Can Eguchi-Kawai reduction provide a practical method for studying large-N_c theories on the lattice?

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Outline

* Introduction & Motivation
* Short history of "volume reduction"
* Application to QCD with 1 & 2 adjoint fermions: adjoint Eguchi-Kawai [AEK] model
* Twisted adjoint Eguchi-Kawai [TAEK] model
* Outlook

Overview: beyond QCD

QCD

$N=3$; $N_f=2+1$

Confinement

Chiral Sym. breaking

Add fermions

Make (nearly) conformal:

$N=3$; $N_f=8$-16

$N=3$, $N_f=2$ sextets

$N=2$, $N_f=2$ adjoints

Add colors

$N \longrightarrow \infty$, $N_f=2+1$

't Hooft limit of QCD

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N=3, N_f=2 sextets
N=2, N_f=2 adjoints

Add colors ↓

$N \longrightarrow \infty$, N_f=2+1
't Hooft limit of QCD

Add both ↘

$N \longrightarrow \infty$
N_f two-index irreps
e.g. N_f=1,2 adjoints
Corrigan-Ramond limit

Why add colors?

* At first sight, this seems foolhardy!
  - Increasing the number of degrees of freedom while still studying a strongly coupled theory
* However, there are important theoretical and computational simplifications
  - Planarity
  - Gauge-gravity duality
  - Volume independence

Planarity ['t Hooft, Witten,...]

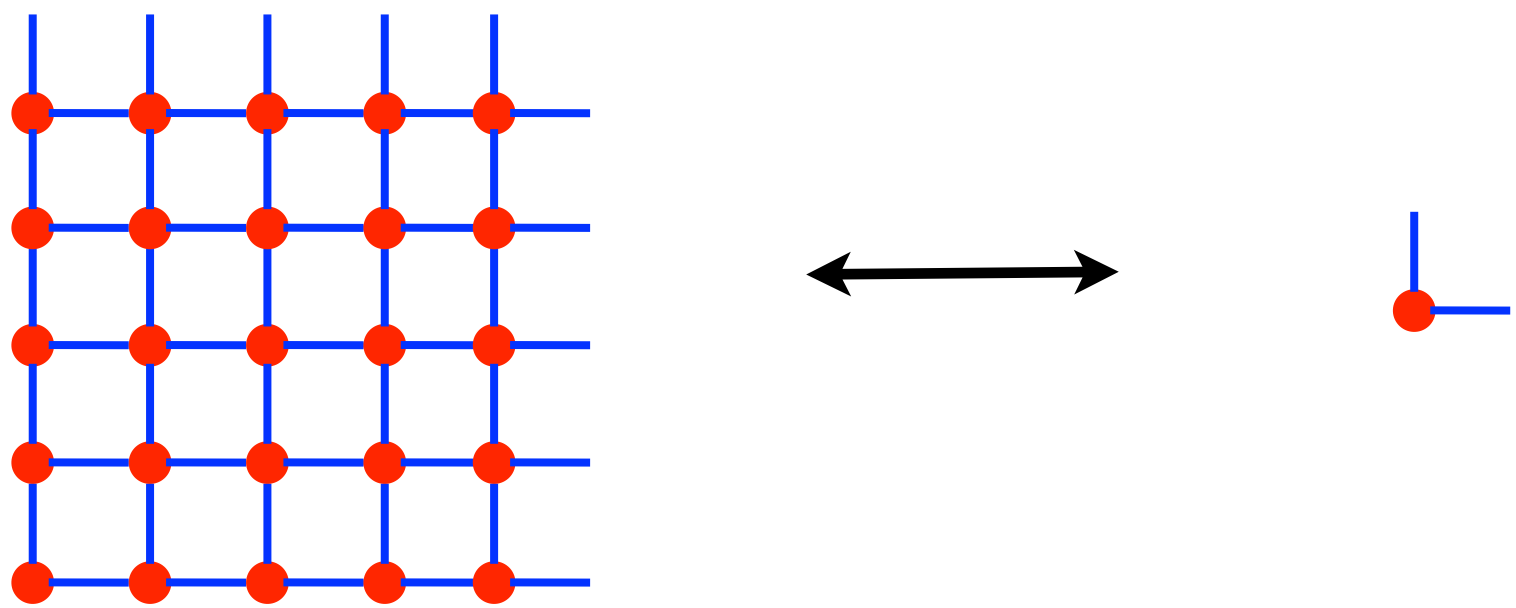
* Limit is $N \longrightarrow \infty$ with $\lambda = g^2 N$ & N_f fixed
* Only planar diagrams contribute in perturbation thy
* Mesons & glueballs are stable (widths ~1/N)
* Expectation values factorize: $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle$

➡ Simplified theory sharing asymptotic freedom, confinement & Chiral SB with QCD

- Long-standing hope that analytic progress is possible
- Lattice calculations can help guide search for string-theory duals

Volume Independence [Eguchi & Kawai]

* Under non-trivial conditions, certain properties of gauge theories at large N are independent of volume



➡ Does this reduction in degrees of freedom provide a practical method to access the theoretical simplicity of large N theories? Are the conditions satisfied?

After a hiatus, much recent interest, e.g.

T. Eguchi & H. Kawai, PRL 48 (1982) 1063 [EK model]

G. Bhanot, U. Heller & H. Neuberger, PL 113B (1982) 49 [QEK model]

A. Gonzalez-Arroyo & M. Okawa, PL 120B (1983) 174 [TEK model]

......

P. Kovtun, M. Unsal & L.G. Yaffe, JHEP 0706 (2007) 109 [Adjoint EK]

B. Bringoltz & S.R. Sharpe, PRD 80 (2009) 065031 [massive Nf=1 AEK works]

A. Heitenen & R. Narayanan, JHEP 1001 (2010) 79, PLB 698 (2011) 171 [massless Nf=1/2 AEK]

T. Azeyanagi, M. Hanada, M. Unsal & R. Yacoby, PRD82 (2010) 125013 [why massive AEK works; ATEK; T > 0]

M. Unsal & L.G. Yaffe, JHEP 1008 (2010) 030 [why massive AEK works]

B. Bringoltz, M. Koren & S.R. Sharpe, PRD85 (2012) 094504 [massive Nf=2 AEK works]

M. Hanada, J.-W. Lee & N. Yamada, arXiv: [chiral symmetry breaking using 2^4 AEK]

A. Gonzalez-Arroyo & M. Okawa, JHEP 1007 (2010) 043 [TEK lives and thrives]

R. Lohmayer & R. Narayanan, arXiv:1305.1279 [AEK problems in weak coupling]

A. Gonzalez-Arroyo & M. Okawa, arXiv:1305.6253 [ATEK for N up to 29^2=841]

I will not discuss:

* Novel simulations of single-site SUSY lattice theories aimed at testing AdS/CFT correspondence and learning about string theories & quantum gravity

  [J. Nishimura, M. Hanada, T. Wiseman, S. Catterall, ...]

* Partial reduction of QCD in ‘t Hooft limit
  * If $L > L_c \approx 1$ fm then results independent of L [Narayanan & Neuberger]

* Obtaining results for large N by extrapolating from N=3,4,5,6 (useful for pure gauge theory) [Teper,...]

* Reduction of one dimension [Cossu & D’Elia]

History of large-N volume independence

First example

VOLUME 48, NUMBER 16 PHYSICAL REVIEW LETTERS 19 APRIL 1982

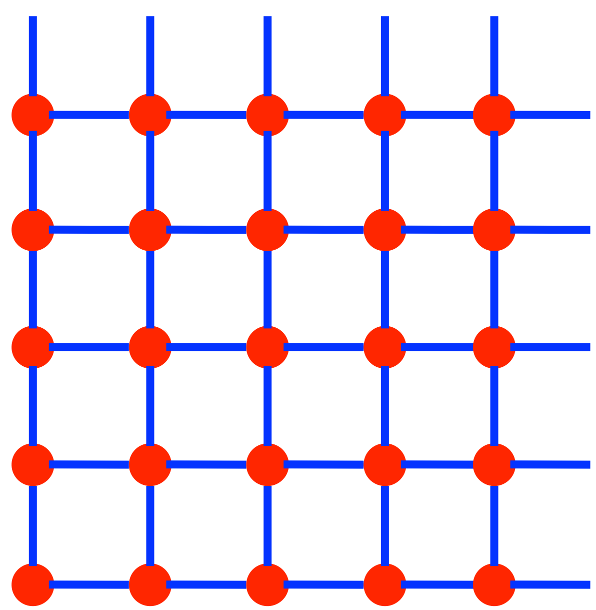
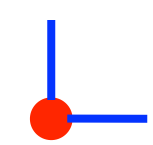
Reduction of Dynamical Degrees of Freedom in the Large-*N* Gauge Theory

Tohru Eguchi and Hikaru Kawai
Department of Physics, Faculty of Science, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan
(Received 19 January 1982)

$$\text{Lattice } SU(N) \text{ on } L^d \overset{N=\infty}{\equiv} \text{Lattice } SU(N) \text{ on } 1^d$$

Now usually called “large-N volume independence”

Lattice $SU(N)$ on $L^d \overset{N=\infty}{\equiv}$ Lattice $SU(N)$ on 1^d

gauge theory	"reduced" or "matrix" model
$U_{n,\mu} \in SU(N)$	$U_\mu \in SU(N)$
$S_{\text{gauge}} = Nb \sum_{\substack{n \\ \mu<\nu}} 2\text{Re}\,\text{Tr}\left(U_{n,\mu} U_{n+\mu,\nu} U^\dagger_{n+\nu,\mu} U^\dagger_{n,\nu}\right)$ $b = (g^2 N)^{-1}$	$S_{EK} = Nb \sum_{\mu<\nu} 2\text{Re}\,\text{Tr}\left(U_\mu U_\nu U^\dagger_\mu U^\dagger_\nu\right)$ $b = (g^2 N)^{-1}$
	

Links all different

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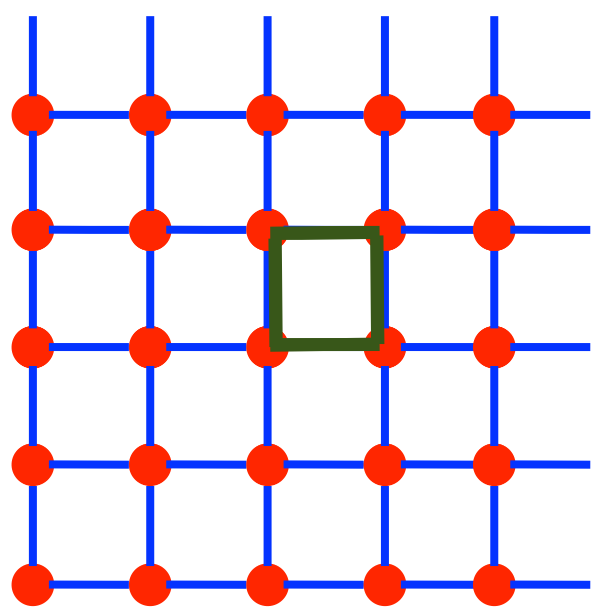
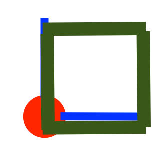
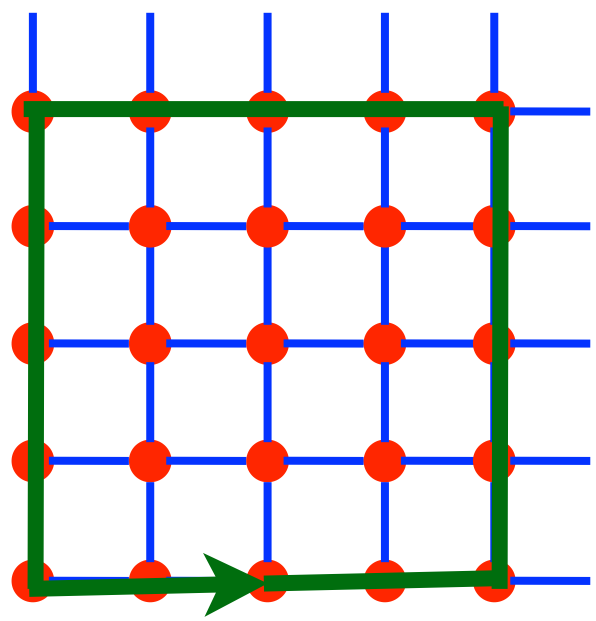


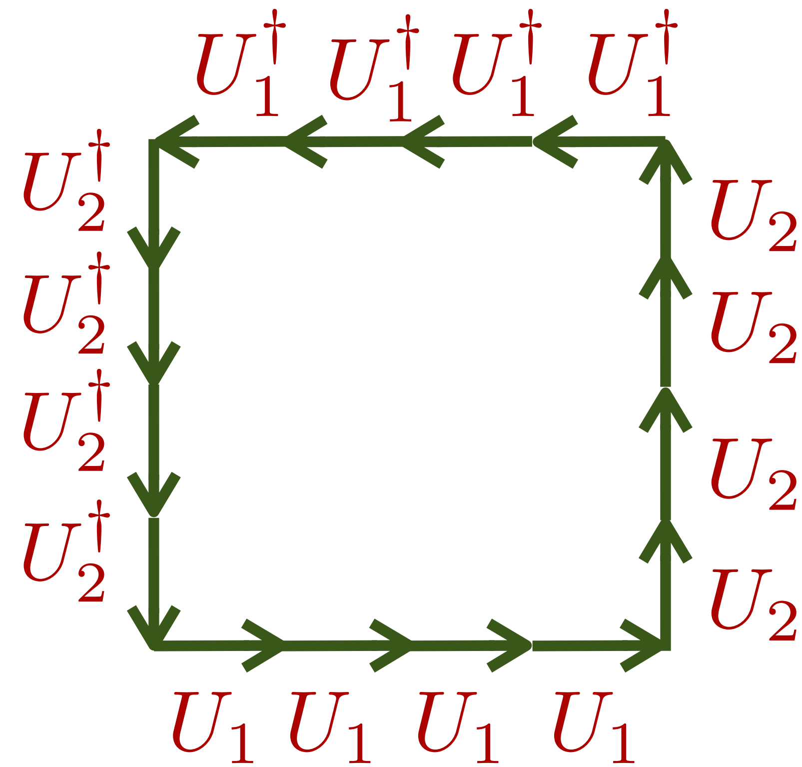


Links all different

Lattice $SU(N)$ on L^d $\overset{N=\infty}{\equiv}$ Lattice $SU(N)$ on 1^d

gauge theory	"reduced" or "matrix" model
$W_C = \frac{1}{N} \mathrm{tr}\, U_{x,\hat\mu} U_{x+\hat\mu,\hat\nu} \cdots U_{x-\hat\nu-\hat\rho,\hat\rho} U_{x-\hat\nu,\hat\nu}\,,$	$W_C^{\mathrm{reduced}} = \frac{1}{N} \mathrm{tr}\, U_\mu U_\nu \cdots U_\rho U_\nu\,.$





$$\langle W_C \rangle_{\text{gauge theory}} = \langle W_C^{\text{reduced}} \rangle_{\text{reduced}} + O(1/N^2).$$

Lattice $SU(N)$ on $L^d \overset{N=\infty}{\equiv}$ Lattice $SU(N)$ on 1^d

gauge theory	"reduced" or "matrix" model
gauge symmetry	
$U_{n\mu} \to \Omega_n U_{n\mu} \Omega^\dagger_{n+\mu} \quad ; \quad \Omega_n \in SU(N)$	$U_\mu \to \Omega U_\mu \Omega^\dagger \quad ; \quad \Omega \in SU(N)$
"center" symmetry	
$U_{[(\vec{n},\tau),\mu]} \to U_{[(\vec{n},\tau),\mu]} z_\mu \quad ; \quad z_\mu \in Z_N$	$U_\mu \to U_\mu z_\mu \quad ; \quad z_\mu \in Z_N$

EK's demonstration of vol. indep.

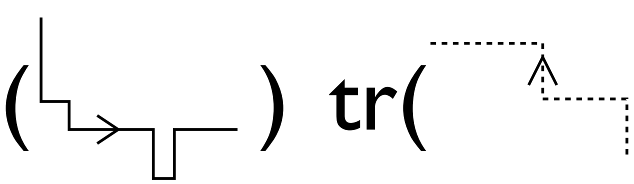
- Show equivalence of Dyson-Schwinger eqs for Wilson loops

gauge	reduced
$U_{n,\mu} \to U_{n\mu}\left(1 + i\epsilon t^a\right)$	$U_\mu \to U_\mu\left(1 + i\epsilon t^a\right)$

- Crucial difference

gauge	reduced
$\mathrm{tr}\left(\cdots \underline{U_{n,\mu}} U_{n+\mu,\nu} \cdots \underline{U^\dagger_{m,\mu}} U_{m-\mu,\rho} \cdots\right)$	$\mathrm{tr}\left(\cdots \underline{U_\mu} U_\nu \cdots \underline{U^\dagger_\mu} U_\rho \cdots\right)$

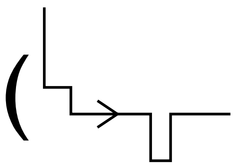
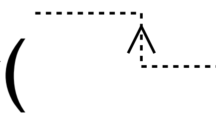
- Get extra terms on the reduced side: must vanish for reduction to hold

e.g. $\Big\langle$ tr() tr( ) $\Big\rangle_{\text{reduced}}$ = 0

- Extra terms correspond to "open loops" in gauge theory

e.g. $$\left\langle \mathrm{tr}\left(U_\mu U_\nu^\dagger\right)\ \mathrm{tr}\left(U_\mu^\dagger U_\nu\right)\right\rangle_{\mathrm{reduced}} = 0$$

EK's demonstration of volume independence

Reduction holds if $\Big\langle \mathrm{tr}($  $)\ \mathrm{tr}($  $)\Big\rangle_{\text{reduced}} = 0$

- Valid if have large-N factorization

$$\langle W_{C_1} W_{C_2}\rangle_{\text{reduced}} = \langle W_{C_1}\rangle_{\text{reduced}}\langle W_{C_2}\rangle_{\text{reduced}} + O(1/N^2),$$

- ... and if center symmetry is unbroken $(Z_N^4 :\ U_\mu \to U_\mu z_\mu)$

$$\langle W_{\text{open}}\rangle_{\text{reduced}} = 0.$$

CONCLUSION: $\mathrm{tr}U_\mu,\ \mathrm{tr}U_\mu U_\nu,\ \text{etc.}$

must all vanish in the reduced model

Alternative view of reduction

- Volume independence is an example of a larger class of equivalences: large-N orbifold equivalences [Kovtun, Unsal & Yaffe]

Restrict to zero-momentum fields

SU(N) gauge theory $\longrightarrow$ SU(N) on single site

SU(N) gauge theory $\longleftarrow$ SU(N) on single site

Orbifold w.r.t. combined gauge and center transfomation

- Orbifold equivalence holds if "orbifolding symmetries" (translation invariance and center symmetry) are unbroken

Reduction fails! [Bhanot, Heller & Neuberger '82]

- Qualitatively: Small L ⇔ High T ⇒ deconfinement ⇒ $\mathrm{tr}(U_\mu) \neq 0$
- Can understand in weak coupling limit as due to clustering of eigenvalues of U_μ [BHN '82, Kazakov & Migdal '82]

$$U_\mu = V_\mu^\dagger \Lambda_\mu V_\mu \qquad \Lambda_\mu = \mathrm{diag}\left[e^{i\theta_\mu^1}, \ldots, e^{i\theta_\mu^N}\right]$$

$$Z_N \text{ symmetry}: \quad \theta_\mu^a \longrightarrow \theta_\mu^a + \frac{2\pi}{N}$$

$$F_{EK} \stackrel{b\to\infty}{\longrightarrow} (d-2)\sum_{a<b} \log\left[\sum_\mu \sin^2\left(\frac{\theta_\mu^a - \theta_\mu^b}{2}\right)\right]$$

➡ Eigenvalues attract for d>2 ⇒ $\theta_\mu^a = \theta_\mu^b$ and so $\mathrm{tr}\ \mathrm{U}_\mu \neq 0$

- For reduction to hold need uniform distribution of eigenvalues, uncorrelated in different directions
- Role of momenta played by $\theta_\mu^a - \theta_\mu^b$

Can reduction be rescued?

't Hooft

QCD (fund) N=3
N_f Dirac

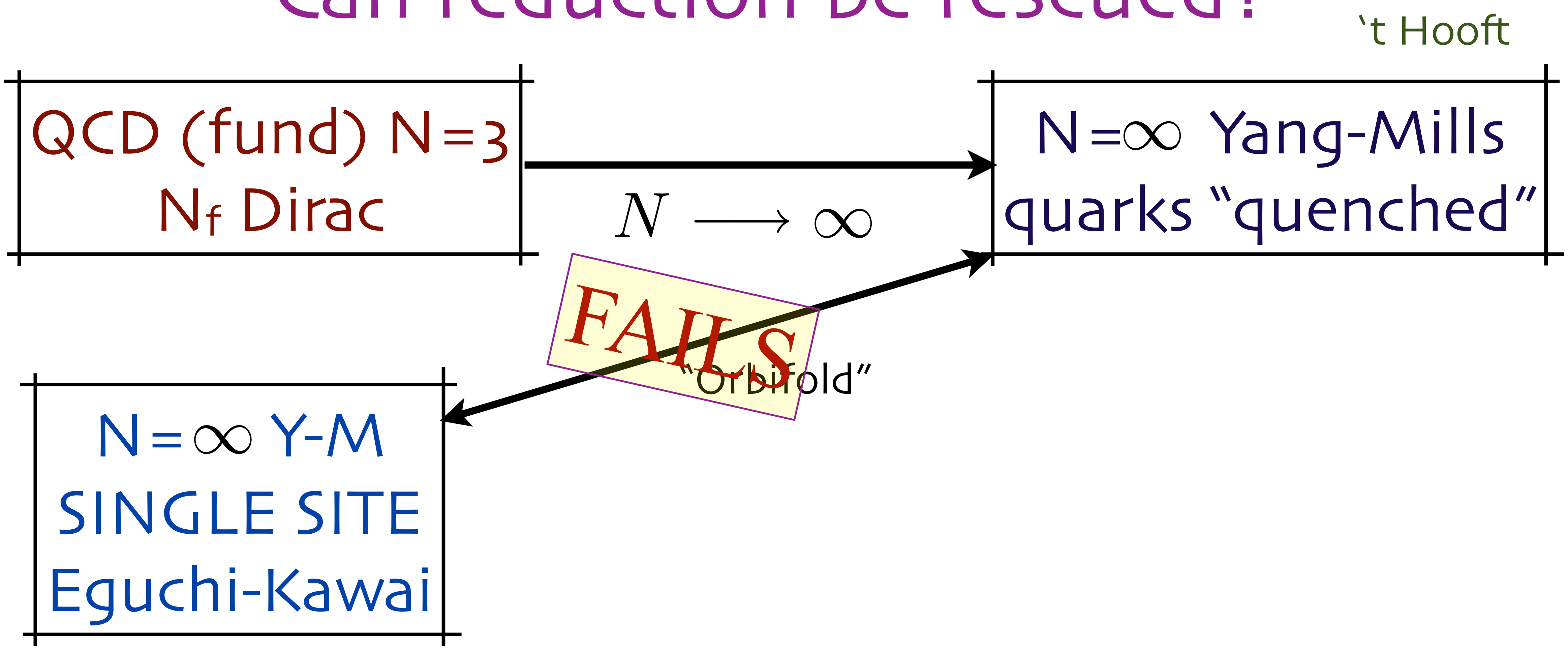
$N \longrightarrow \infty$

$N=\infty$ Yang-Mills
quarks "quenched"

"Orbifold"

$N=\infty$ Y-M
SINGLE SITE
Eguchi-Kawai

Can reduction be rescued?



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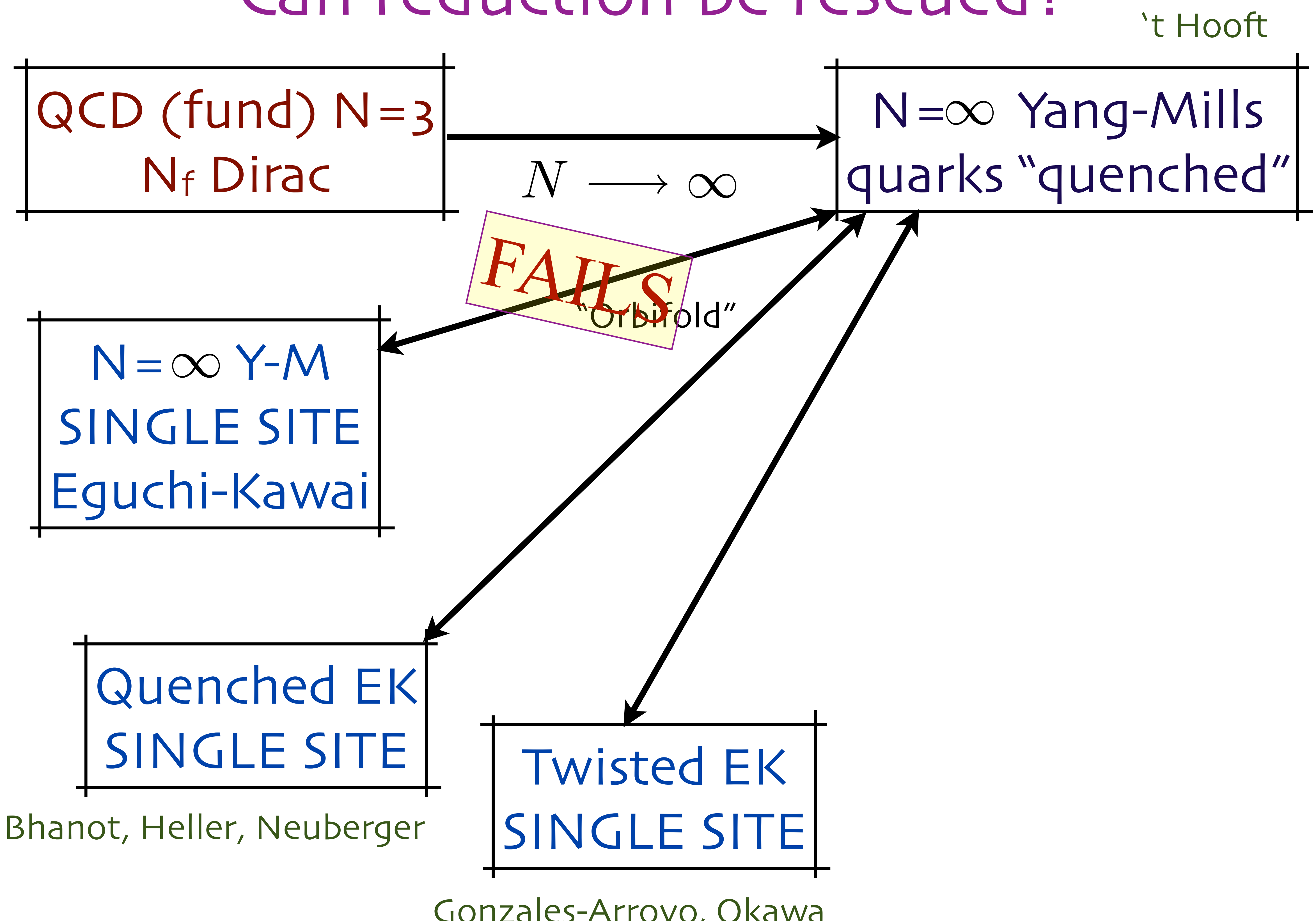

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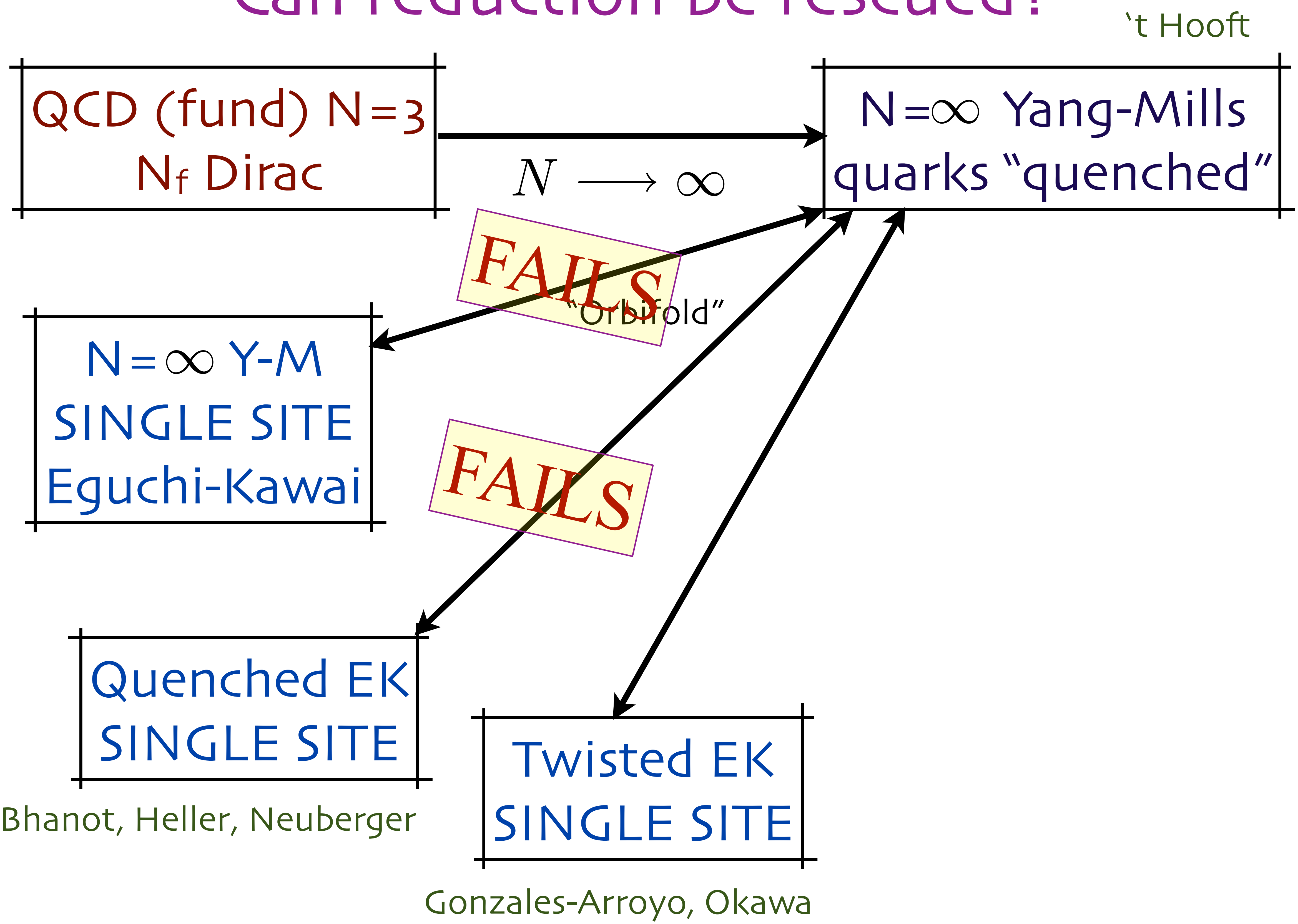

Can reduction be rescued?

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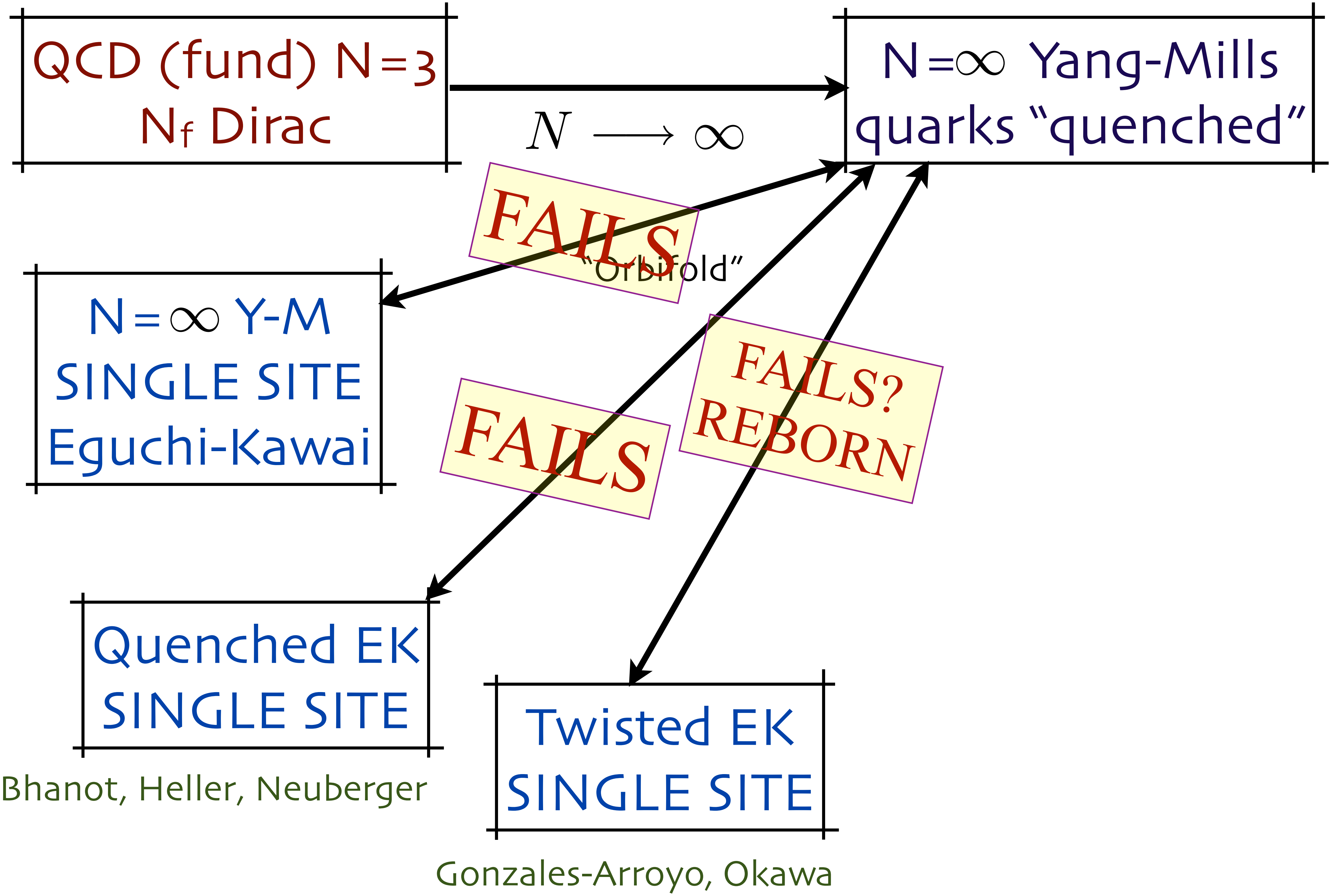

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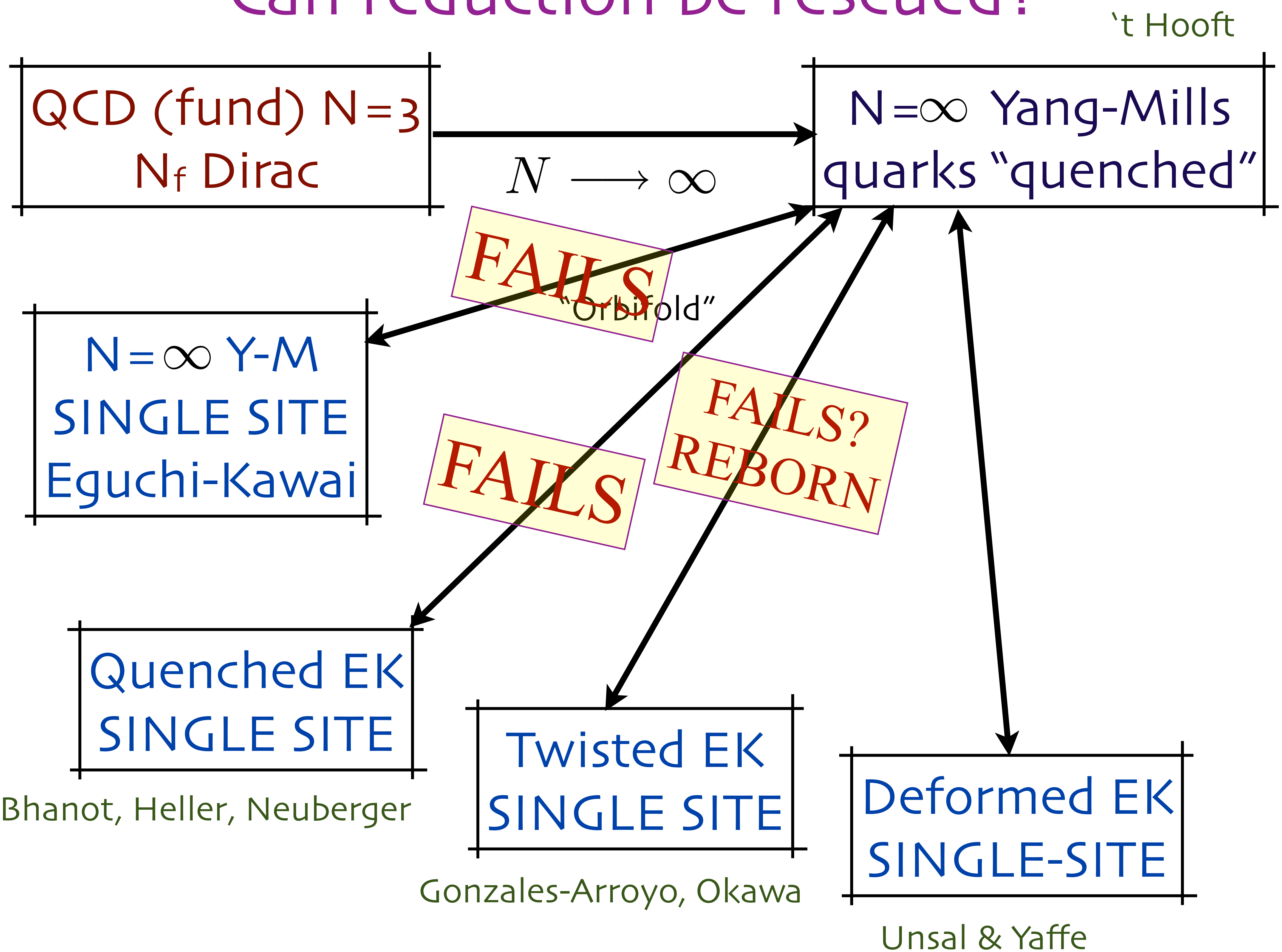

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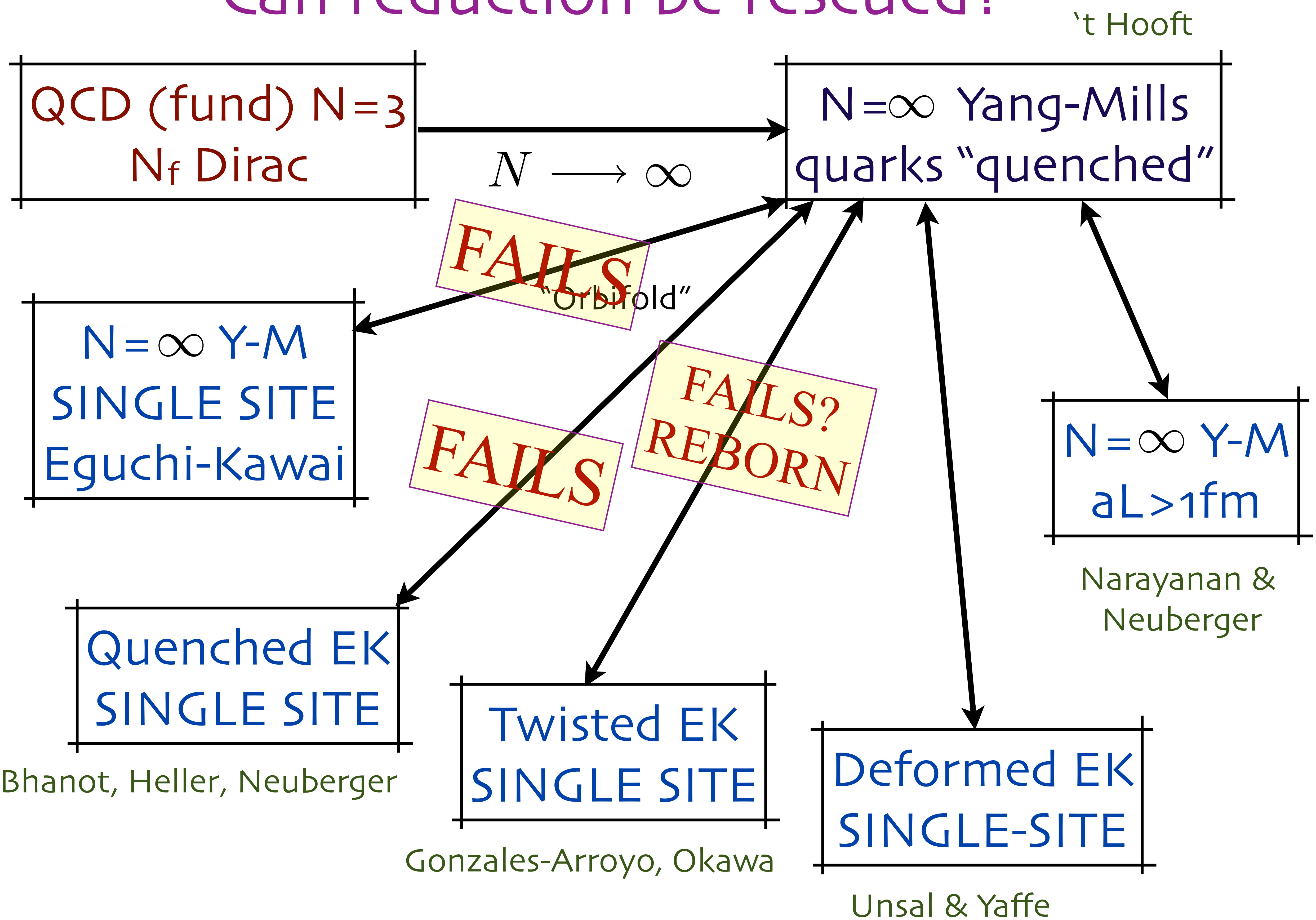

An alternative approach: AEK

QCD (N=3)
$2N_f$ Dirac fermions
in AS irrep (q^{ab})
infinite volume

An alternative approach: AEK

[Corrigan-Ramond
Armoni-Shifman-Veneziano]

QCD (N=3)		QCD (N=infinity)
$2N_f$ Dirac fermions	$N \longrightarrow \infty$	$2N_f$ Dirac fermions
in AS irrep (q^{ab})		in AS irrep (q^{ab})
infinite volume		infinite volume

An alternative approach: AEK

[Corrigan-Ramond
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$N \longrightarrow \infty$

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"Orientifold" equivalence
(in even C sectors)
[ASV]

QCD (N=infinity)
N_f Dirac fermions
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SINGLE SITE

"Orbifold" equivalence
[Kovtun,Unsal,Yaffe]

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Agree within
1/N !

"Orientifold" equivalence
(in even C sectors)
[ASV]

QCD (N=infinity)
N_f Dirac fermions
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SINGLE SITE

"Orbifold" equivalence
[Kovtun,Unsal,Yaffe]

QCD (N=infinity)
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Why do adjoint fermions help?

- Adjoint fermions survive in large N limit (unlike fundamentals)
- At one-loop order, fermions lead to repulsion between link eigenvalues, as long as use periodic (non-thermal) BC [K,U & Y]
- Repulsion wins for $N_f > 1/2$ massless Dirac fermions

➡ Usually leads to uniform distribution of θ_μ, but depends on details of fermion action [Lohmayer & Narayan, 2013]

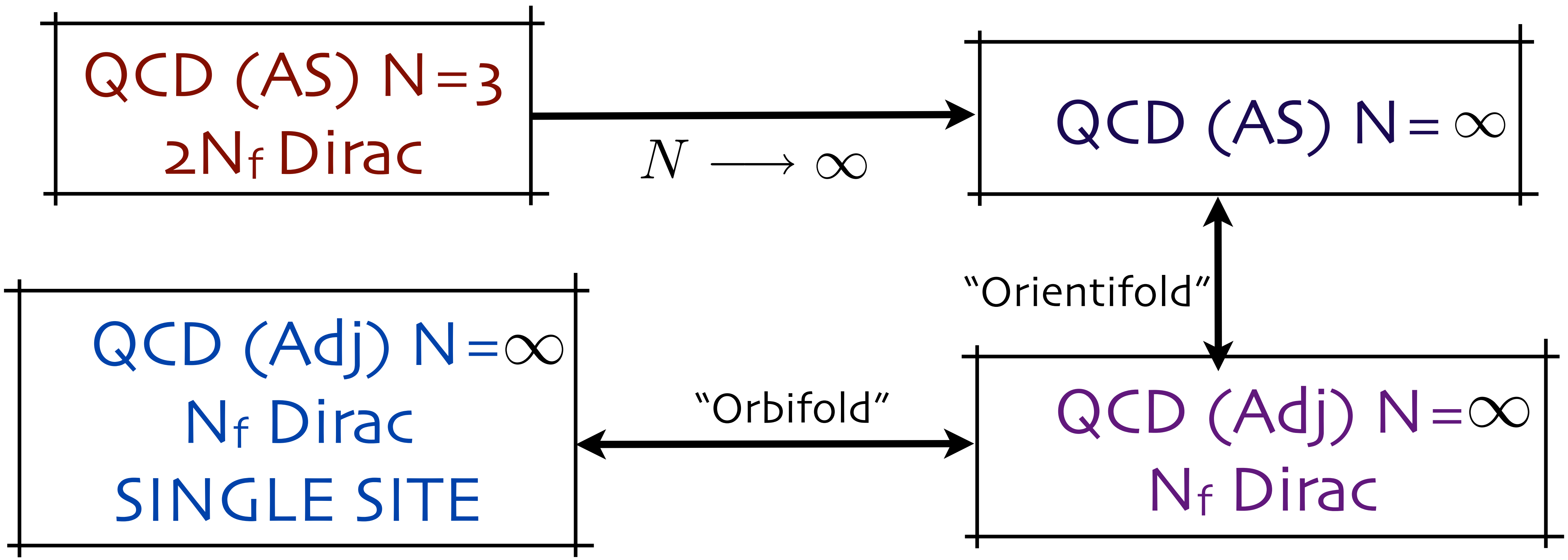
- Any non-zero mass $[|m_{phys}| > 1/(aN)]$ leads to attraction at small $\theta^a_\mu - \theta^b_\mu$ and thus to center-symmetry breaking

➡ Need massless fermions?

- However, perturbation theory not reliable for small $|\theta^a_\mu - \theta^b_\mu|$, nor in stronger coupling region of interest

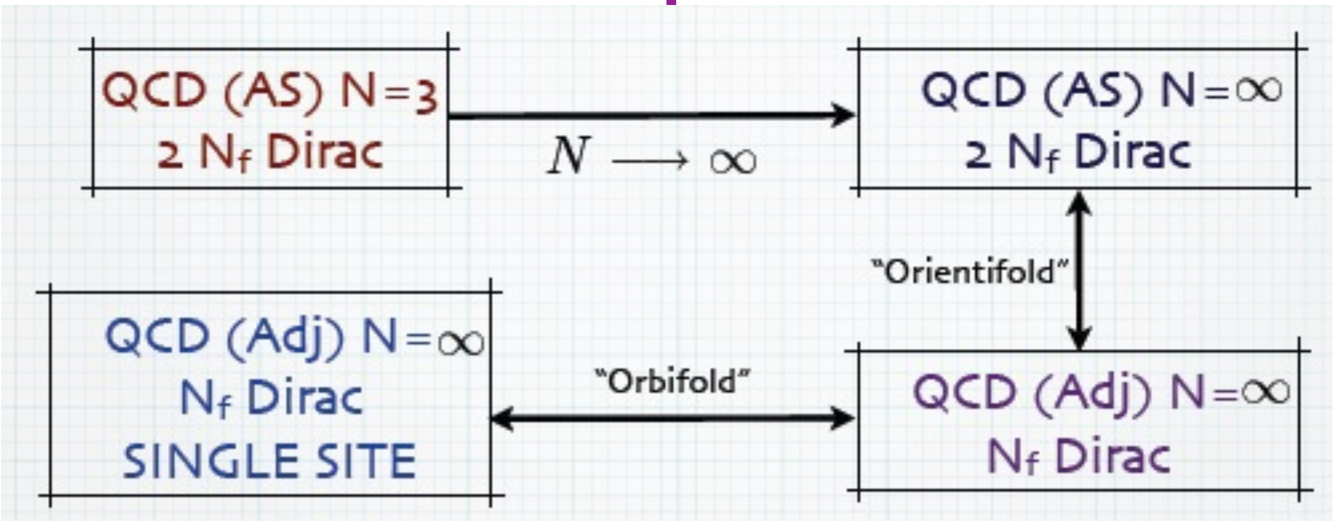
➡ Need non-perturbative simulation

What would we learn?



* Use single-site QCD(Adj) for N large to learn about 3 theories of great interest
  - $N_f=1$: learn about QCD with 2 flavors in Corrigan-Ramond large-N limit
  - $N_f=2$: alternative window on "minimal" walking technicolor theory
  - [$N_f=1/2$: equivalent to SYM, for which exact results are known]
* Even though "matrix model" lives on a single site, one can calculate many physical quantities (string tension, pion mass, ...)

Conditions for equivalences to hold



1. Large-N factorization holds
2. Orientifold: C not broken in QCD(AS,Adj)
3. Orbifold: Translation invariance unbroken in QCD(Adj.) in infinite volume
4. Orbifold: $(Z_N)^4$ center symmetry unbroken in QCD(Adj.) on a single site

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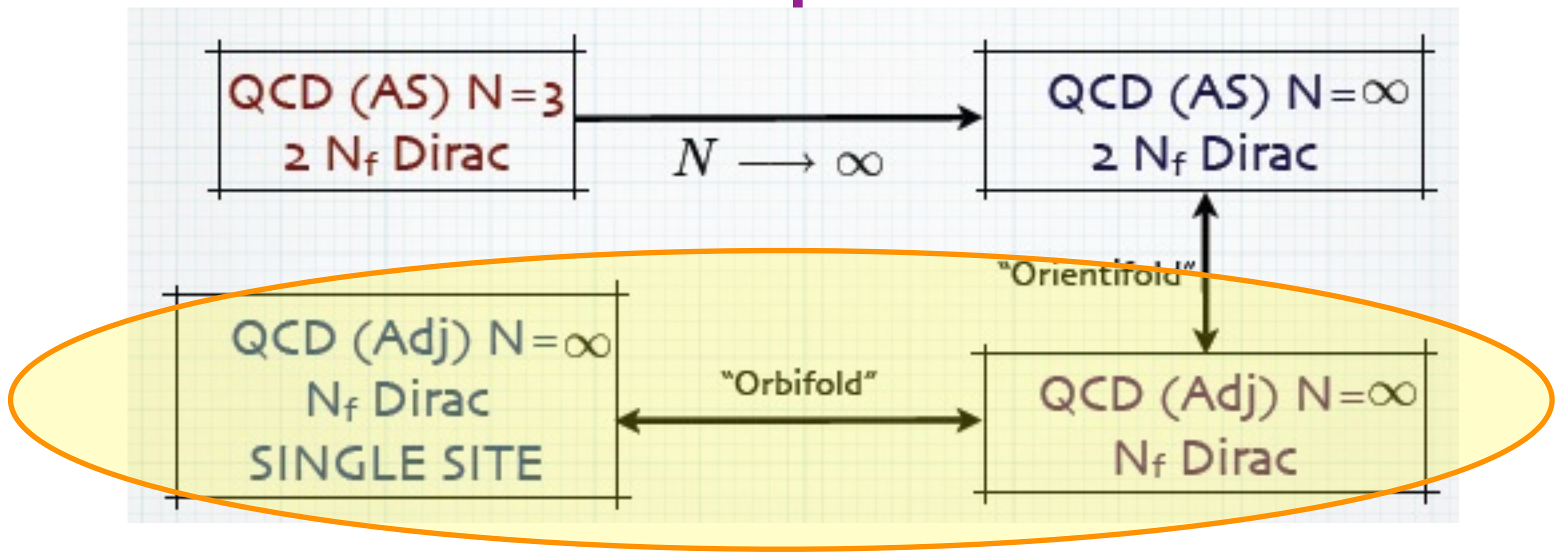

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IN THIS TALK:

We assume the first three hold and study the last

Results for N_f=1&2 adjoint Eguchi-Kawai (AEK) model

B. Bringoltz & S.R. Sharpe, PRD 80 (2009) 065031 [arXiv:0906.3538]

B. Bringoltz, M. Koren & S.R. Sharpe, PRD 85 (2012) 094504 [arXiv:1106.5538]

A. Gonzalez-Arroyo & M. Okawa, arXiv: 1305.6253

Action of AEK model

Wilson gauge and fermion action

$$S_{\mathrm{gauge}} = 2Nb \sum_{\mu<\nu} \mathrm{ReTr} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \,, \qquad b = 1/(g^2 N)$$

Parameters

$$S_F = \sum_{j=1,N_f} \bar{\psi}_j D_W \psi_j \qquad \kappa \sim 1/m$$

$$D_W = \mathbf{1} - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_\mu^{\mathrm{adj}} + (1+\gamma_\mu) U_\mu^{\dagger\,\mathrm{adj}} \right]$$

Symmetries:

gauge: $U_\mu \longrightarrow \Omega U_\mu \Omega^\dagger \quad (\text{all } \mu) \quad \Omega \in SU(N)$

center $(Z_N)^4$: $U_\mu \longrightarrow U_\mu e^{2\pi i n_\mu / N} \quad n_\mu \in Z_N$

Scaling of CPU with N

* Original studies used Metropolis algorithm

$$P(U) = e^{S_{\rm EK}(U)} \left(\det D_W\right)^{N_f}$$

  * Determinant real & positive; evaluate explicitly
  * Scaling is $\sim(N^2)^3 \times N^2 \sim N^8 \Rightarrow$ can reach $N \approx 15$ on PC

* Present studies use rHMC (HMC) for $N_f=1$ (2)
  * Using $U^{\rm adj} \sim U \cdot U^\dagger$, scaling is $\sim(N^3) \times N^{1\text{-}1.5} \sim N^{4\text{-}4.5}$
  * Can reach N=53 on PC, N=289 on supercomputer

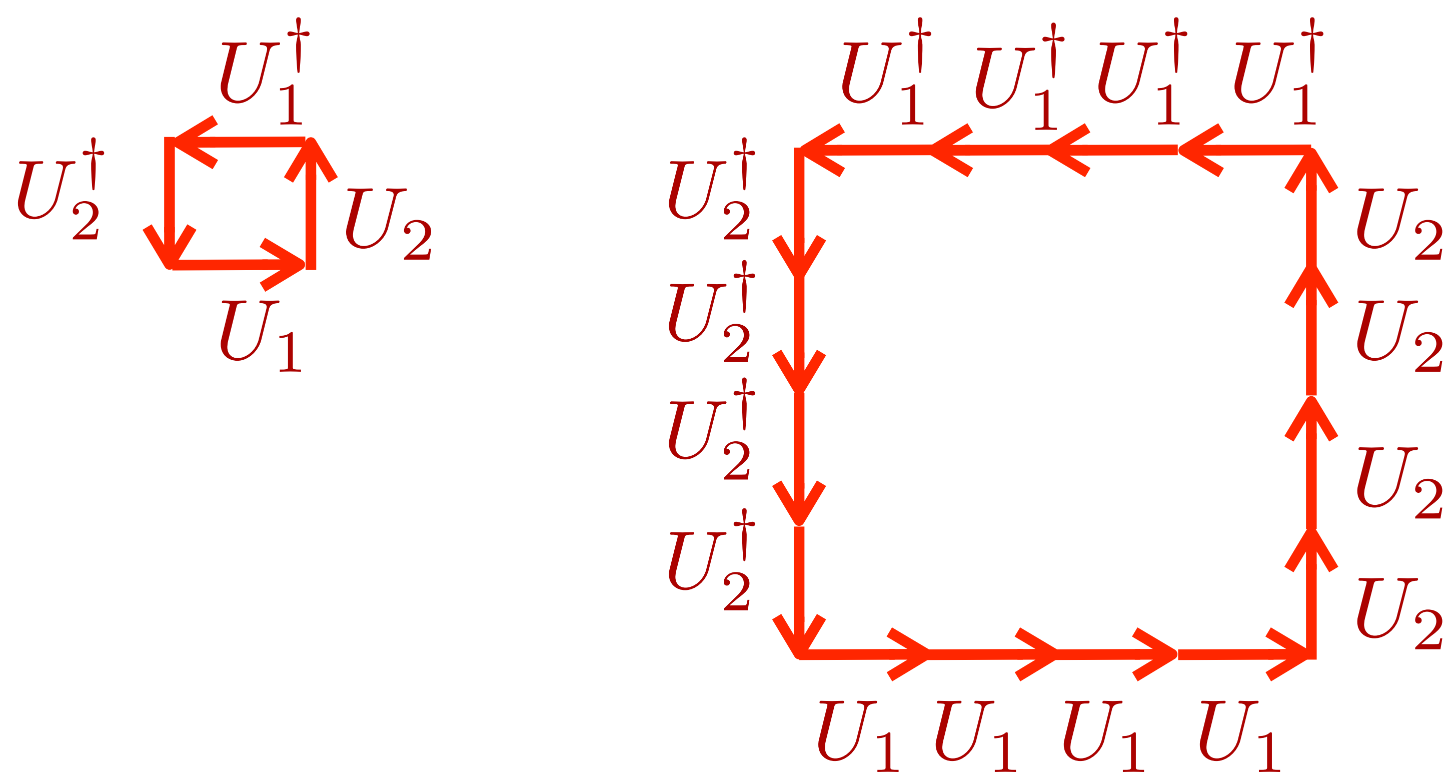
Order params for symm breaking

* traces of "open" loops

$$\mathrm{tr}\,(U_\mu)\,,\ \ \mathrm{tr}\,(U_\mu U_\nu)\,,\ \ \mathrm{tr}\left(U_\mu U_\nu^\dagger\right)\,,\ \ \mathrm{tr}\,(U_1^{n_1} U_2^{n_2} U_3^{n_3} U_4^{n_4})\,,\ldots$$

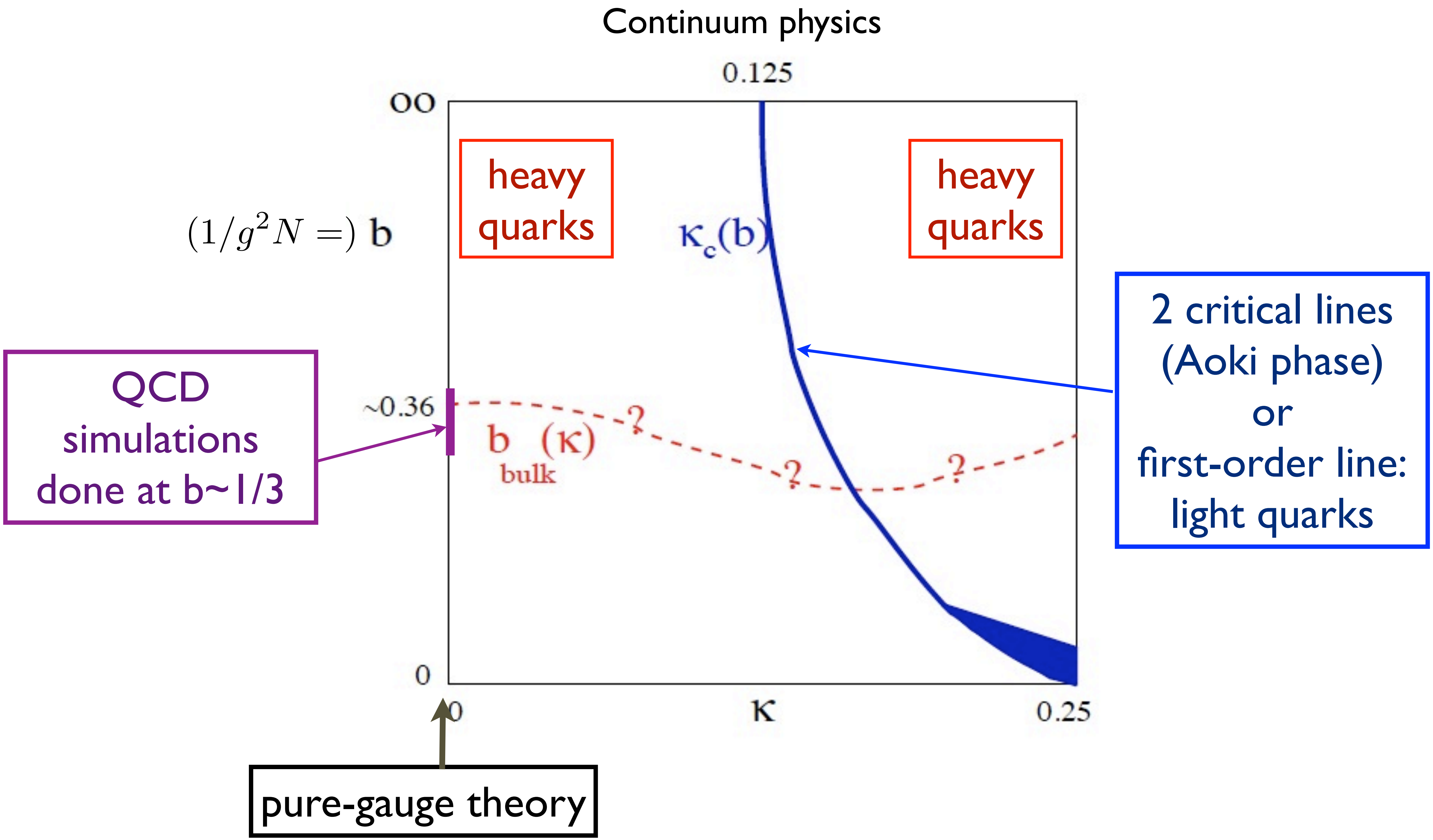
* histograms of eigenvalues of links: θ_μ^a

* also calculate plaquette and larger Wilson loops

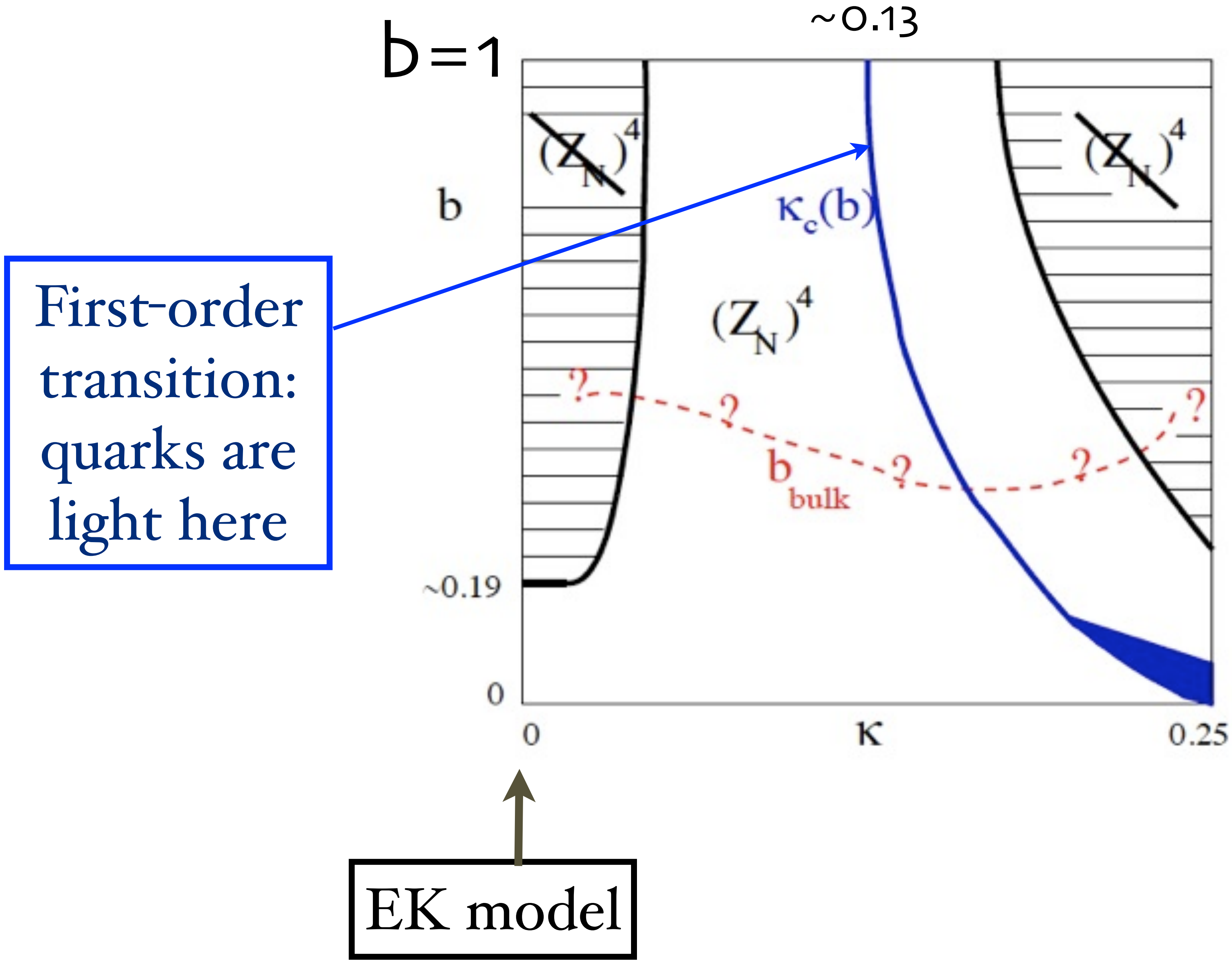


Expected phase diagram (infinite volume)

$N_f=1$, N=infinity

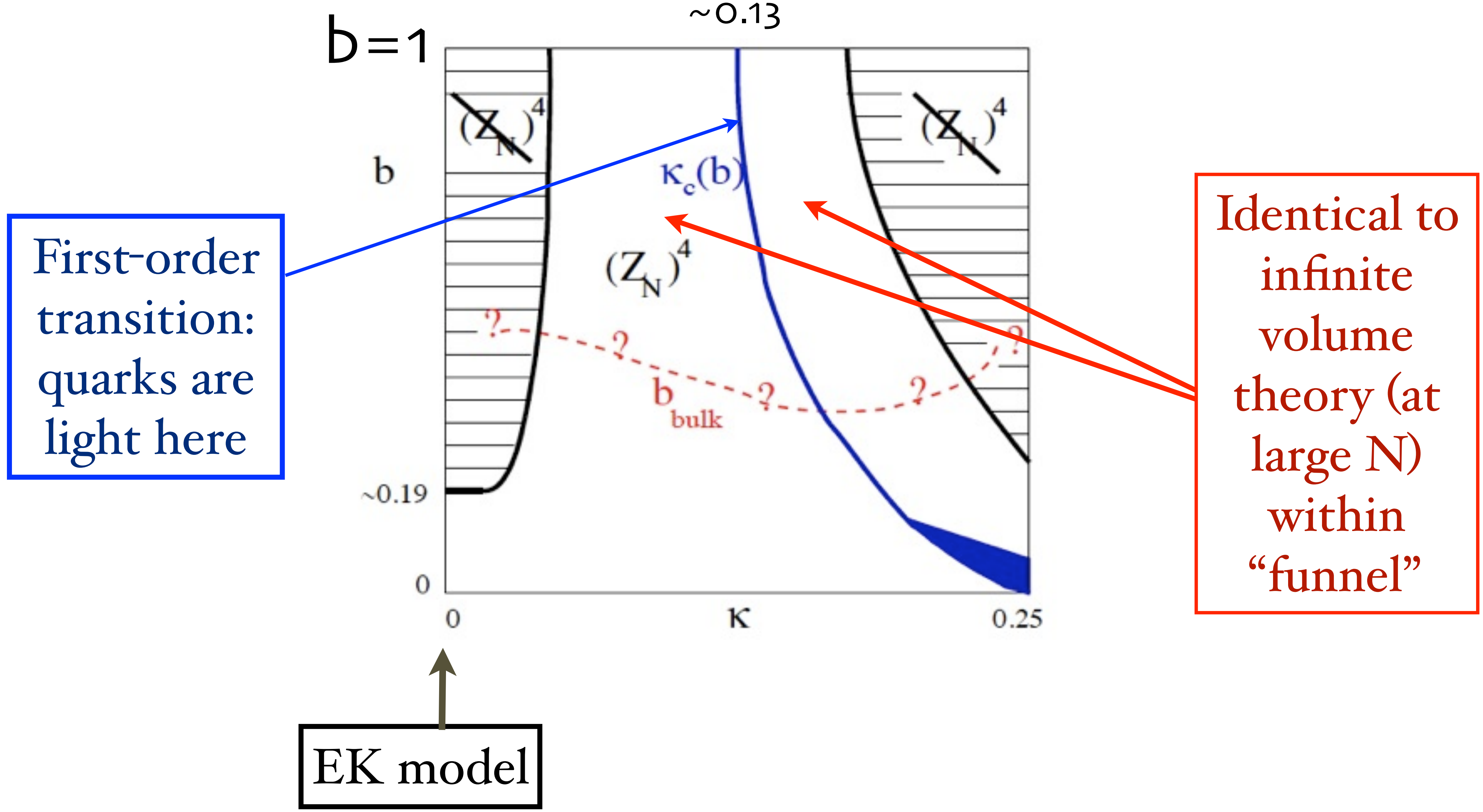


Conclusion for $N_f=1$ AEK model [B&S]



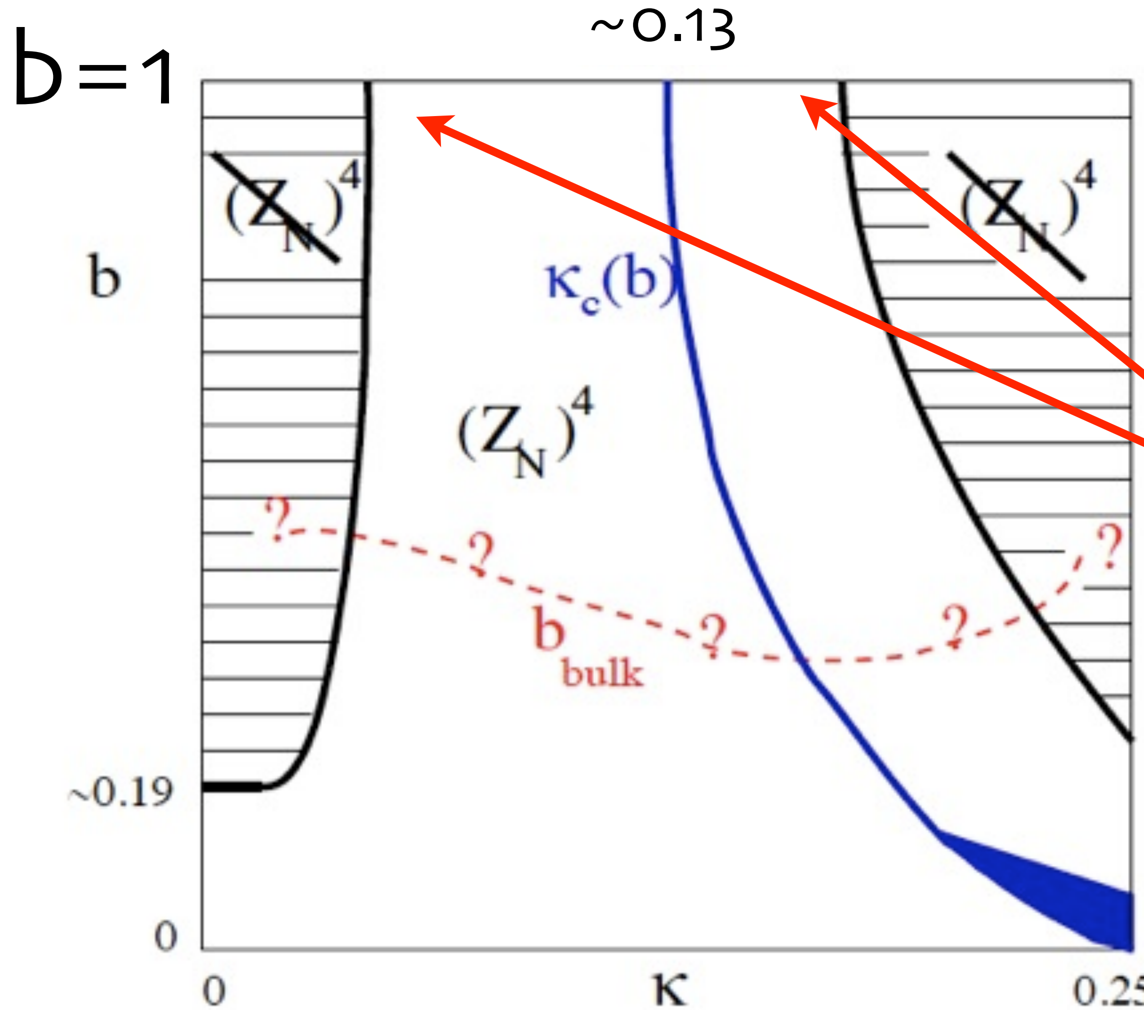
Based on $N \leq 53$; shows weak N dependence

Conclusion for N_f=1 AEK model [B&S]



Based on N≤53; shows weak N dependence

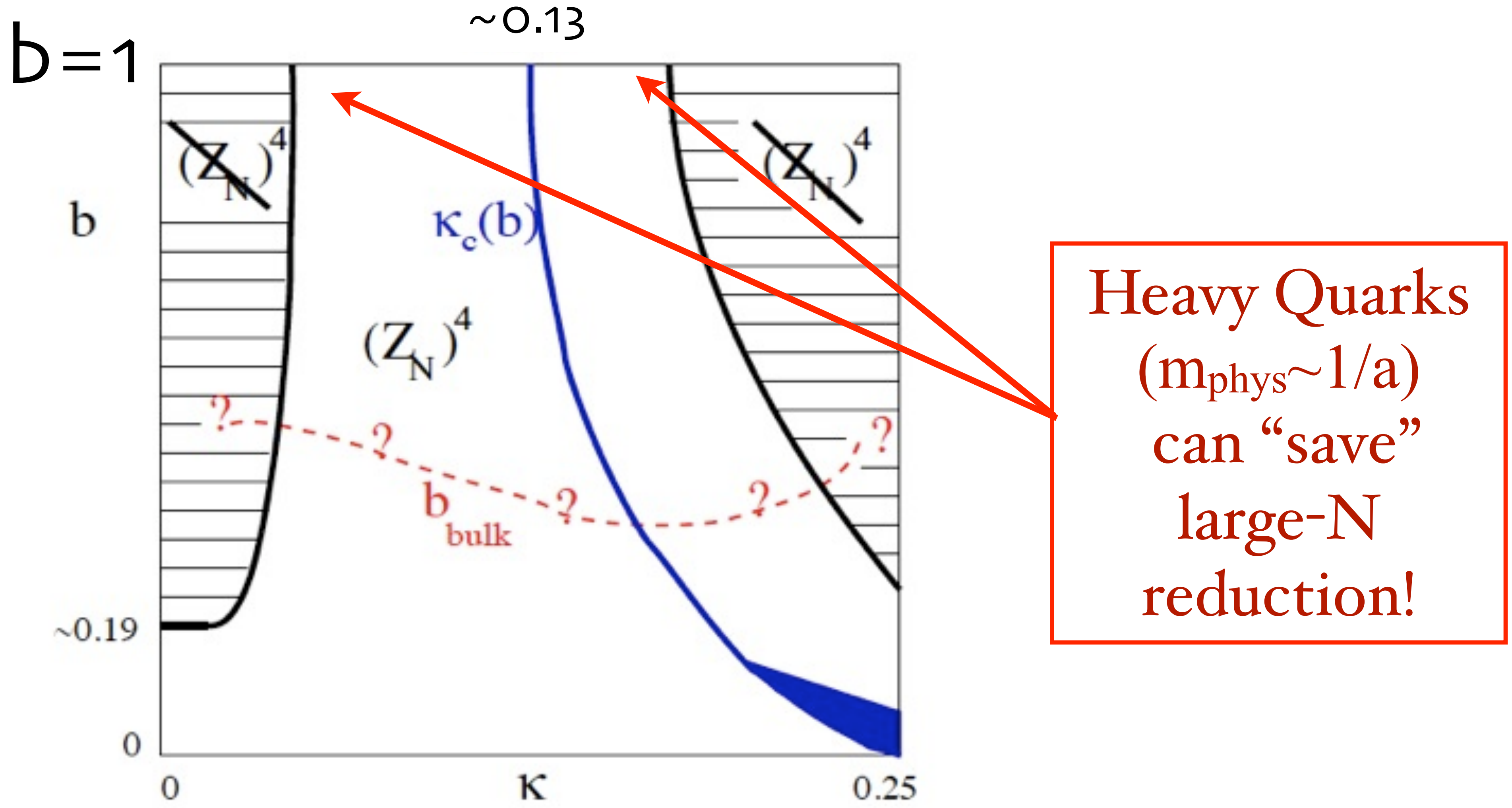
Very surprising feature:



Heavy Quarks (m_{phys}~1/a) can "save" large-N reduction!

- Inconsistent with pert. thy (requires m_{phys}=0 in general)
- Violates naive decoupling of heavy quarks

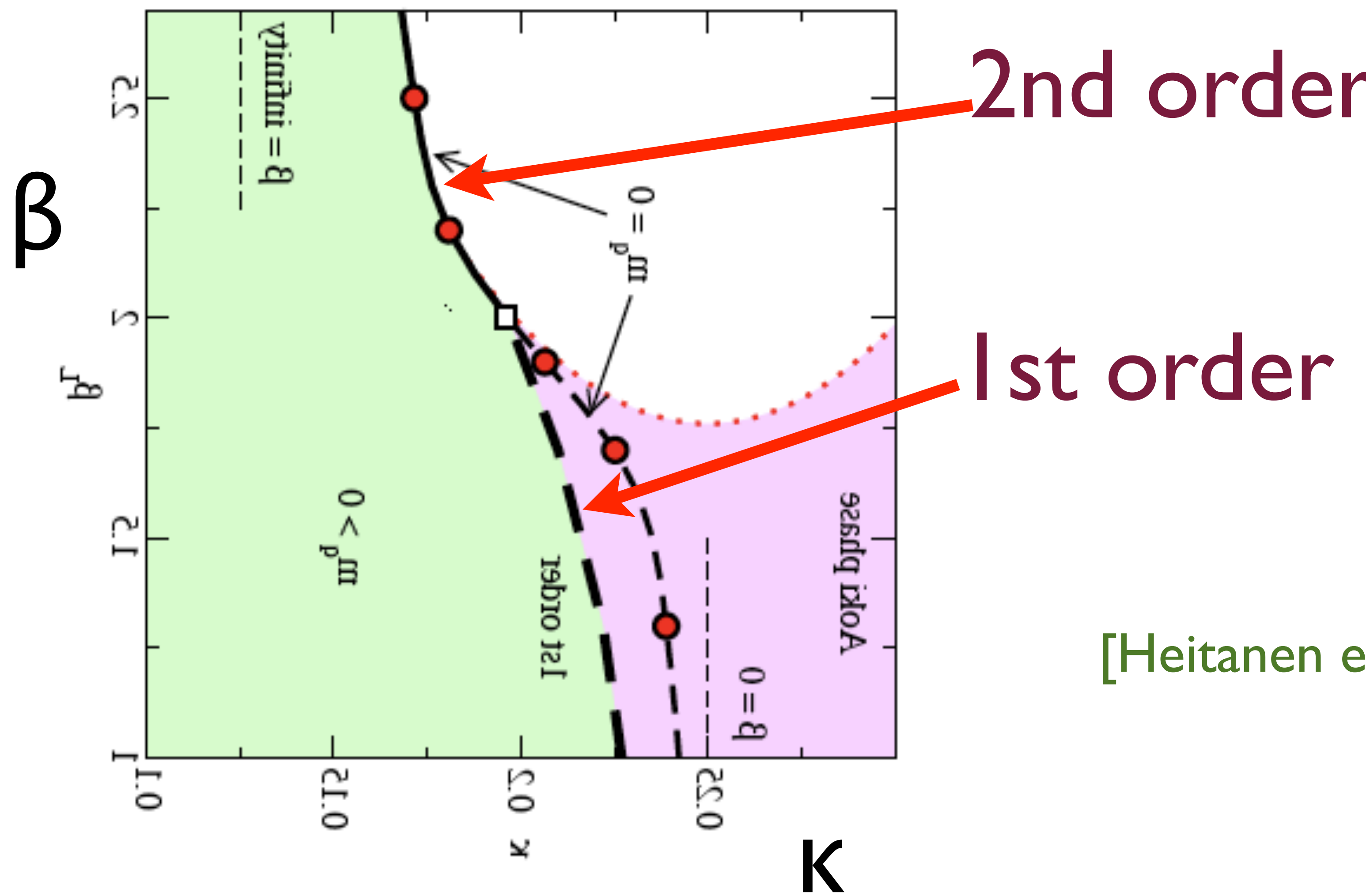
Very surprising feature:



- Checked using rHMC [Azeyanagi, Hanada, Unsal & Yacoby; Koren & SS]
- Supported by analytic arguments going beyond PT [AHUY, Unsal & Yaffe]
  - ➡ Predicts that funnel closes as $|am_{\rm phys}| < \dfrac{1}{b^{1/4}}$

Infinite volume expectation for $N_f=2$?

* N=2 gauge theory ("minimal walking technicolor") subject of many recent studies

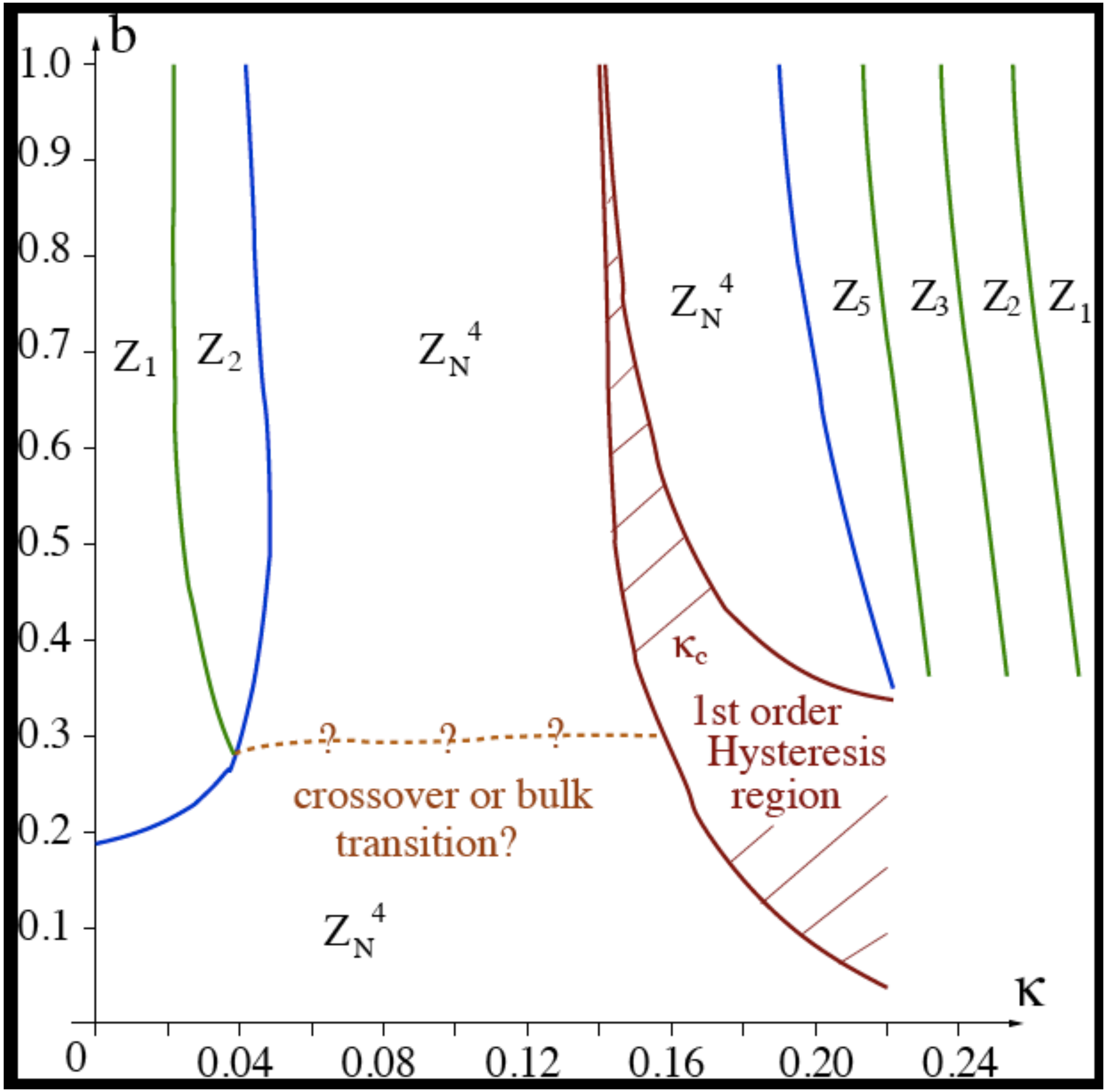


[Heitanen et al]

* Dependence on N not known

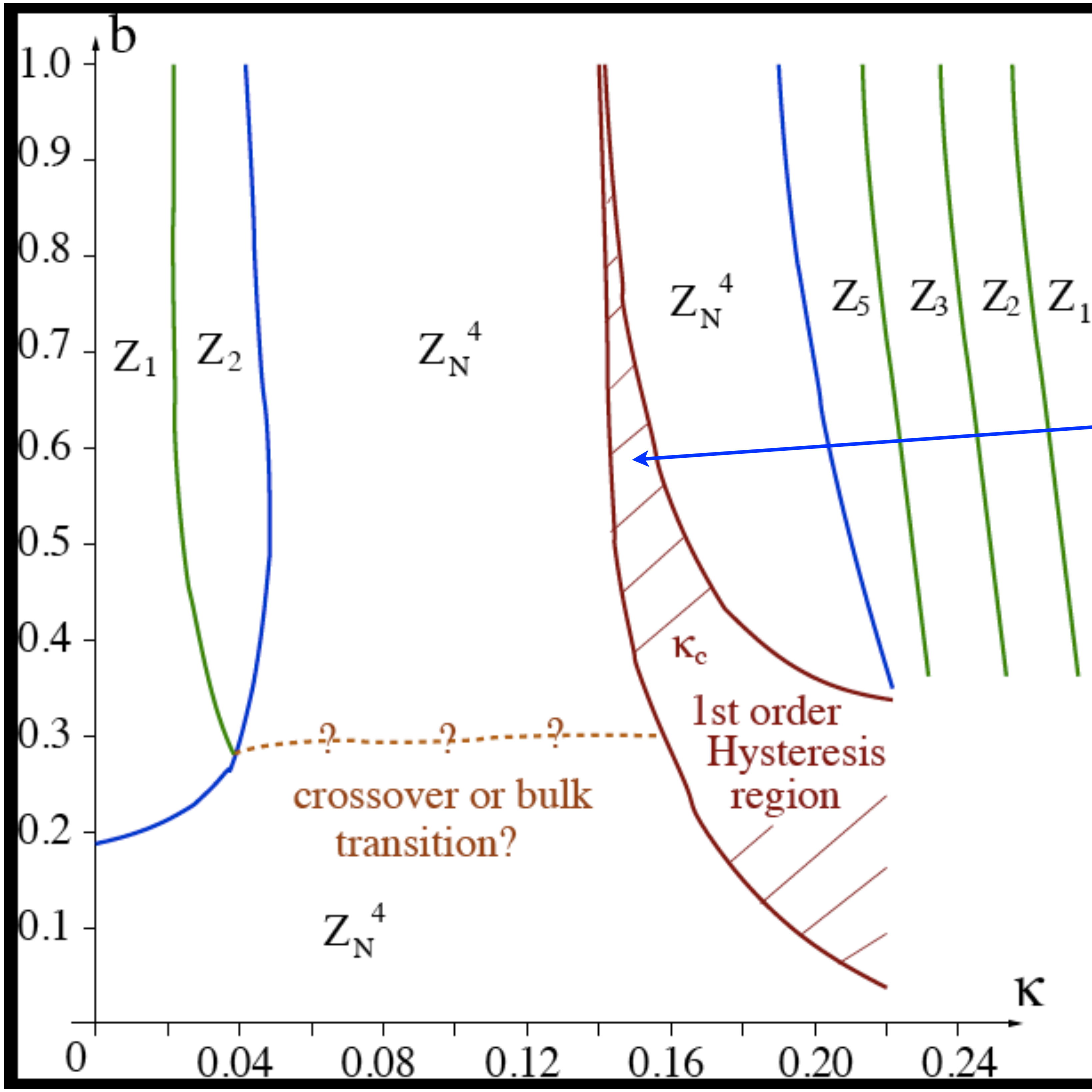
Phase diagram of $N_f=2$ AEK model [B,K&S]

$16 \leq N \leq 53$



Phase diagram of N_f=2 AEK model [B,K&S]

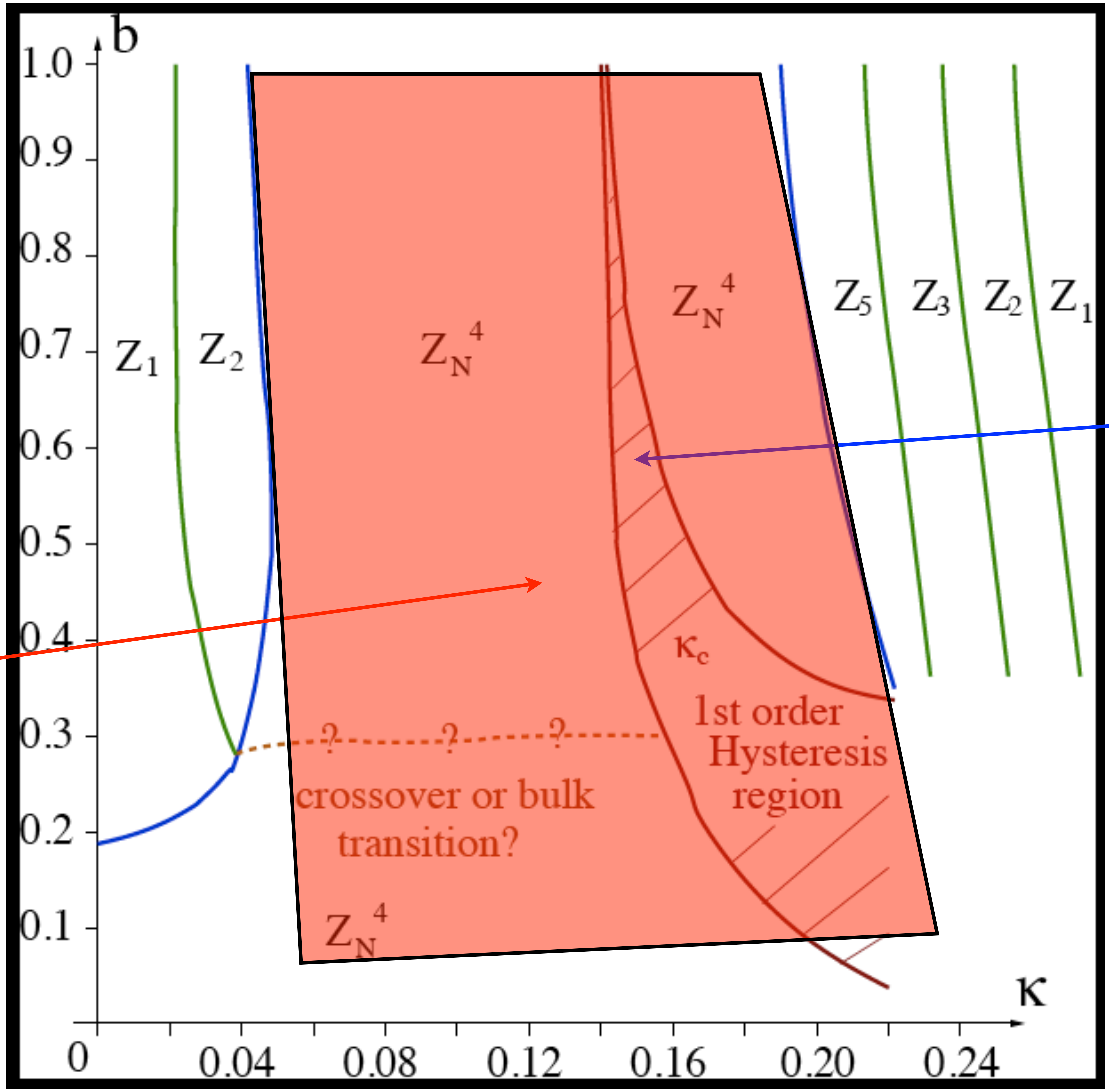
$16 \leq N \leq 53$



First-order transition for all b

Phase diagram of $N_f=2$ AEK model [B,K&S]

$16 \leq N \leq 53$

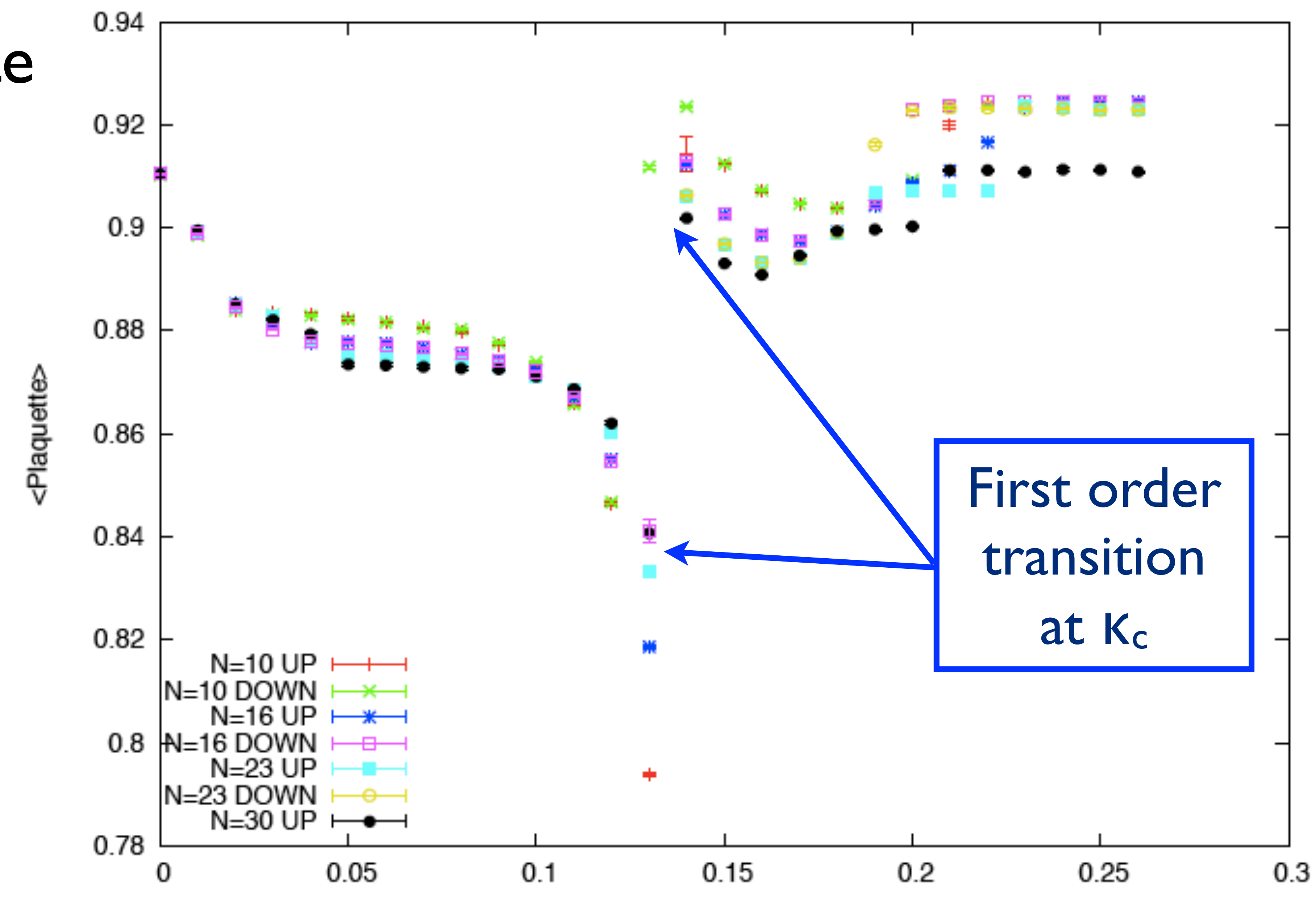


Funnel in which volume indep. holds

First-order transition for all b

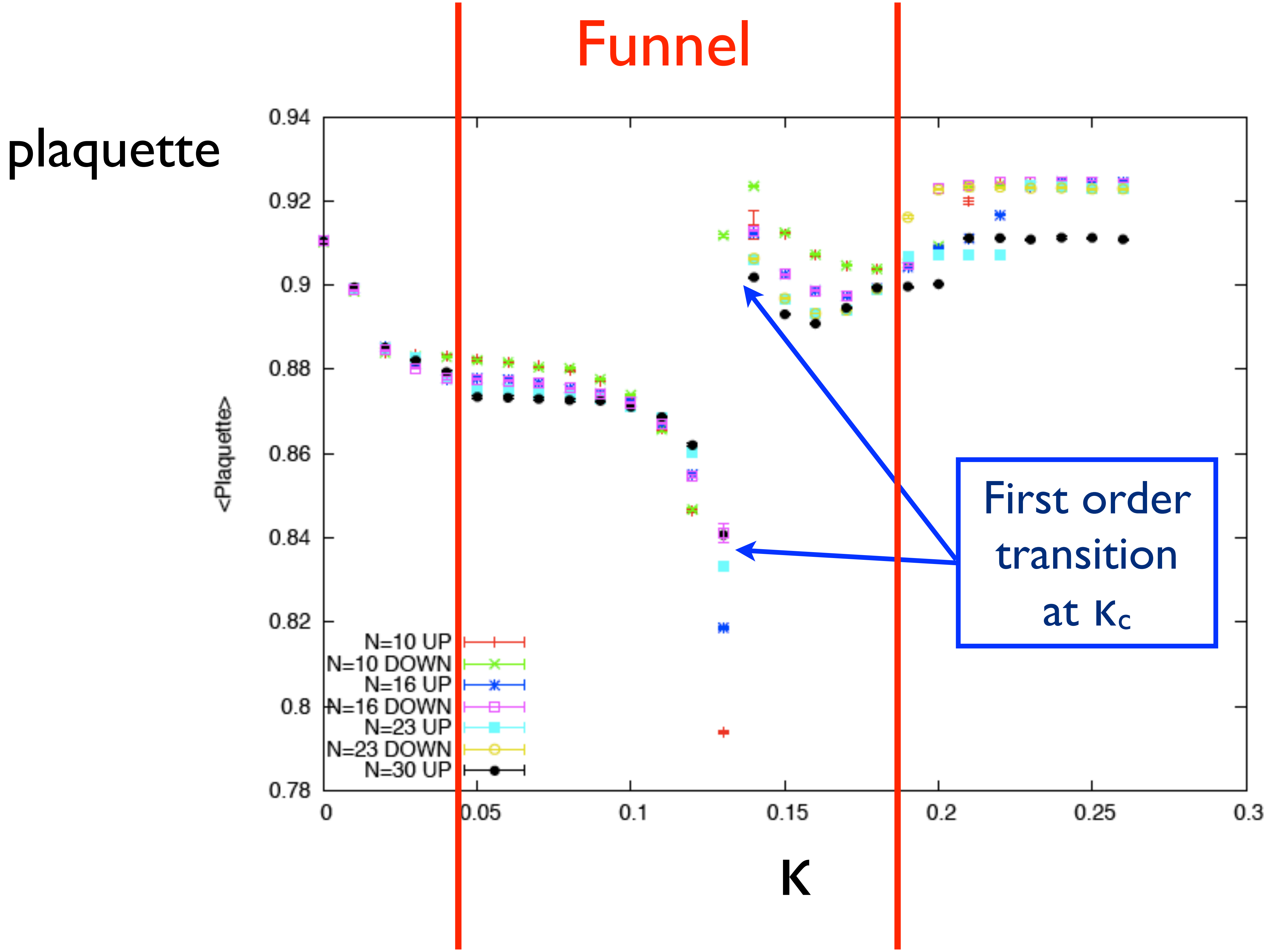
Hysteresis scans at b=1 (N=10,16,23,30)

plaquette



κ

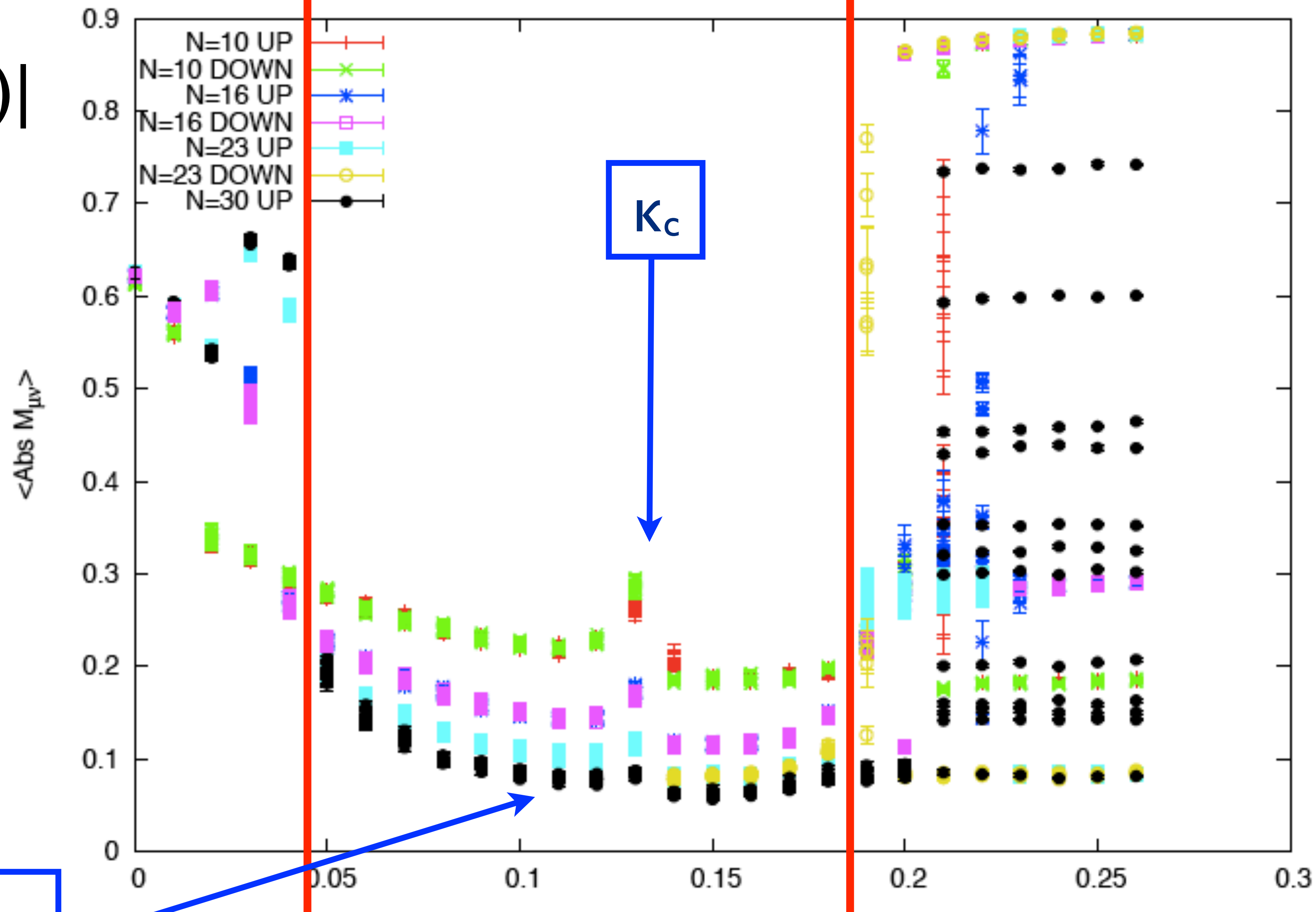
Hysteresis scans at b=1 (N=10,16,23,30)



Hysteresis scans at b=1 (N=10,16,23,30)

Funnel

$|tr(U_\mu U_\nu)|$



K_c

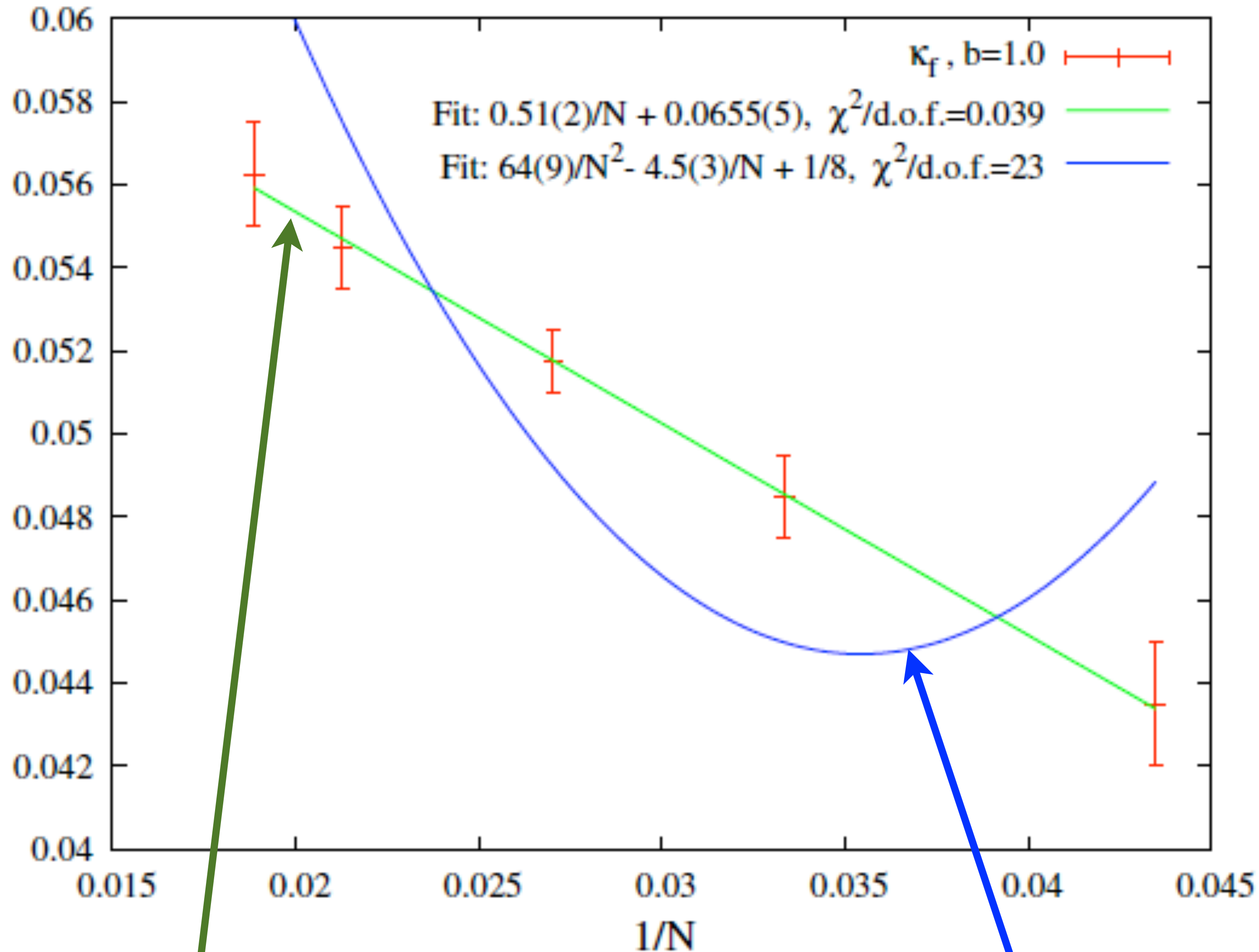
Scaling to zero ~1/N

K

Funnel width finite as N→∞

b=1

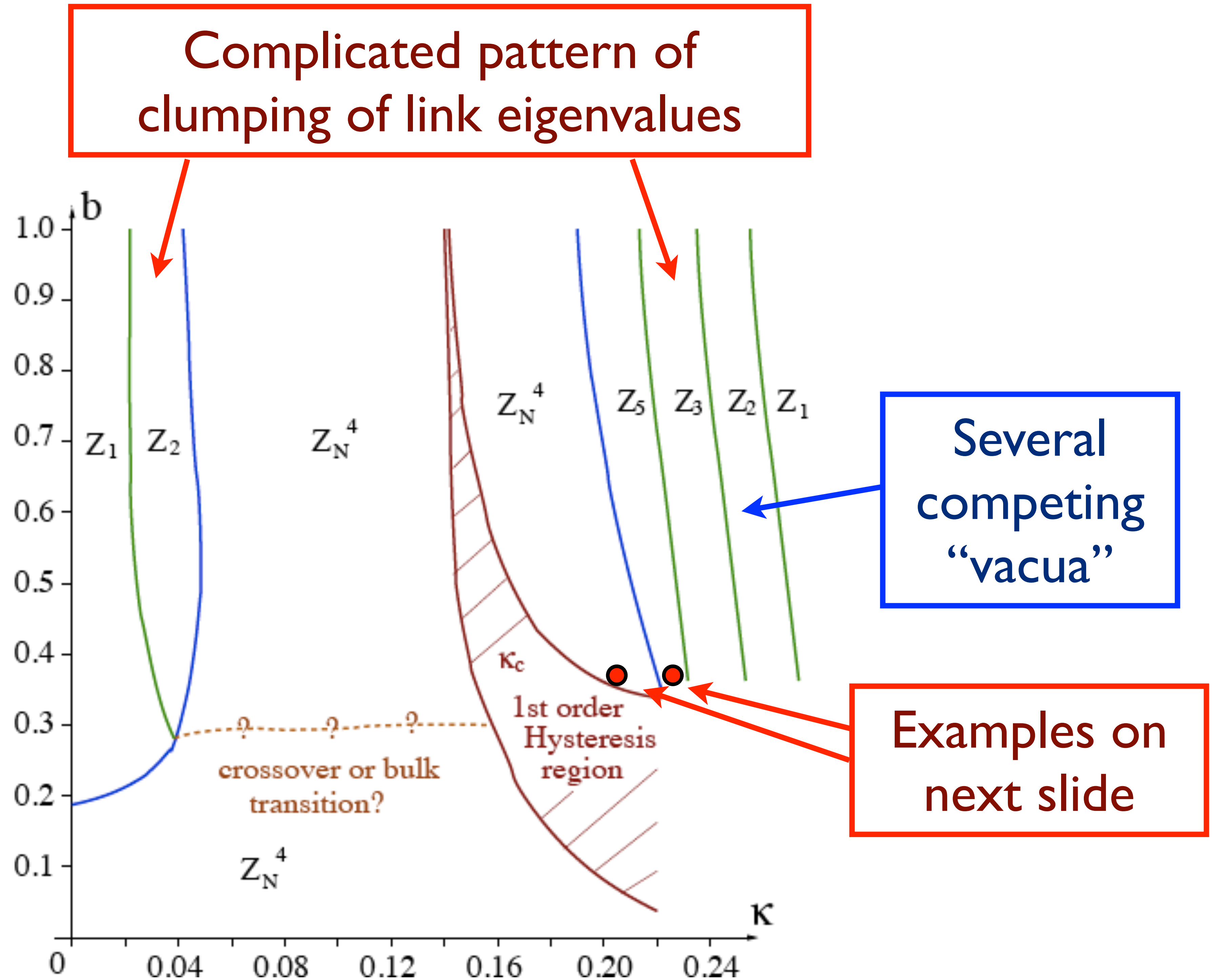
κ_f



Good fit gives intercept
κ_f=0.0655≪κ_c

V. poor fit if force κ_f to equal
κ_c at N=∞

Outside the "funnel"



* Qualitatively consistent with analytic arguments

Distribution of link eigenvalues

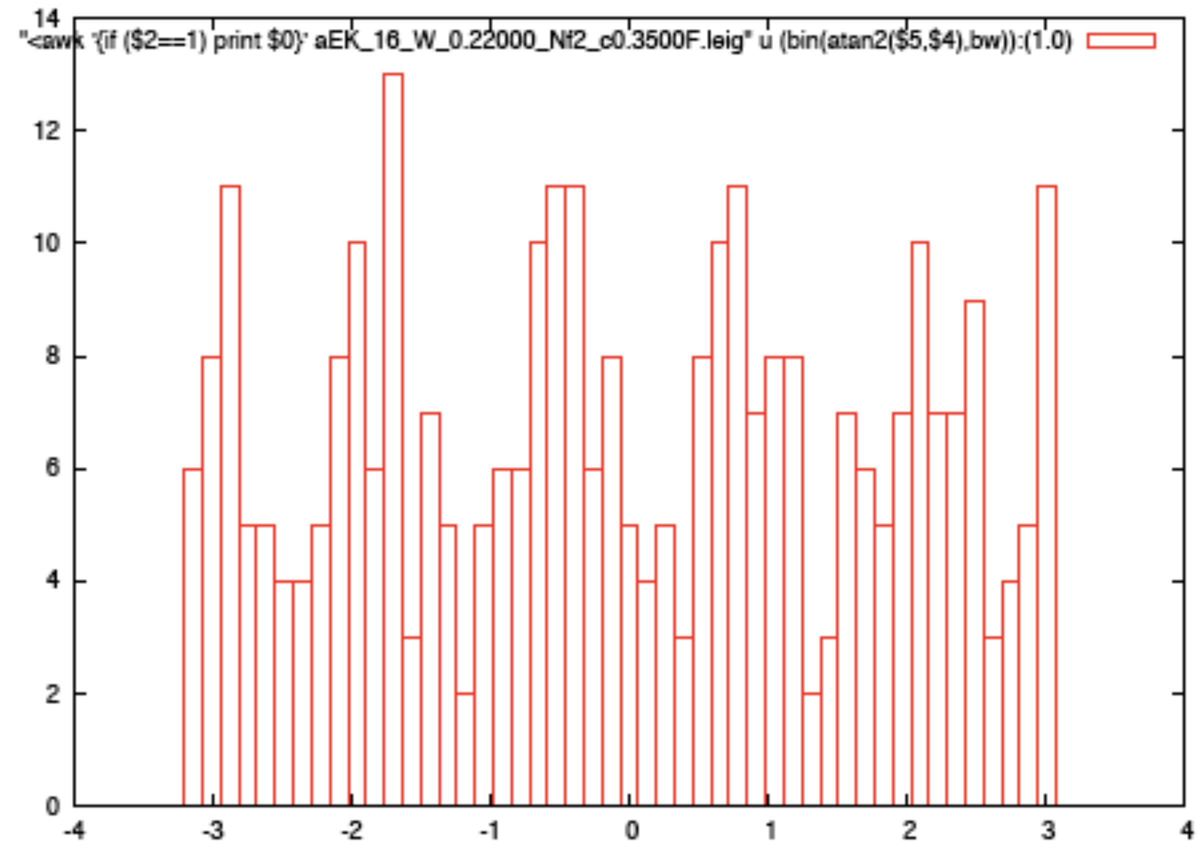
$$U_\mu = V_\mu^\dagger \Lambda_\mu V_\mu \qquad \Lambda_\mu = \mathrm{diag}\left[e^{i\theta_\mu^1}, \ldots, e^{i\theta_\mu^N}\right]$$

N=24, b=0.35, κ=0.19

-π θ_μ^a π

$(Z_{24})^4$ invariant inside funnel

N=16, b=0.35, κ=0.22

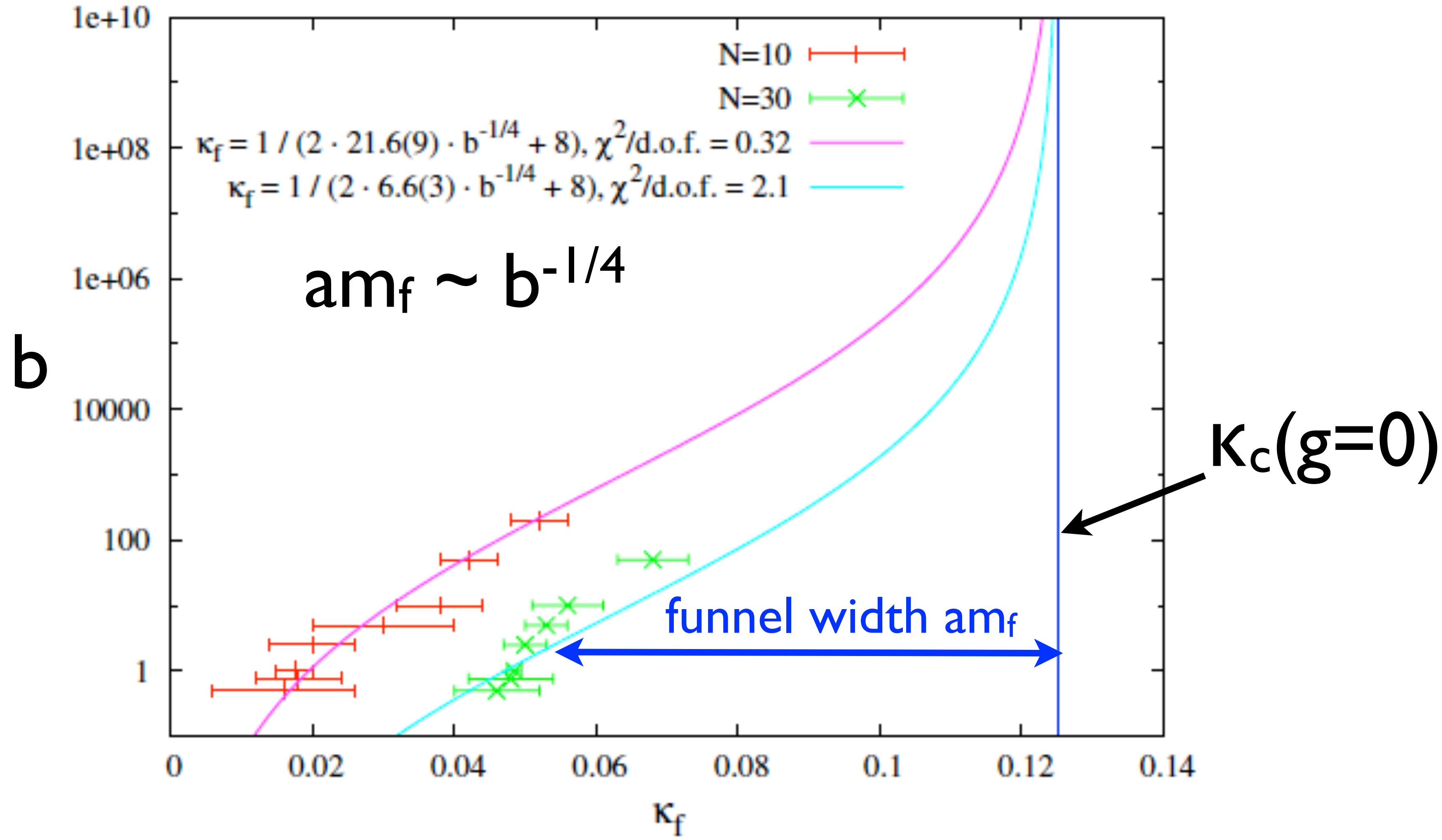


-π θ_μ^a π

5 clumps (e.g. 4,3,3,3 & 3)
all 4 links have "locked" clumping

Extreme weak coupling

* Funnel narrows in accord with [AHUY] prediction



* In fact, funnel closes before $b=\infty$ due to non-universal UV effect: $\mathrm{tr}(U_1U_2U_3U_4)\neq 0$ [Lohmayer & Narayanan]

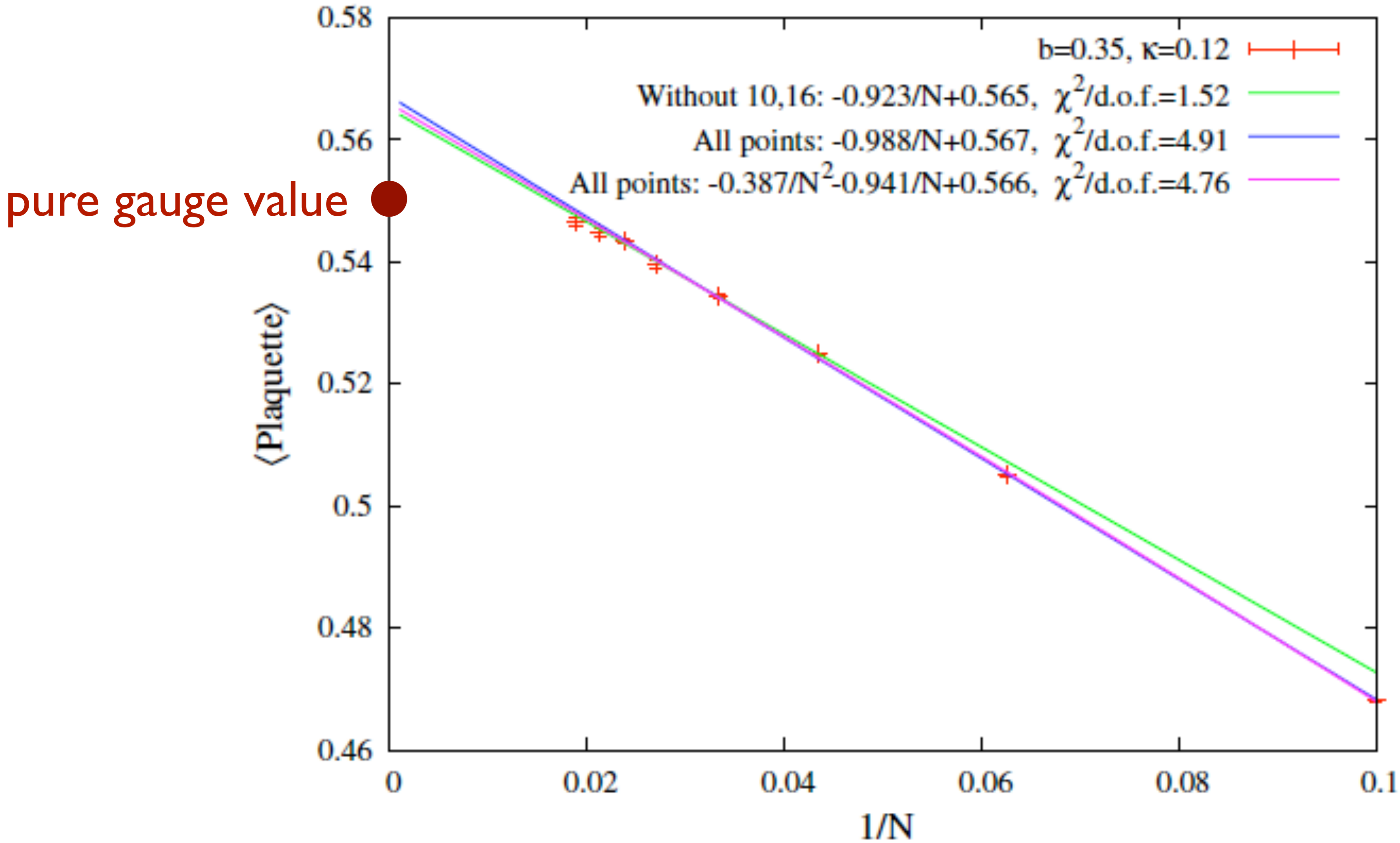
* Can fix by small change to fermion action.

Conclusions for $N_f=2$ AEK model

* In range of interesting values of b (and beyond) volume independence works for $|m| < O(1/a)$
  * Crucial first test of reduction has been passed
  * Also seen on 2^4 lattice by [Catterall, Galvez & Unsal, JHEP 1008 (2010) 010]
  * By tuning quark mass can use reduction to study both pure gauge theory and (nearly) conformal theory
  * Semi-analytic understanding of phase diagram
* Phase diagram similar to that for $N_f=1$
  * No sign of 2nd-order transition seen for N=2

Problems at very large N ?

- Extrapolate average plaquette to N=∞ using N≤53
- Extrapolation requires 1/N term
- Result should lie close to pure gauge value

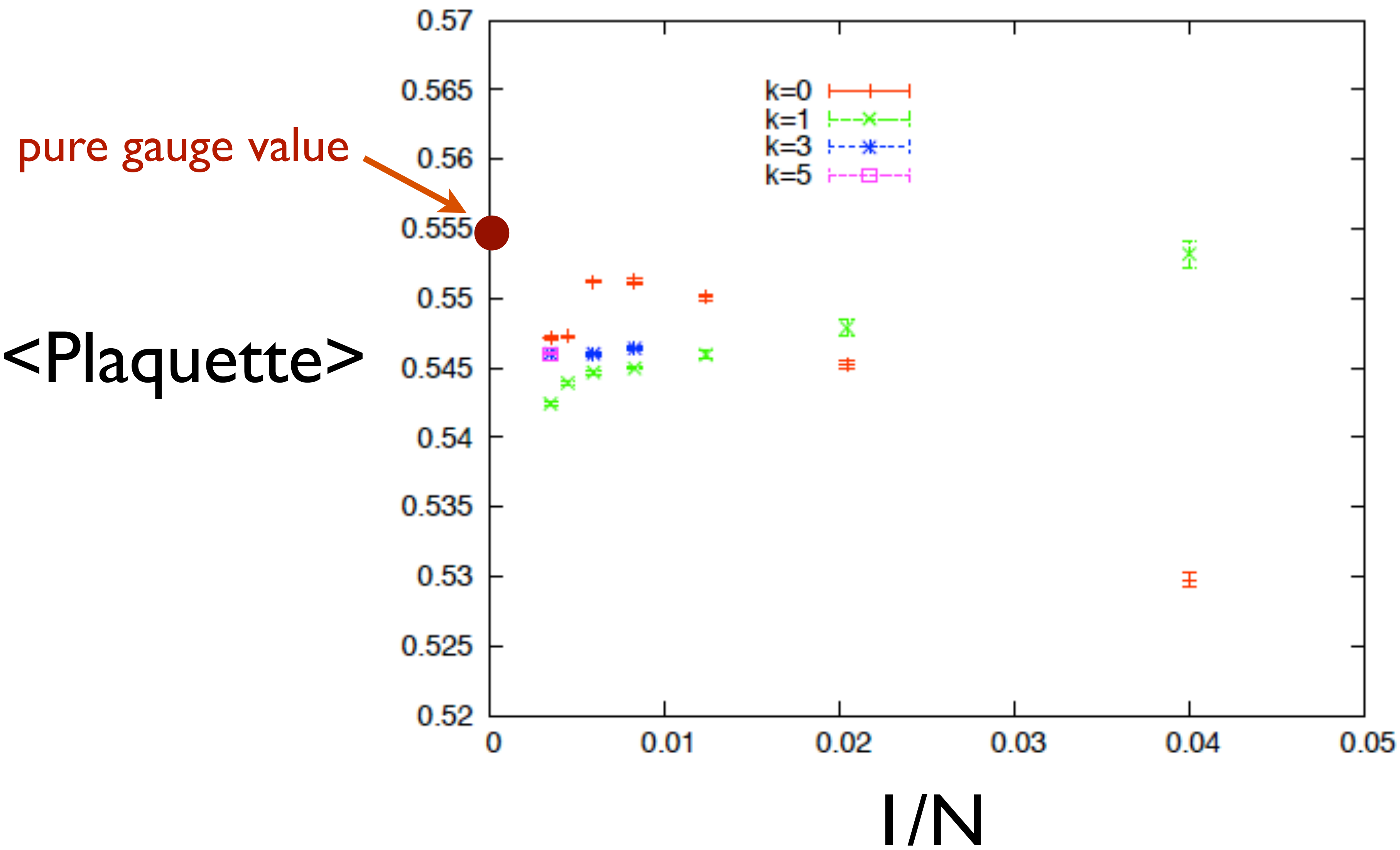


$N_f = 2$, b=0.35, κ=0.12 (heavy adjoint)

[Bringoltz, Koren & SS]

Problems at very large N ?

- New results with N up to 289 [Gonzalez-Arroyo & Okawa]
- Non uniform behavior in N !? (k=0 points in plot)



N_f =2, b=0.35, κ=0.12 (heavy adjoint)

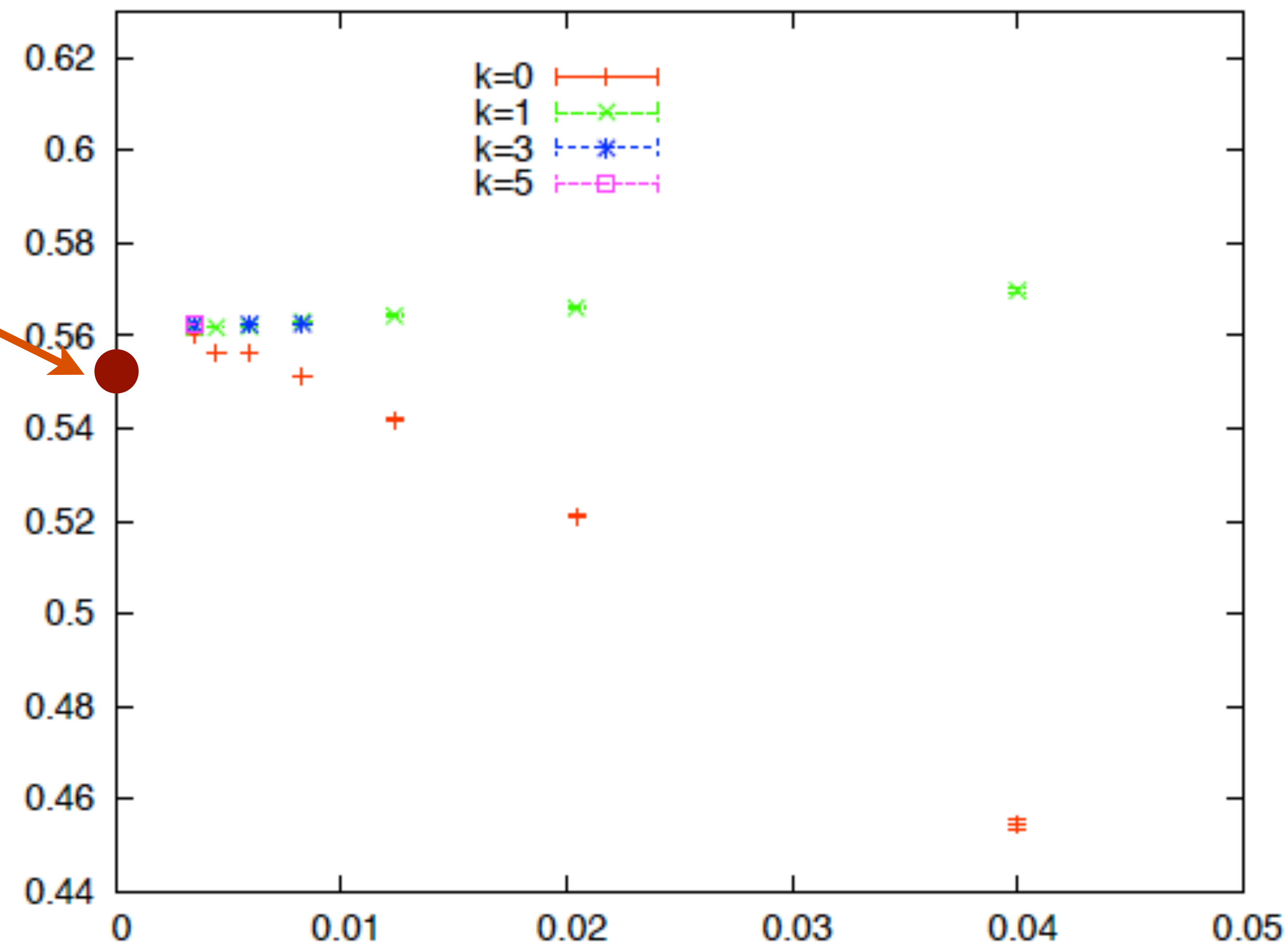
- k=1,3,5 points are with Twisted AEK model---have better behavior

Problems at very large N ?

- New results with N up to 289 [Gonzalez-Arroyo & Okawa]
- Form of N dependence varies with parameters

pure gauge value

<Plaquette>



1/N

$N_f = 2$
b=0.35,
κ=0.14
(lighter adjoints)

- k=1,3,5 points are with Twisted AEK model---have better behavior

Twisted Adjoint Eguchi-Kawai (TAEK) model: recent results

A. Gonzalez-Arroyo & M. Okawa, arXiv: 1304.0306, 1305.6253

Action of single-site TAEK model

Only change from AEK is twist in gauge action:

$$S_{\text{gauge}} = 2Nb \sum_{\mu<\nu} \mathrm{Re}\mathrm{Tr} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \, z_{\mu\nu}$$

['t Hooft; Gonzalez-Arroyo & Okawa]

$$\boxed{z_{\mu\nu} = e^{2\pi i k/L} \qquad L^2 = N}$$

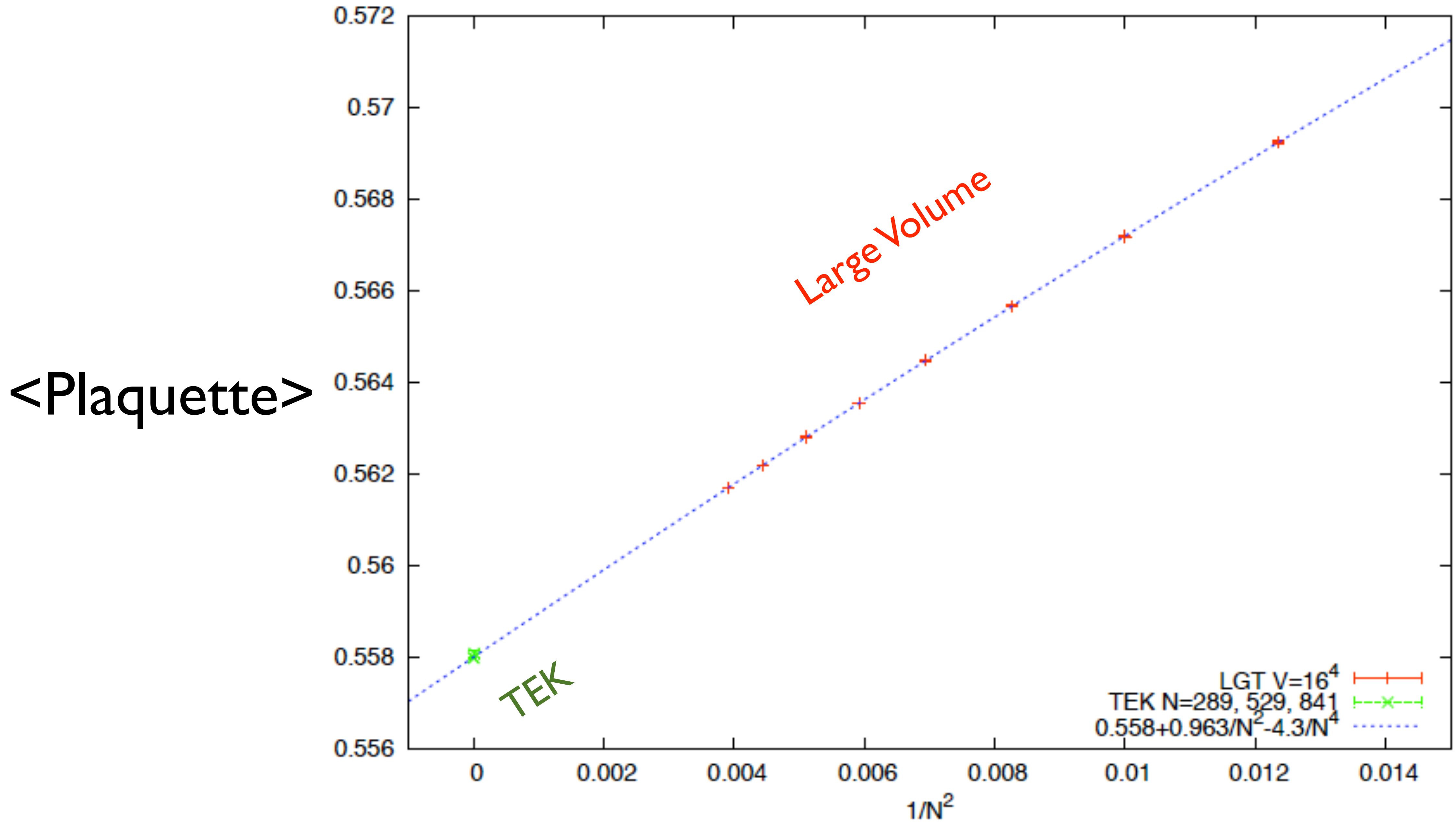
$$S_F = \sum_{j=1,N_f} \bar{\psi}_j D_W \psi_j$$

$$D_W = \mathbf{1} - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_\mu^{\text{adj}} + (1+\gamma_\mu) U_\mu^{\dagger\,\text{adj}} \right]$$

- Weak coupling: $Z_N{}^4$ broken to $Z_L{}^4$; perturbation theory as $L \longrightarrow \infty$ reproduces that on L^4 lattice
- Spectrum of D_W in weak coupling identical to that on an L^4 lattice
- Pure gauge: k=1 theory fails at large N; revived by using k/L > 0.1
- Adjoints not necessary for reduction---used because of physics interest

Reduction works for TEK model

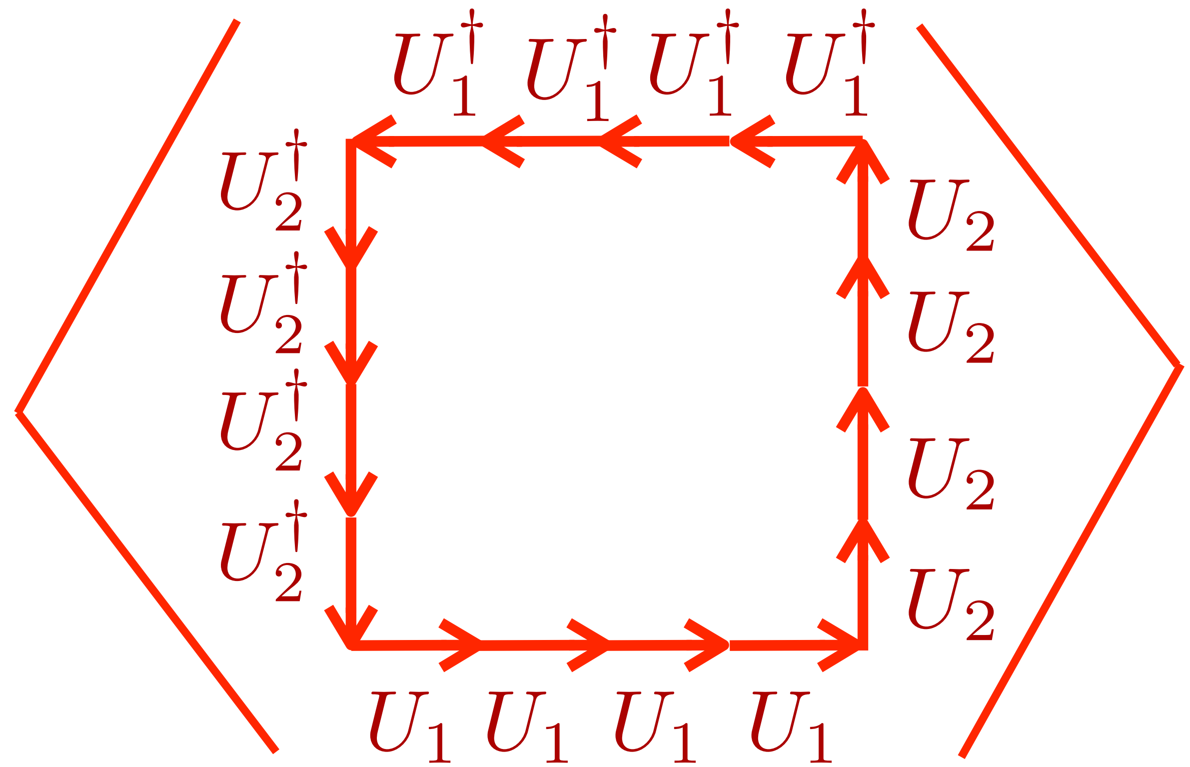
[Gonzalez-Arroyo & Okawa]



- Pure gauge: 16^4 with N<16 vs 1^4 with N=289, 529, 841 (L=17, 23,29) k=5,7,9

Wilson loops and string tension

W(4,4) =



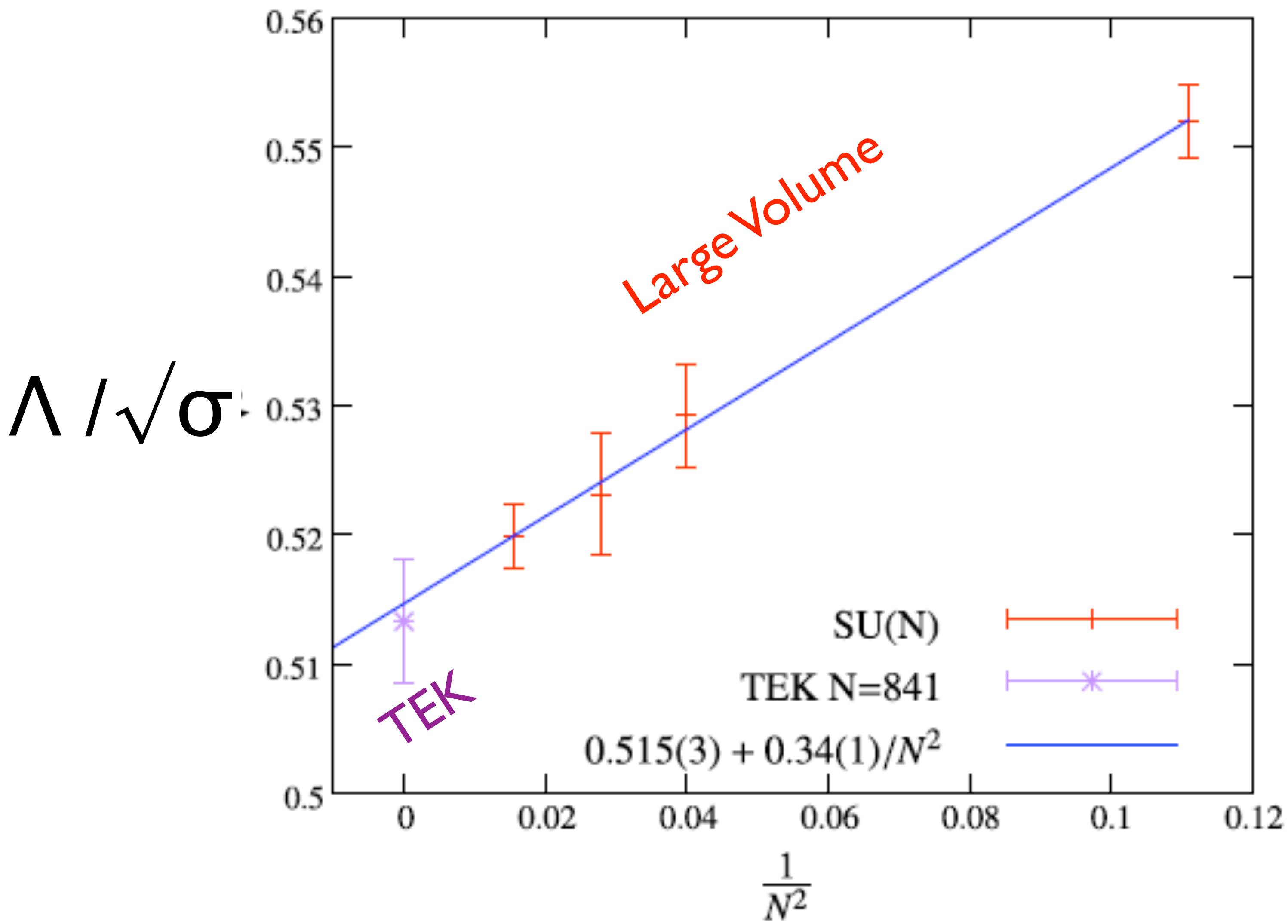
$$W(M,M') \overset{M'\to\infty}{\propto} e^{-V(M)M'}$$

$$V(M) \xrightarrow{M\to\infty} \sigma M$$

- Can reach loops of size L/2 x L/2 (since "volume" is L^4)
- Use smearing to get good signal for large loops (standard method)

Reduction works for TEK model

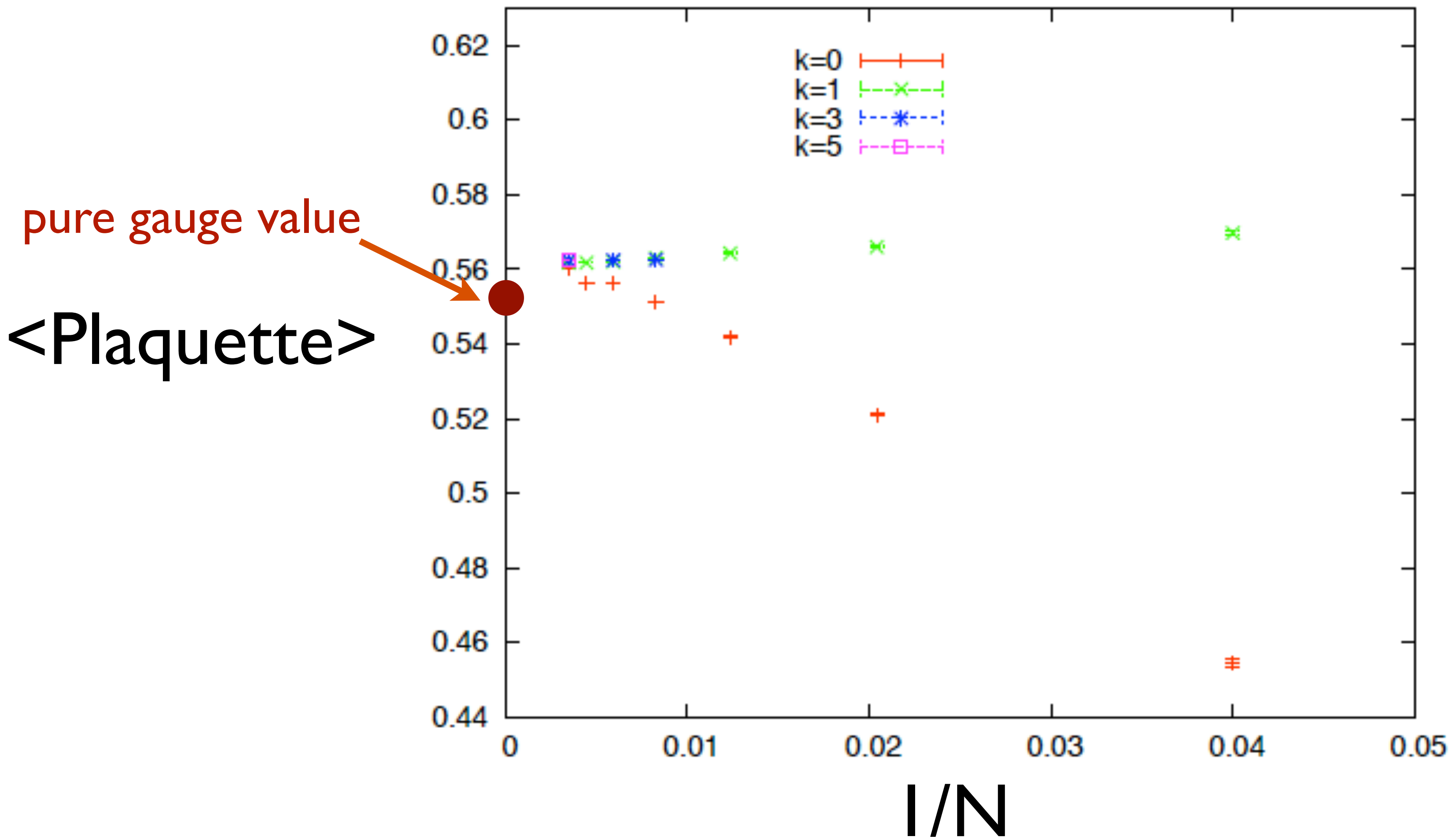
[Gonzalez-Arroyo & Okawa]



- Pure gauge: 32^4 with 3, 5, 6, 8 vs 1^4 with N=841, k=9 (L=29)

Improved N dependence for TAEK model

[Gonzalez-Arroyo & Okawa]

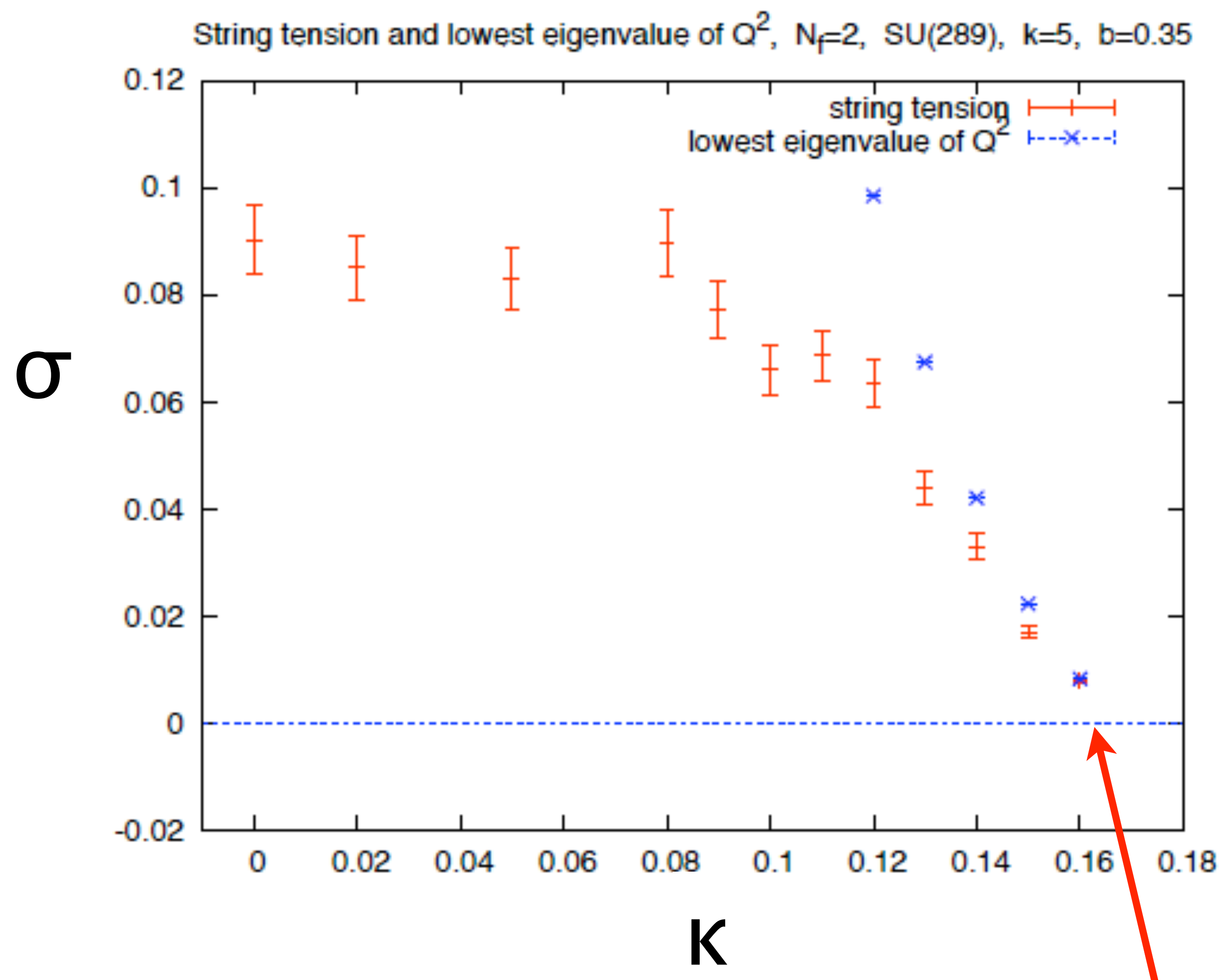


$N_f = 2$,
b=0.35,
κ=0.14

(Same plot as shown above)

Search for conformality

[Gonzalez-Arroyo & Okawa, arXiv:1304.0306]



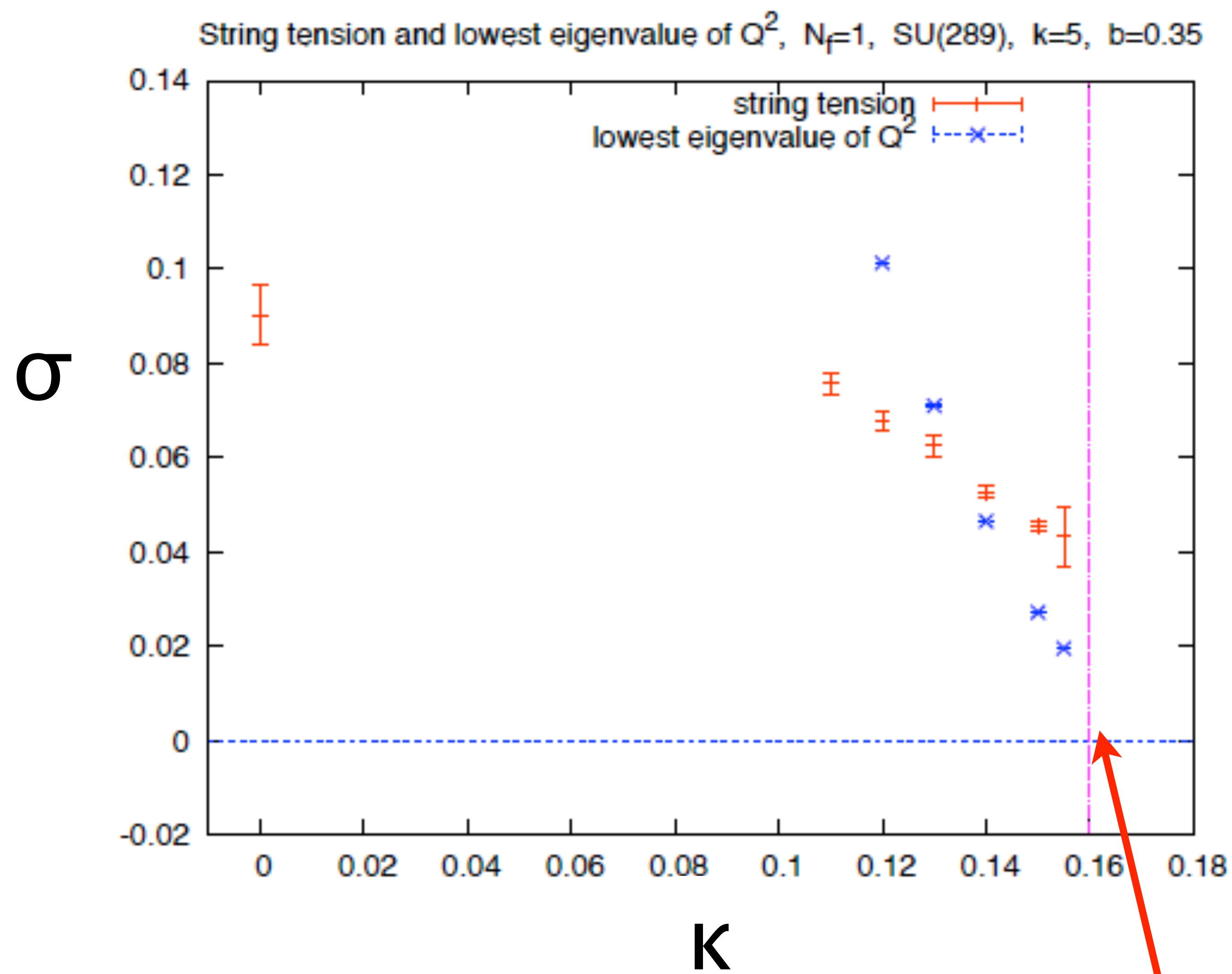
TAEK model
N_f=2
N=289, k=5

Preliminary

Quark mass & string tension vanish!

Cross-check

[Gonzalez-Arroyo & Okawa, arXiv:1304.0306]



TAEK model

$N_f=1$

$N=289$, $k=5$

Preliminary

Quark mass vanishes but string tension does not!

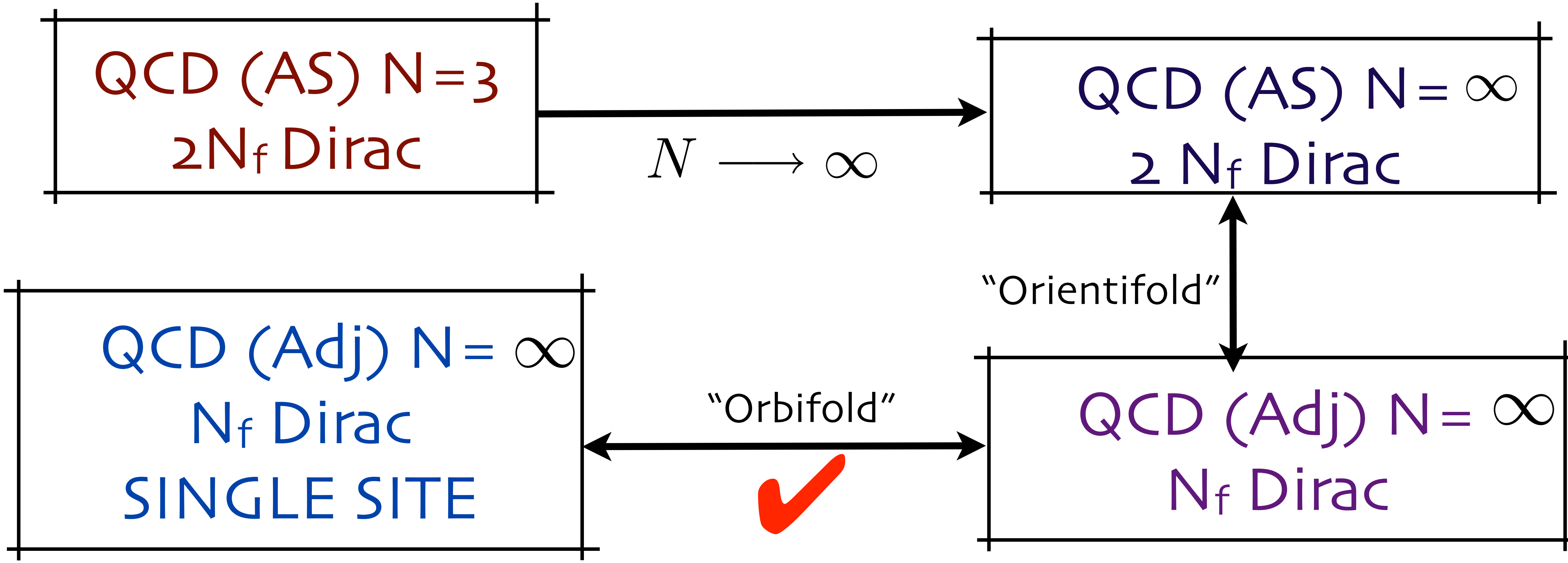
Conclusions & Outlook

EK reduction appears practical

- Need large values of N (e.g. $289=17^2$, $841=29^2$)
  - Not surprising once accept that $L=\sqrt{N}$ (no free lunch!)
- Twisted model appears to be the “model of choice”
  - Only downside is that it is difficult to include fundamental fermions
  - Without twisting, can use heavy adjoints to stabilize center symmetry
- Successful calculation of string tension
  - First application: indications of conformal fixed-point for 2 adjoints

Future directions & issues

* Calculation of hadron properties in $N_f=1$ TAEK
  - In principle, can calculate hadron masses, glueball-qq-bar mixing, ... on a single site, although it may be easier to extend in one direction
  - Window into hadron resonances where decays widths are small
* Efficient implementation on supercomputers?
  - Use 2^4 (or larger) to allow parallelism
* Scaling vs standard large N extrapolation?
  - We find CPU$\sim N^{4.5} \sim L^5 N^2$ vs. standard CPU$\sim L^5 N^3$
* Extend calculation to $N_f=1/2$ using overlap fermions
  - [Heitanen & Narayanan] have taken first steps



Thank you!
Any questions?