &= 26.1\% \end{aligned}$$

$$\text{whereas } \delta^{18}\text{O}_{\text{Coral-SMOW}} = 26.0\%$$

difference in δ values of magnetite and quartz as the "oxygen isotopic fractionation" between them. The following exercise will help you understand the relationship between α and δ values and how we can put measurements of $\delta^{18}\text{O}$ to work to estimate equilibration temperatures:

Famous geochemist Bob Clayton runs some experiments in which he synthesizes magnetite plus quartz at various temperatures. He separates the two minerals from the reaction products and measures their $\delta^{18}\text{O}$ values relative to the SMOW standard. He gets the following results:

T °C	T K	$10^6/T^2$	$\delta^{18}\text{O}_{\text{qtz-SMOW}}$	$\delta^{18}\text{O}_{\text{mt-SMOW}}$	$\alpha_{\text{qtz-mt}}$	1000 ln α	$\delta^{18}\text{O}_{\text{qtz}} - \delta^{18}\text{O}_{\text{mt}}$
500	773	1.674	+1.02	-9.16	1.0027	10.22	10.18 ‰
700	973	1.056	+0.64	-5.79	1.00647	6.45	6.43 ‰
900	1173	0.727	+0.45	-3.98	1.00445	4.44	4.43 ‰
1100	1373	0.530	+0.33	-2.91	1.00325	3.24	3.24 ‰

Calculate the exact value of the equilibrium constant α at each temperature and add it to the table. You will need to convert the $\delta^{18}\text{O}$ values to isotopic ratios. Take $(^{18}\text{O}/^{16}\text{O})_{\text{SMOW}} = 0.0020052$. Fill in the rest of the table. Compare the value of $1000 \ln \alpha_{\text{qtz-mt}}$ at each temperature to the difference between the δ values of the two minerals. What do you notice?

Draw a plot of $1000 \ln \alpha_{\text{qtz-mt}}$ vs $10^6/T^2$ and draw a line through the data. Note that a plot of $\delta^{18}\text{O}_{\text{qtz}} - \delta^{18}\text{O}_{\text{mt}}$ would have given you essentially the same diagram.

Plots of this type allow you to predict the difference in δ values between minerals, fluids and gases in equilibrium. Any pair of substances give a line on the diagram. For example, the line for quartz-pyroxene would have approximately 1/3 the slope of the quartz-magnetite line. At any given temperature, the difference in $\delta^{18}\text{O}$ values between pyroxene and quartz will be ~1/3 as large as the difference in $\delta^{18}\text{O}$ values of magnetite and quartz.

We can use these kinds relationships to determine temperatures of equilibration:

2 (9) Table: $(^{18}\text{O}/^{16}\text{O})_{\text{SMOW}} = 0.0020052$

(4)

At $500^\circ\text{C} = 773\text{K}$, $\frac{10^6}{T^2} = 1.674$

$$\delta^{18}\text{O}_{\text{qtz-SMOW}} = 1.02, \text{ so } \left(\frac{^{18}\text{O}}{^{16}\text{O}}\right)_{\text{qtz}} = 1.00102 \times 0.0020052 = 0.002007$$

$$\delta^{18}\text{O}_{\text{mt-SMOW}} = -9.16, \text{ so } \left(\frac{^{18}\text{O}}{^{16}\text{O}}\right)_{\text{mt}} = 0.99084 \times \text{"} = 0.0019868$$

$$\Rightarrow \alpha_{\text{qtz-mt}} = \frac{(^{18}\text{O}/^{16}\text{O})_{\text{qtz}}}{(^{18}\text{O}/^{16}\text{O})_{\text{mt}}} = 1.01027$$

$$1000 \ln \alpha_{\text{qtz-mt}} = 10.22$$

$$\underline{\underline{\text{cf}}} \quad \delta^{18}\text{O}_{\text{qtz}} - \delta^{18}\text{O}_{\text{mt}} = 1.02 - (-9.16) = 10.18 \text{ ‰}$$

↑ ↑
BOTH RELATIVE TO SMOW

At $700^\circ\text{C} = 973\text{K}$; $\frac{10^6}{T^2} = 1.056$

$$\delta^{18}\text{O}_{\text{qtz-SMOW}} = 0.64, \text{ so } \left(\frac{^{18}\text{O}}{^{16}\text{O}}\right)_{\text{qtz}} = 0.0020065$$

$$\delta^{18}\text{O}_{\text{mt-SMOW}} = -5.79, \text{ so } \left(\frac{^{18}\text{O}}{^{16}\text{O}}\right)_{\text{mt}} = 0.0019936$$

$$\alpha_{\text{qtz-mt}} = 1.00647$$

$$1000 \ln \alpha_{\text{qtz-mt}} = 6.45$$

$$\underline{\underline{\text{cf}}} \quad \delta^{18}\text{O}_{\text{qtz}} - \delta^{18}\text{O}_{\text{mt}} = 0.64 - (-5.79) = 6.43 \text{ ‰}$$

$$\text{At } 900^\circ\text{C} = 1173\text{ K} ; \frac{10^6}{T^2} = 0.727$$

(5)

$$\delta^{18}\text{O}_{\text{qtz-smow}} = 0.45\text{‰} \Rightarrow \left(\frac{18\text{O}}{16\text{O}}\right)_{\text{qtz}} = 0.0020061$$

$$\delta^{18}\text{O}_{\text{mt-smow}} = -3.98\text{‰} \Rightarrow \left(\frac{18\text{O}}{16\text{O}}\right)_{\text{mt}} = 0.0019972$$

$$\Rightarrow \alpha_{\text{qtz-mt}} = 1.004448$$

$$1000 \ln \alpha_{\text{qtz-mt}} = 4.44$$

$$\underline{\text{cf}} \quad \delta^{18}\text{O}_{\text{qtz-smow}} - \delta^{18}\text{O}_{\text{mt-smow}} = 0.45 - (-3.98) = 4.43$$

$$\text{At } 1100^\circ\text{C} = 1373\text{ K} ; \frac{10^6}{T^2} = 0.530$$

$$\delta^{18}\text{O}_{\text{qtz-smow}} = 0.33 \Rightarrow \left(\frac{18\text{O}}{16\text{O}}\right)_{\text{qtz}} = 0.0020059$$

$$\delta^{18}\text{O}_{\text{mt-smow}} = -2.91 \Rightarrow \left(\frac{18\text{O}}{16\text{O}}\right)_{\text{mt}} = 0.0019994$$

$$\alpha_{\text{qtz-mt}} = 1.00325$$

$$1000 \ln \alpha_{\text{qtz-mt}} = 3.24$$

$$\text{cf} \quad \delta^{18}\text{O}_{\text{qtz-smow}} - \delta^{18}\text{O}_{\text{mt-smow}} = 0.33 - (-2.91) = 3.24$$

Regressing these gives:

$$\left(1000 \ln \alpha_{qtz-mt}\right) = 6.10 \left(\frac{10^6}{T^2}\right)$$

Which we can read as:

$$\left(\delta^{18}O_{qtz-snow} - \delta^{18}O_{mt-snow}\right) = 6.10 \left(\frac{10^6}{T^2}\right)$$

A pair of oxygen isotope analyses, one on quartz and one on co-existing magnetite gives a value for T.

2b) (i) At 700°C ; $1000 \ln \alpha_{qtz-mt} = 6.45$

ie $\delta^{18}O_{qtz-snow} - \delta^{18}O_{mt-snow} \approx 6.45\text{‰}$

if $\delta^{18}O_{qtz-snow} = 10\text{‰}$; $10 - \delta^{18}O_{mt-snow} = 6.45$

$\Rightarrow \delta^{18}O_{mt-snow} \approx 3.55\text{‰}$

if $\delta^{18}O_{qtz-snow} = 15\text{‰}$; $\delta^{18}O_{mt-snow} = 8.55\text{‰}$

2 b (ii) $\left. \begin{aligned} \delta^{18}\text{O}_{\text{qtz-snow}} &= 11.1\text{‰} \\ \delta^{18}\text{O}_{\text{mt-snow}} &= 7.0\text{‰} \end{aligned} \right\}$ What temperature did they equilibrate at? (7)

$$\Rightarrow \delta^{18}\text{O}_{\text{qtz-snow}} - \delta^{18}\text{O}_{\text{mt-snow}} = 4.1\text{‰}$$

Using the approximation derived on the handout (and confirmed by filling in the values in the table)

$$\delta^{18}\text{O}_{\text{qtz-snow}} - \delta^{18}\text{O}_{\text{mt-snow}} \approx 1000 \ln \alpha_{\text{qtz-mt}} \approx 4.1\text{‰}$$

And from our regression of the experimental data in the table

$$1000 \ln \alpha_{\text{qtz-mt}} = 6.10 \left(\frac{10^6}{T^2} \right)$$

$$\Rightarrow \frac{4.1}{6.10} = \frac{10^6}{T^2} = 0.672$$

$$\Rightarrow T^2 = \frac{10^6}{0.672} = 1.488 \times 10^6$$

$$T = 1220 \text{ K}$$

$$= 948^\circ\text{C}$$

(iii) Did the feldspar equilibrate its O isotope composition with qtz and magnetite at 948°C?

$$\text{Given } \delta^{18}\text{O}_{\text{qtz-snow}} = 11.1 \text{‰}$$

$$\delta^{18}\text{O}_{\text{albite-snow}} = 10.45 \text{‰}$$

$$\delta^{18}\text{O}_{\text{qtz-snow}} - \delta^{18}\text{O}_{\text{albite-snow}} \approx 1000 \ln \alpha_{\text{qtz-albite}} = 0.65$$

From the qtz-albite thermometer equation, $T_{\text{equil.}}$ is given by

$$0.65 = 0.97 \left(\frac{10^6}{T^2} \right)$$

$$\Rightarrow T^2 = \frac{0.97 \times 10^6}{0.65} = 1.492 \times 10^6$$

$$\Rightarrow T = 1222 \text{ K}$$

$$= 949 \text{ °C} \quad \dots \text{ the same } T \text{ as indicated}$$

by quartz-magnetite isotopic compositions. Suggests qtz, mt and albite all came to O-isotope equilibrium at ~950°C.

(iv) In the vein assemblage

$$\delta^{18}\text{O}_{\text{qtz-snow}} - \delta^{18}\text{O}_{\text{snow}} = 4.1 - 2.25 = 1.85 \text{‰}$$

Substituting for $1000 \ln \alpha_{\text{qtz-albite}}$;

$$1.85 = 0.97 \left(\frac{10^6}{T^2} \right)$$

$$\Rightarrow T^2 = \frac{0.97 \times 10^6}{1.85} = 5.24 \times 10^5$$

$$T = 724 \text{ K}$$

$$= 451^\circ \text{C}$$

Veins were emplaced during cooling of the granite.

- (v) Bulk $\delta^{18}\text{O}_{\text{snow}}$ value of the granite is between 7.0 - 11.1‰
 (NB magnetite is almost always the "lightest" (i.e. most ^{16}O -enriched) mineral in a rock, quartz is usually the "heaviest" (i.e. most ^{18}O -enriched).)

Bulk $\delta^{18}\text{O}_{\text{snow}}$ value of qtz-fsp veins is between 2.25 - 4.1‰

Bulk values are different, so the veins must have crystallised from a different source material to the granite.

$$(vi) 1000 \ln \alpha_{\text{qtz-water}} \approx \delta^{18}\text{O}_{\text{qtz-snow}} - \delta^{18}\text{O}_{\text{water-snow}} = 4.1 \left(\frac{10^6}{T^2} \right) - 3.7$$

Solve for $\delta^{18}\text{O}$ of water ...

$$4.1 - \delta^{18}\text{O}_{\text{water-snow}} = 4.1 \left(\frac{10^6}{T^2} \right) - 3.7 ; T = 451^\circ \text{C}$$

$$\begin{aligned} \delta^{18}\text{O}_{\text{qtz-snow}} - \delta^{18}\text{O}_{\text{water-snow}} &= (4.1 + 3.7) - \frac{4.1 \times 10^6}{(724)^2} \\ &= 7.8 - 7.8 \end{aligned}$$

$$\delta^{18}\text{O}_{\text{water-SMOW}} = 0.0 \text{‰}$$

Similarly, $\delta^{18}\text{O}_{\text{albite-SMOW}} - \delta^{18}\text{O}_{\text{water-SMOW}} = 3.13 \left(\frac{10^6}{72} \right) - 3.7$

$$\Rightarrow \delta^{18}\text{O}_{\text{water-SMOW}} = (2.25 + 3.7) - \frac{3.13 \times 10^6}{(72)^2}$$

$$= 5.95 - 5.97$$

$$= -0.02 \text{‰}$$

Hence $\delta^{18}\text{O}_{\text{water}}$ indicated by both minerals ~~is~~

$$= 0.0 \text{‰}$$

i.e. the water isotopic composition is the same as SMOW - standard mean ocean water.

(vii) This suggests the veins may have formed from a fluid in equilibrium with seawater