ESS 312 Geochemistry Week 5
GEOCHRONOLOGY

This lab deals with the radioactive decay equation. First we'll consider the equation in its *differential* form, in order to understand the significance of the radioactive decay constant. Then we'll use Excel™ to simulate the decay of a parent isotope and use it to calculate a simple *radiometric* age. Part 2 walks you through an age calculation based on the *isochron* method, which we covered in lectures.

Next week we'll expand this treatment to consider an open geochemical system, in which the parent element and its daughter isotope are transported between different geochemical reservoirs – an approach that can be adapted to model differentiation of the Earth. The results will tell us about the timing of differentiation, element transport rates, and exchange of elements between the mantle, crust, oceans and atmosphere. We'll encounter the method again later in the course as a way to evaluate the response times of complex natural systems to perturbations in one or more of the reservoirs.

RADIOACTIVE DECAY AND DAUGHTER ISOTOPE BUILD-UP IN A CLOSED SYSTEM

Radioactive decay is one of the best-known examples of a truly random, probabilistic process, in which the chance of an individual atom decaying in a period of time is constant, regardless of the number of atoms present or the time elapsed since radioactive decay began. The decay probability applies to each atom, but it is measured by looking at a very large sample of atoms and determining the fraction that decays in a given time period. We call this fraction the "decay constant" λ. Its dimensions are t^{-1}, or "per unit time". A decay constant of 0.1 per year (0.1 yr^{-1}) tells us that, on average, 1/10 of the atoms present in a system will decay each year. Mathematically, λ is the constant that relates the decay rate of an isotope to its abundance N:

$$\frac{dN}{dt} = -\lambda N \qquad (1)$$

A sample that initially contains 10^6 atoms of an isotope with a decay constant of 0.1 yr^{-1} decreases by 10^5 atoms in the first year, leaving 9 x 10^5 atoms. The next year, 9 x 10^4 atoms decay, leaving 8.1 x 10^5 atoms, and so on. (A bit like having a 10% annual service charge on your bank account). Most of the isotopes used to study differentiation of the Earth have decay constants on the order of $10^{-9} – 10^{-11}$ yr^{-1}. (Why must we use isotopes with such small decay constants?)

The radioactive decay law given in equation (1) above is in *differential* form. In other words, the equation is a statement about the *rate of change* of N as a function of time. To get an exact solution for the *value* of N as a function of time, we need to integrate equation (1). If we specify that the abundance of the isotope at time zero $= N_0$, the solution is:

$$N = N_0 \, e^{-\lambda t} \tag{2}$$

or as a statement about the fraction of the parent isotope remaining at time t:

$$\frac{N}{N_0} = e^{-\lambda t} \tag{3}$$

(1) From equation (3), derive an expression for the half-life – the value of t at which $N/N_0 = 1/2$.

To illustrate the decay of a radioactive parent isotope such as ^{14}C, set up a simple spreadsheet:

(i) Set up columns to keep track of t and N. Enter values for t from 0 to 20,000 years, in 50 increments of 400 years.

(ii) Set up a cell for the decay constant λ, and give it a value of 1.2097×10^{-4} yr^{-1}. This is the established value for the decay constant of ^{14}C.

(iii) Enter 100 as the initial value of N (i.e. N_0 at $t = 0$).

This represents "percentage of modern carbon-14 abundance" which is the standard way of expressing ^{14}C measurements. Think of it as a value for the ^{14}C content of atmospheric CO_2, from which living organisms ultimately derive their carbon. Once an organism dies, it ceases to exchange its carbon with the atmosphere, and the percentage of ^{14}C begins to drop due to radioactive decay.

(iv) Now use equation (3) to fill in the remaining values of N corresponding to the values of t.

(2) Graph the abundance of the parent radioisotope through time on a linear N vs linear t plot.

(3) Now produce the same graph on a log(N) vs linear t plot (Create a new plot, click on the y-axis to get the "Format axis" dialog box, and click on the 'logarithmic scale' button under the 'Scale' tab). This should remind you of the relationship between the logarithm and exponential functions.

(4) At what time does the abundance reach 1/2 of the initial value? 1/4? 1/8? Is the interval between these points the same in each case? Estimate the "half life" of ^{14}C and compare it to an exact value derived using the expression you came up with for question (1).

(5) In 2000 a mammoth tusk was uncovered by construction workers remodeling the playground at Lakeview Elementary School in Kirkland. Subsequent ^{14}C measurements on the tusk gave a value of 12.47% compared to modern ^{14}C activity. Use your spreadsheet or graphs to calculate the age of the tusk.

(News links to this are mostly dead now, but another good site for mammoth excavations is the La Brea Tar Pits, on Wilshire Boulevard in Los Angeles, just down the hill from Hollywood. See http://www.tarpits.org/education/guide/flora/mammoth.html).

(6) What assumptions do you have to make in order to estimate this age from the ^{14}C data?

Part 2: Isochron Diagrams for dating co-genetic rocks or minerals

For most radioactive isotope systems, we cannot make simple assumptions about the initial parent abundance, nor can we distinguish between atoms of the daughter isotope produced by decay of the parent, and those that were present at the time the rock formed. Faced with more than one unknown, we make use of the "isochron" equation (derived in class):

$$\frac{D}{S} = \left(\frac{D}{S}\right)_0 + \frac{N}{S}\left(e^{\lambda t} - 1\right) \tag{4}$$

Here D is the daughter isotope, N the parent and S is a stable, non-radiogenic isotope of the same element as the daughter. So for the ^{87}Rb-^{87}Sr system, this would read:

$$\frac{^{87}Sr}{^{86}Sr} = \left(\frac{^{87}Sr}{^{86}Sr}\right)_0 + \frac{^{87}Rb}{^{86}Sr}\left(e^{\lambda_{87} t} - 1\right) \tag{5}$$

This predits that a family of samples that formed with the same ^{87}Sr/^{86}Sr ratio, but different ^{87}Rb/^{86}Sr ratios, should plot on a line in a diagram with co-ordinates y = ^{87}Sr/^{86}Sr and x = ^{87}Rb/^{86}Sr. The slope of the line will depend on the time elapsed since the samples formed, and is given by: slope = $(e^{\lambda t} - 1)$. Thus by measuring ^{87}Sr/^{86}Sr and ^{87}Rb/^{86}Sr ratios in a family of co-genetic rocks or minerals and fitting a line through the data, we can determine their age. Try it:

The following data are analyses of chondritic meteorites:

^{87}Rb/^{86}Sr	^{87}Sr/^{86}Sr
0.4716	0.7300
0.6222	0.7399
0.8312	0.7536
0.3387	0.7213
0.8088	0.7522
0.5426	0.7346
0.3543	0.7222
0.5063	0.7322
0.7160	0.7460
0.0013	0.6989

0.9462	0.7613
0.9103	0.7588
0.8950	0.7578
0.9264	0.7599

(7) Use Excel to plot these data in co-ordinates x = ^{87}Rb/^{86}Sr and y = ^{87}Sr/^{86}Sr . You can use the "Add Trendline" command in the "Chart" menu to fit a line through the data. Select "Display equation on chart" from the trend-line options to find the slope of the line. Use this to calculate the age of this family of meteorites. You will need the decay constant for ^{87}Rb: λ = 1.42 x 10^{-11} yr^{-1}.

(8) What assumptions are you making in calculating the age? How can you tell from the data if these assumptions are likely to be valid?

(9) What was the initial ^{87}Sr/^{86}Sr ratio of Sr in the meteorites at the time of formation?

(10) Rocks in the Earth's crust typically contain Sr with a ^{87}Sr/^{86}Sr ratio of ~ 0.710. We think the bulk Earth average ^{87}Sr/^{86}Sr ratio is ~ 0.7047. How do these values compare to the *initial* ^{87}Sr/^{86}Sr ratio you calculated for the meteorites in question (9)? What can you infer about the average ^{87}Rb/^{86}Sr ratio of the bulk Earth and the crust? How must Rb fractionate *relative* to Sr in the melting processes that form continental crust?