Kaons and long-distance meson mixing from lattice QCD

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Based partly on 2013 white paper

LATTICE QCD AT THE INTENSITY FRONTIER

Thomas Blum, Michael Buchoff, Norman Christ, Andreas Kronfeld,
Paul Mackenzie, Stephen Sharpe, Robert Sugar and Ruth Van de Water

(USQCD Collaboration)

SUMMARY

Lattice QCD calculations now play an essential role in the search for new physics at the intensity frontier. They provide accurate results for many of the hadronic matrix elements needed to realize the potential of present experiments probing the physics of flavor. The methodology has been validated by comparison with a broad array of measured quantities, several of which had not been well measured in experiment when the first good lattice calculation became available. In the US, this effort has been supported in an essential way by hardware and software support provided to the USQCD Collaboration.

This document has laid out an ambitious five year vision for future LQCD calculations, explaining how they can provide essential and timely information for upcoming experiments at the intensity frontier, by undertaking calculations of new, more computationally challenging, quantities. In addition, steady improvements in lattice results for matrix elements which are already well calculated will ensure that existing experimental results are fully utilized in the search for new physics. Our plans rely on continuing hardware and software support at similar levels to those of the last decade.

www.usqcd.org/documents/13flavor.pdf

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Very helpful input from experimentalists and phenomenologists!

We gratefully acknowledge suggestions and comments from Marina Artuso, Brendan Casey, Tim Gershon, Enrico Lunghi, Bob Tschirhart and Jure Zupan.

Master Formula

Expt = (CKM)(pQCD)(non-pert QCD)
+ BSM(non-pert QCD)

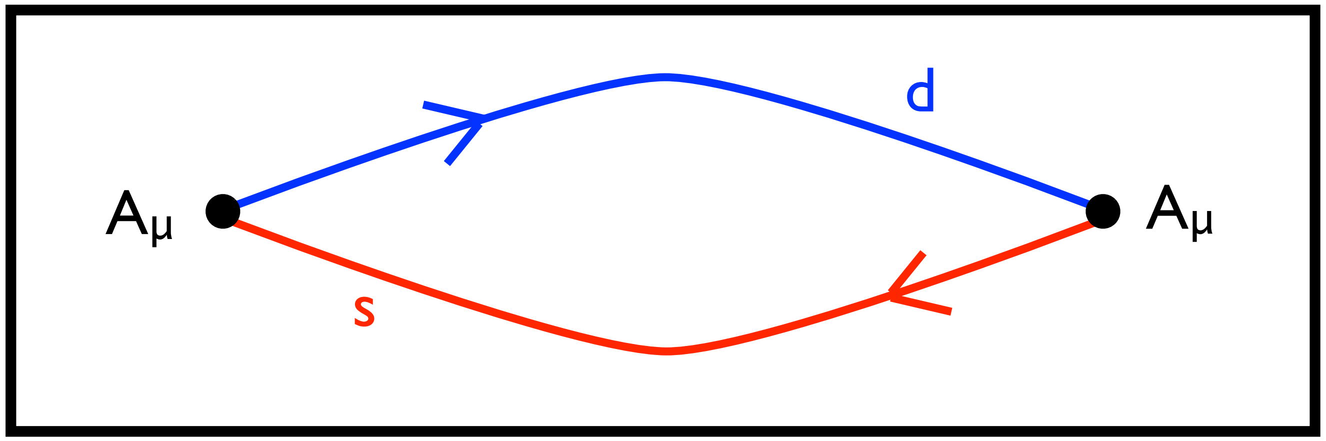
LQCD provides first-principles method to calculate (some) non-perturbative QCD matrix elements

Outline

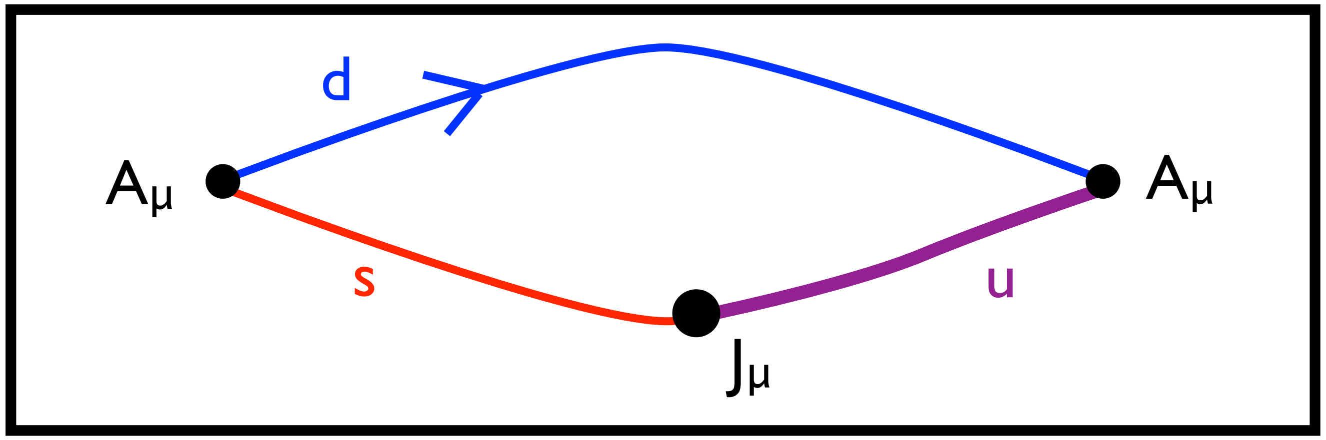
- Standard vs non-standard/novel/new lattice calculations
- Results for standard quantities
- Results & prospects for new quantities
  - $K \to \pi\pi$ decays
  - ΔM_K (long distance)
  - $K \to \pi\nu\nu$ and other rare decays
  - $D \to \pi\pi$, $D \to KK$ and D-Dbar mixing
- Summary & Outlook

Standard vs non-standard quantities

- Standard means matrix elements involving single particles



$\Rightarrow f_K^2$ (similarly f_π, f_D, f_B)



$\Rightarrow K\rightarrow\pi$ form factor

(similarly $B\rightarrow D$, etc)

Standard vs non-standard quantities

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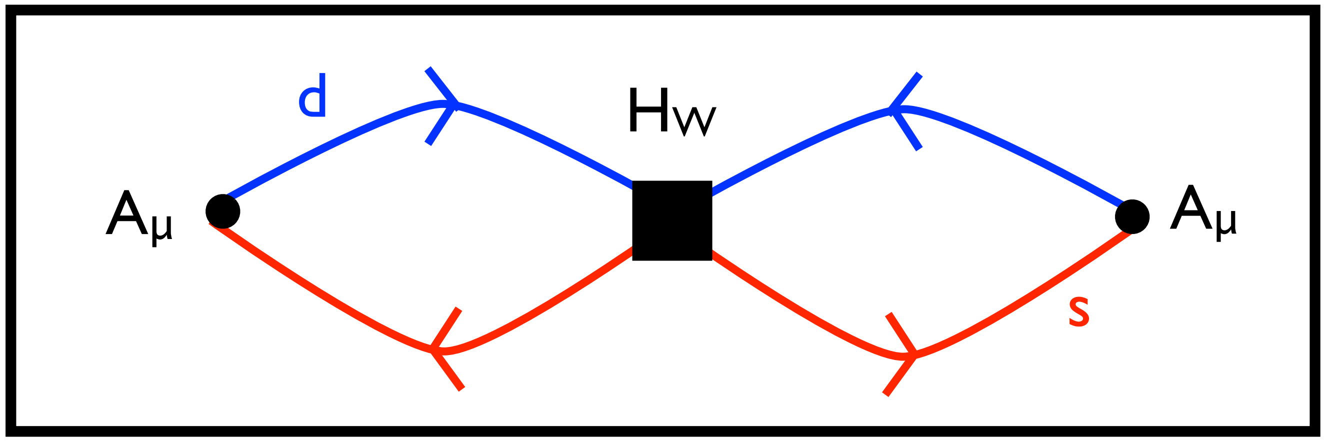
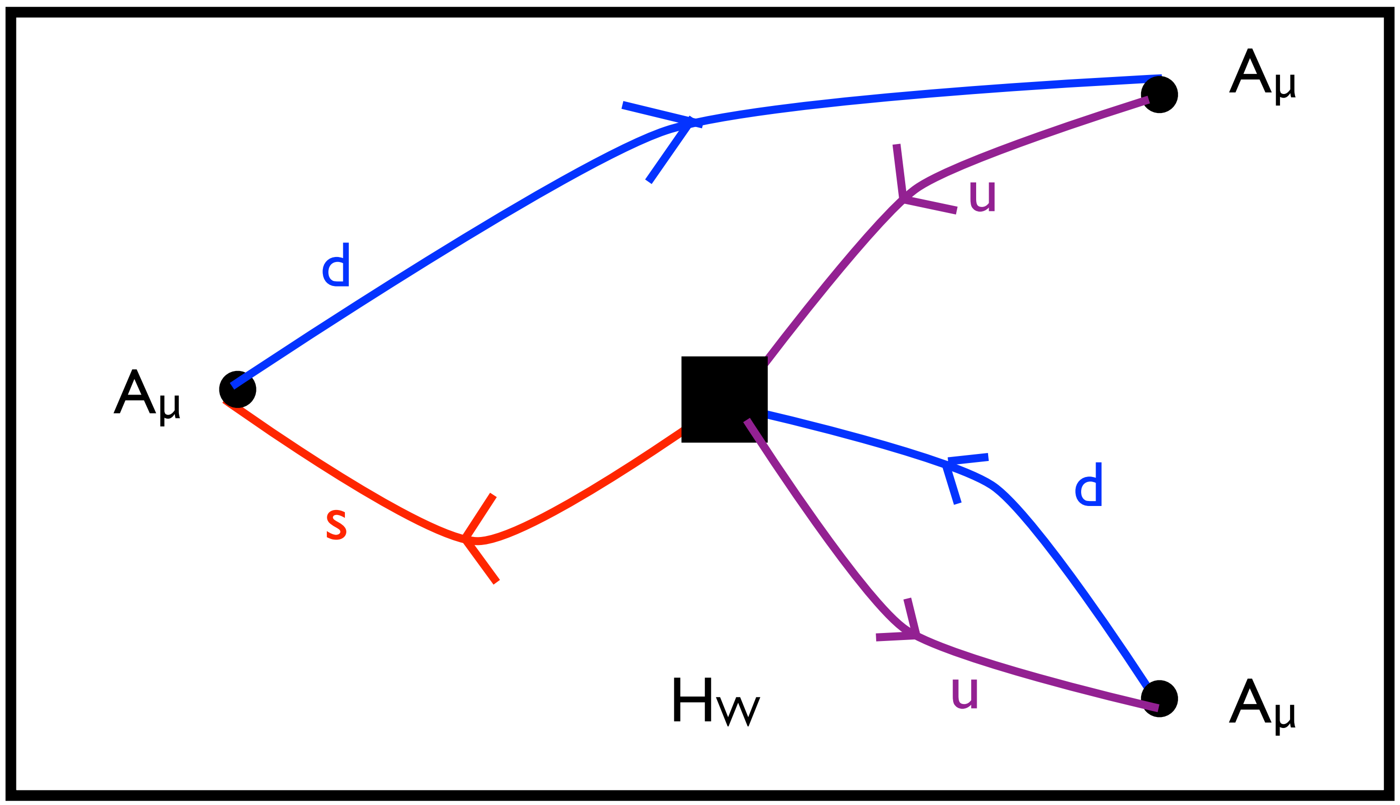


$\Rightarrow B_K$ (similarly B_D, B_B)

- Nearly 20 standard matrix elements are fully controlled with small errors
  - Decay constants: f_π, f_K, f_D, f_{Ds}, f_B, f_{Bs}
  - Form factors: $K\to\pi$, $D\to K$, $D\to\pi$, $B\to D$, $B\to D^*$, $B_s\to D_s$ & $B\to\pi$
  - Mixing matrix elements: B_K, B_B, B_{Bs}

Standard vs non-standard quantities

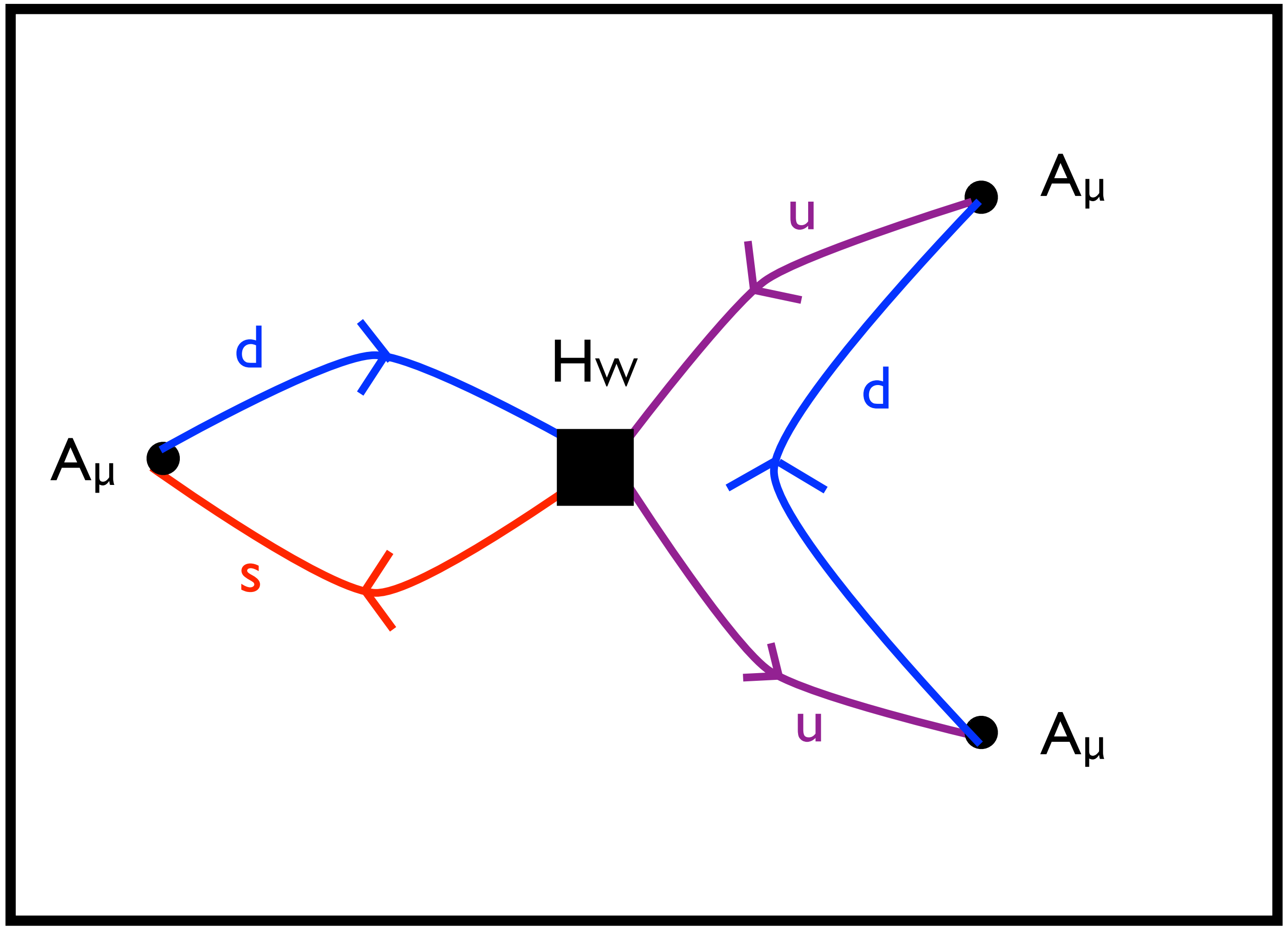
- Non-standard: matrix elements involving two or more particles



$\Rightarrow K \to \pi\pi$ (I=2)

Standard vs non-standard quantities

- Non-standard: matrix elements involving two or more particles and/or quark-disconnected contributions



$\Rightarrow K \rightarrow \pi\pi$ (I=0)

Standard vs non-standard quantities

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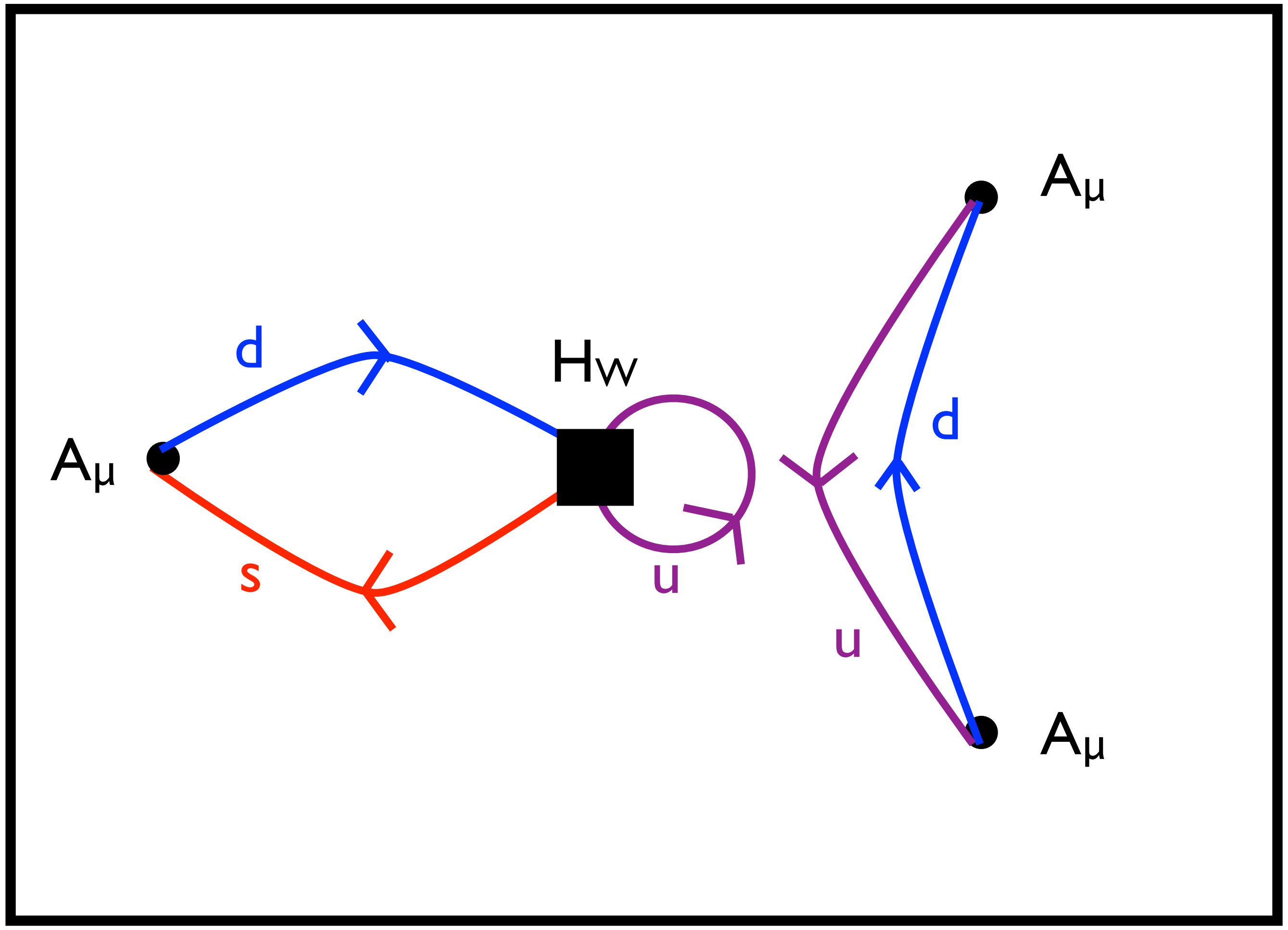

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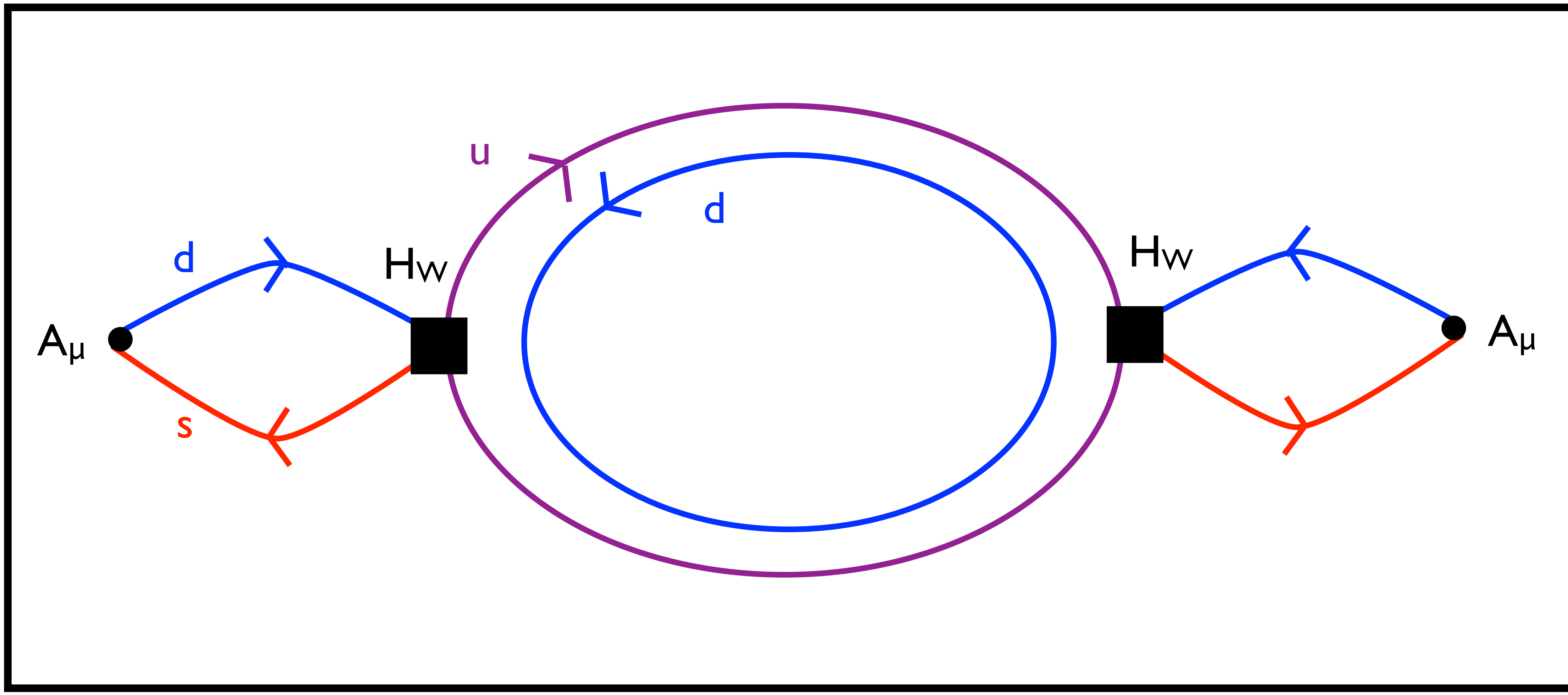
Standard vs non-standard quantities

- Non-standard: matrix elements involving two or more particles and/or quark-disconnected contributions and/or two insertions of H_W



$\Rightarrow$ Long distance part of ΔM_K

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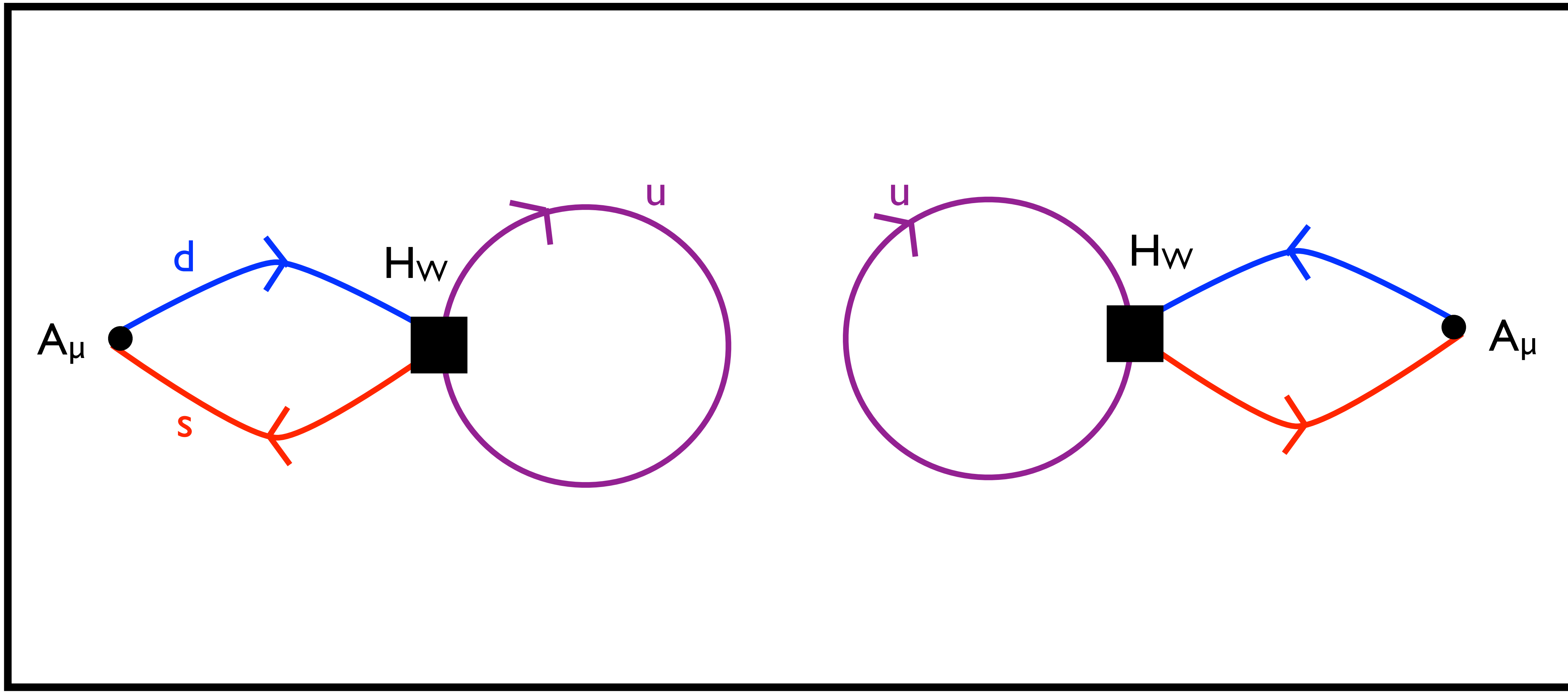

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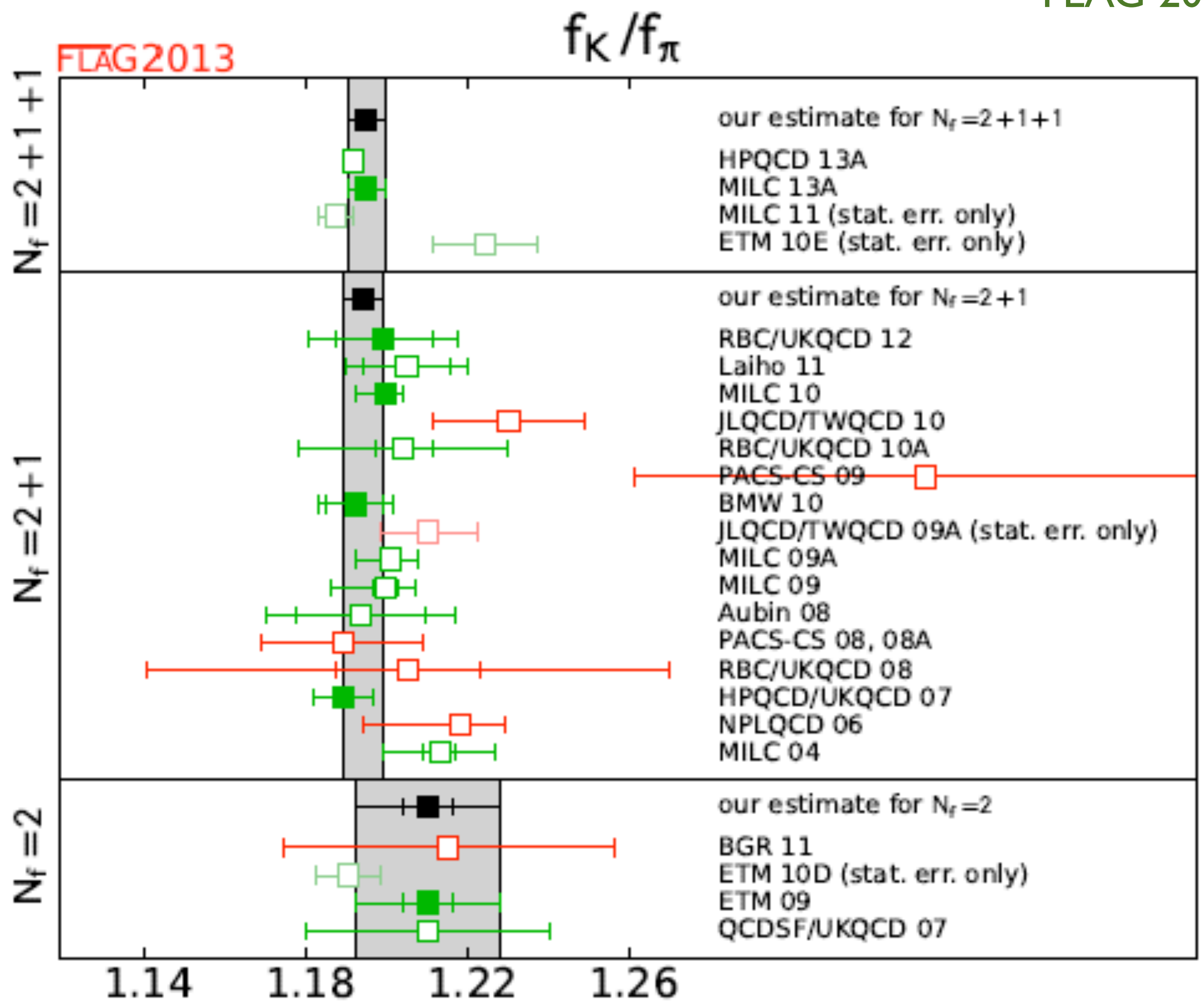
Status of f_K/f_π

FLAG 2013 (arXiv:1310.8555)



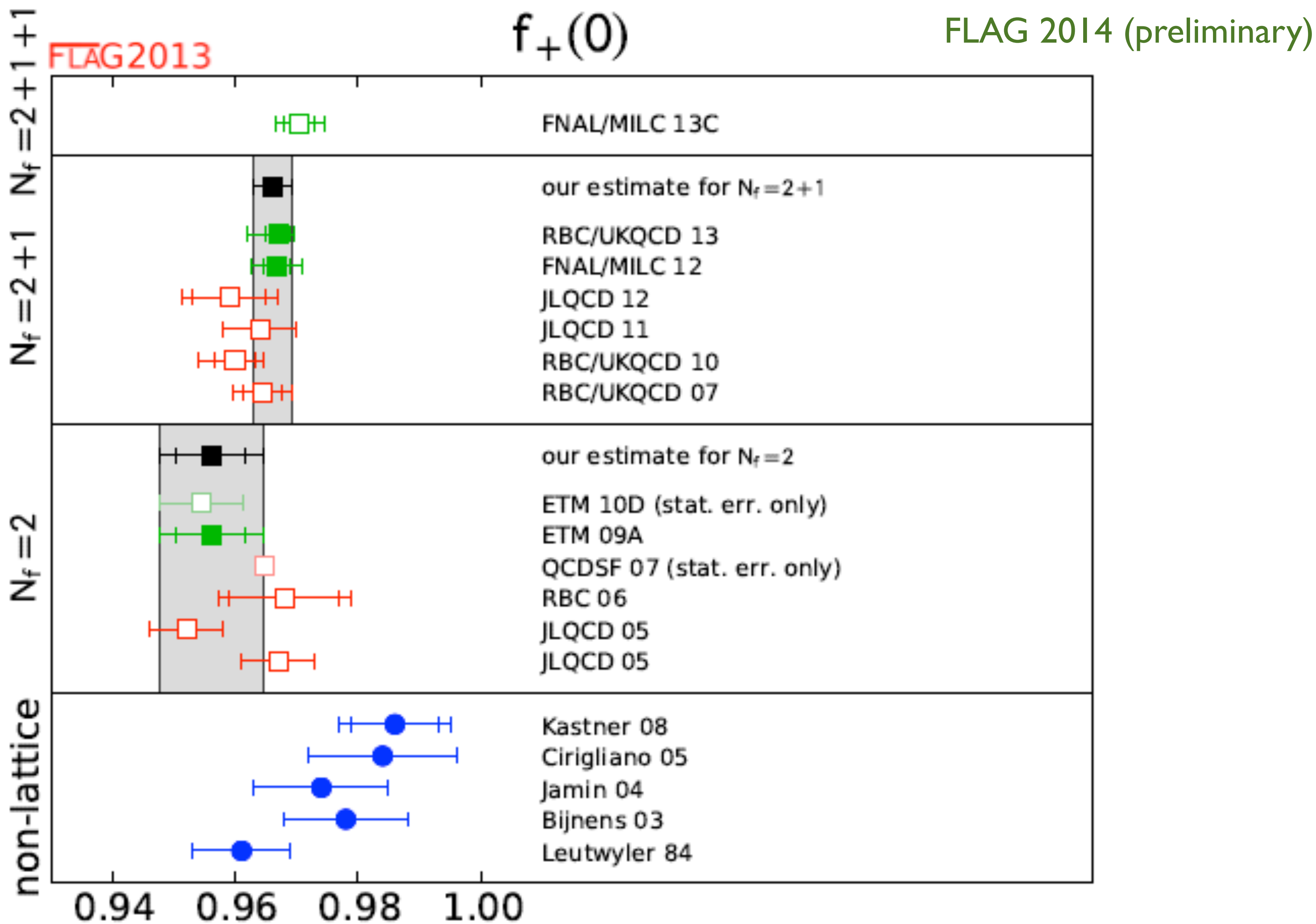
$$f_{K^\pm}/f_{\pi^\pm} = 1.195(3)(4), \quad (\text{direct}, N_f = 2+1+1),$$
$$f_{K^\pm}/f_{\pi^\pm} = 1.192(5), \quad (\text{direct}, N_f = 2+1),$$

0.4% error (really 2% on difference from 1)

$$\Rightarrow |V_{us}| = 0.2256(3)_{\text{exp}}(2-4)_{\text{EM}}(10)_{\text{lat}}(1)_{V_{ud}}$$

- Further lattice improvement underway, but will eventually require inclusion of EM effects. This is not easy since axial current not EM gauge invariant.

Status of K→π form factor



$$f_+(0) = 0.9661(32) \qquad (N_f = 2+1)$$

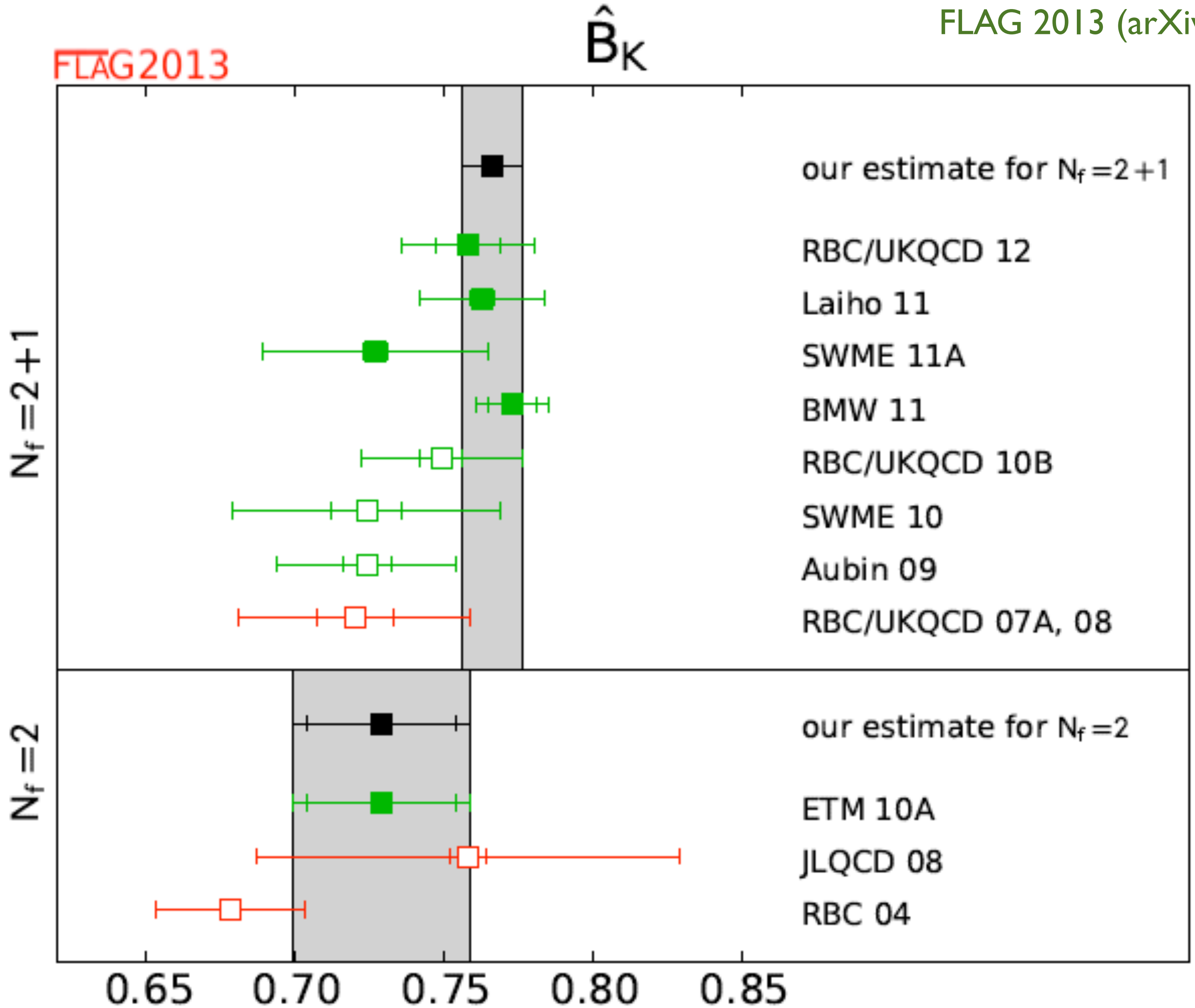
$$\Rightarrow |V_{us}| = 0.2239(5)_{\rm exp}(2)_{\rm EM}(7)_{\rm lat}$$

0.3% error
[really 10% on $f_+(0)-1$]

- Lattice errors will be reduced using physical light-quark masses (underway)
  "EM wall" not far away

Status of B_K

FLAG 2013 (arXiv:1310.8555)



$$N_f = 2+1: \qquad \hat{B}_K = 0.766(10)$$ 1.3% error

- Can be further improved, but already other errors dominate when using B_K in unitarity triangle analysis (V_{cb}, long distance in ε_K, PT errors)

Reasons for future improvement

- Steadily improve calculations of standard matrix elements, in particular using:
  - Physical light-quark masses
  - Isospin breaking & EM effects
  - Charmed sea
  - Finer lattice spacings & improved actions (heavy quarks)
  - Improved statistical errors
  - Improved methods of normalizing operators (e.g. SMOM)
- Already in use for some quantities, will soon become widespread

Expanding portfolio of standard quantities

- Contributions of BSM physics to K (and D & B) mixing
- $B \to K\, l^+l^-$, $\Lambda_b \to \Lambda\, l^+l^-$ and related form factors
- Tensor form factors for $K \to \pi$; needed to constrain BSM theories
- Others?

Can achieve few-10% accuracy on few year timescale, which is commensurate with experimental program, and significantly enhances search for BSM physics

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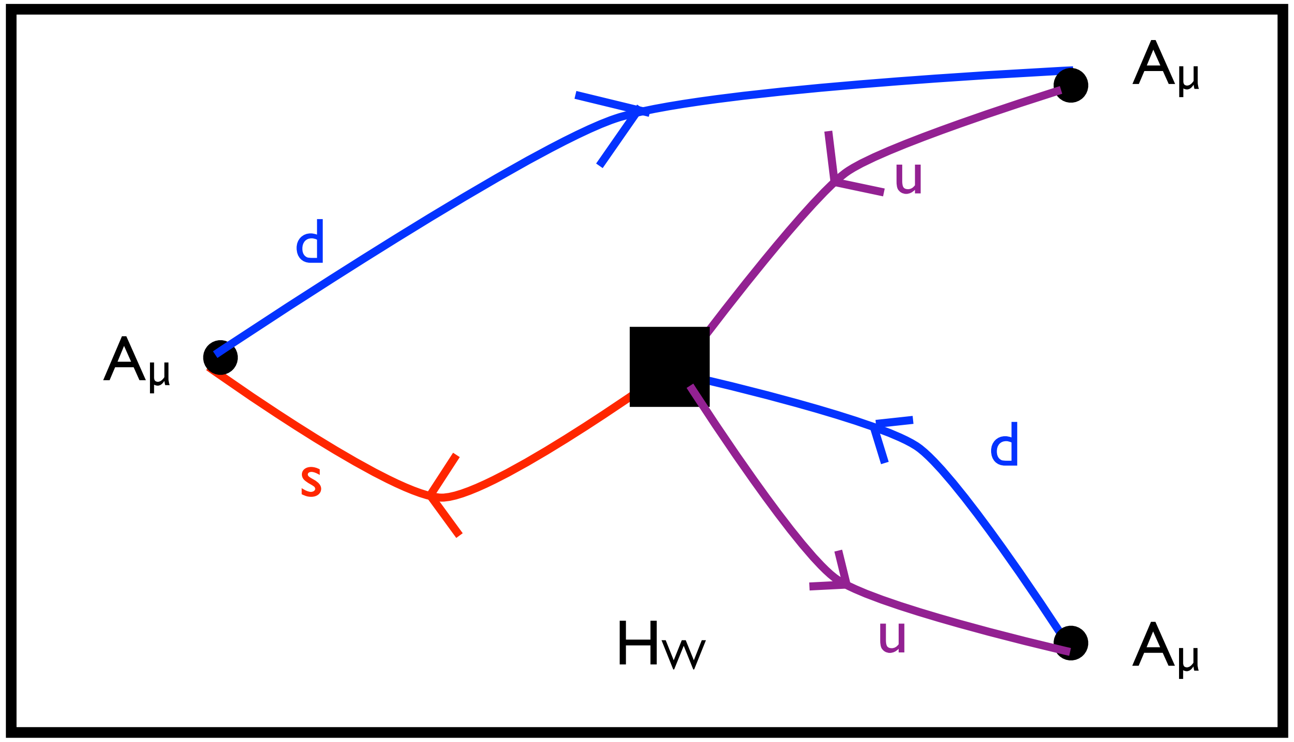
$K\to\pi\pi(I=2)$ decay amplitudes

[RBC/UKQCD arXiv:1111.1699, 1206.5142, **1311.3844(Lattice 2013)**]

- First controlled result for an amplitude involving two particles
- Isospin 2 ⇒ no quark-disconnected contributions
- Uses physical kinematics (physical quark masses, moving pions so $M_K=2E_\pi$) which requires box with $L\approx5.5$ fm
- Original result at $a\approx0.14$ fm; two new (prelim.) results at $a\approx0.12$ & 0.9 fm, allowing continuum extrapolation
  - Systematic error now ≈11% (down from 19%); statistical errors 1-2%

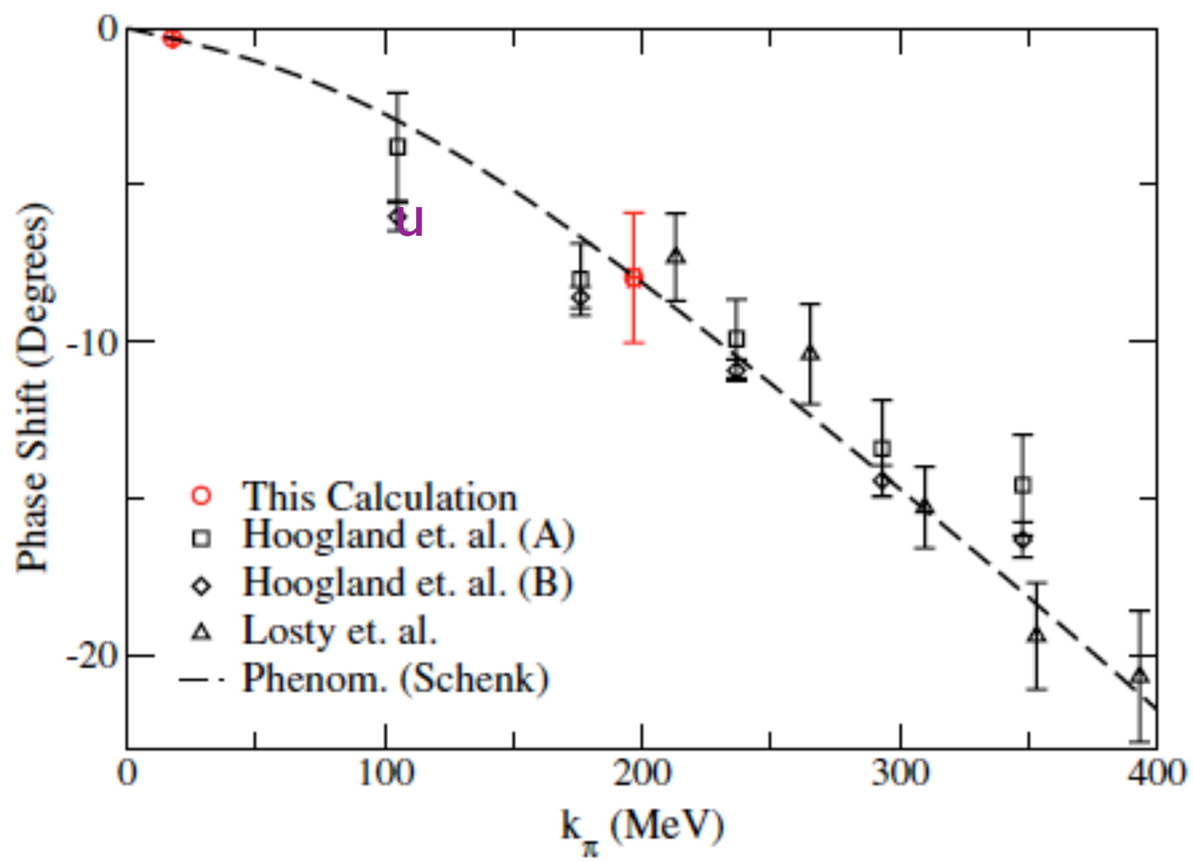


ππ(I=2) phase shift

[RBC/UKQCD arXiv:1111.1699, 1206.5142]

- Auxiliary calculation needed to make finite volume correction (few % effect)
- First calculation of phase shift with physical quark masses

I=2 phase shift



Lattice

Experiment

Comparison with experiment

Lattice Results (2012) [RBC/UKQCD arXiv:1111.1699, 1206.5142]

$\mathrm{Re}\, A_2 = 1.38\ (5)_{stat}\ (26)_{syst}\ 10^{-8}$ GeV ⟷ expt. $1.479(4)\ 10^{-8}$ [K^+]
$1.57(6)\ 10^{-8}$ [K_S]

$\mathrm{Im}\, A_2 = -6.5\ (5)_{stat}\ (12)_{syst}\ 10^{-13}$ GeV ⟷ New information!
Can use with expt result for ε’
to determine $\mathrm{Im}\, A_0$

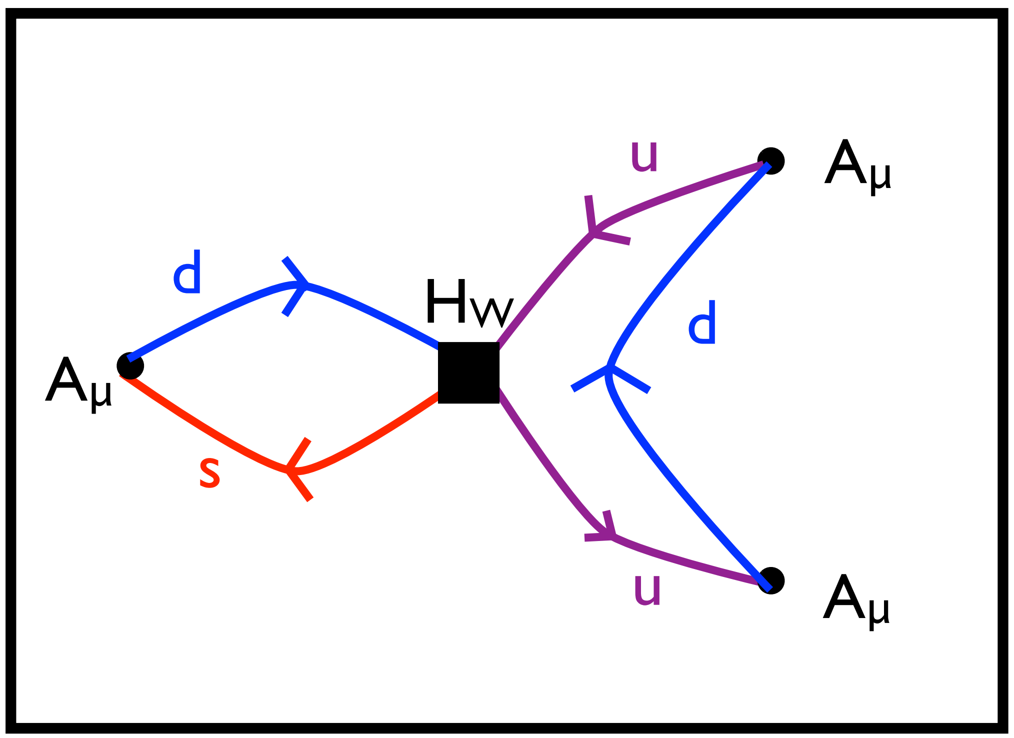
Present lattice errors about half this size [arXiv:1311.3844; RBC/UKQCD article in progress]

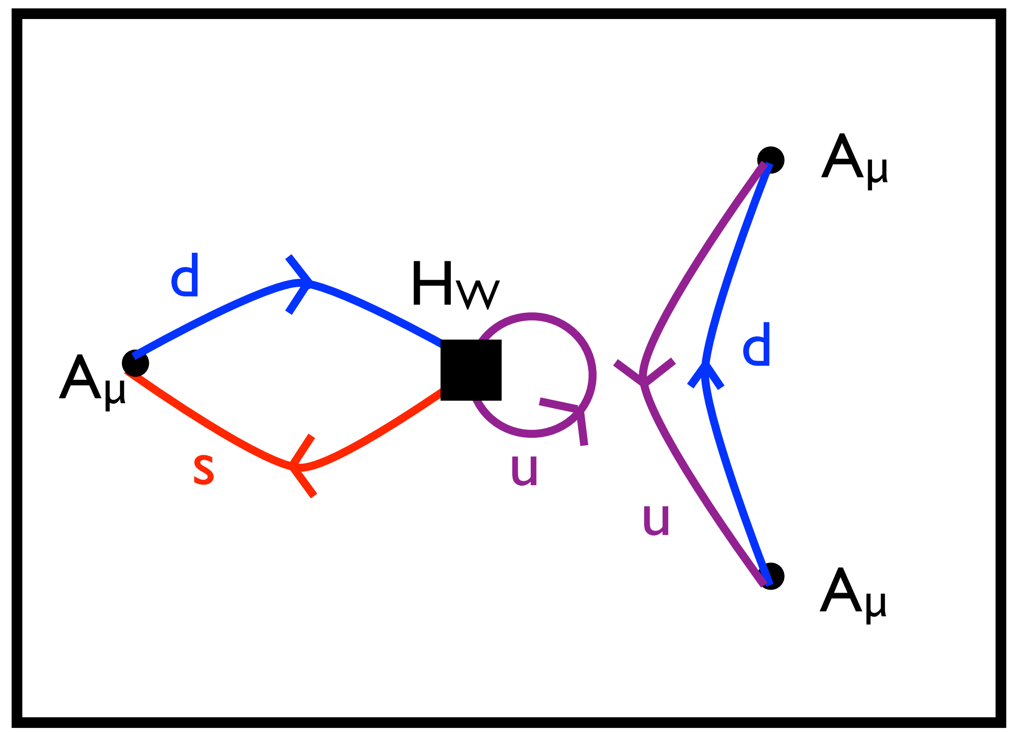
K→ππ(I=0) amplitude & ΔI=1/2 rule

- I=0 involves disconnected contractions ⇒ numerics much more challenging
  - Several other technical challenges too. Fermions with chiral symmetry essential.
- Pilot calculation in 2012: decay at threshold for M_K~660, 880MeV

[RBC/UKQCD arXiv:1212.1474]

  - Demonstrates that technology (& related theory) exists. Statistical errors ~ 15%.





Present status

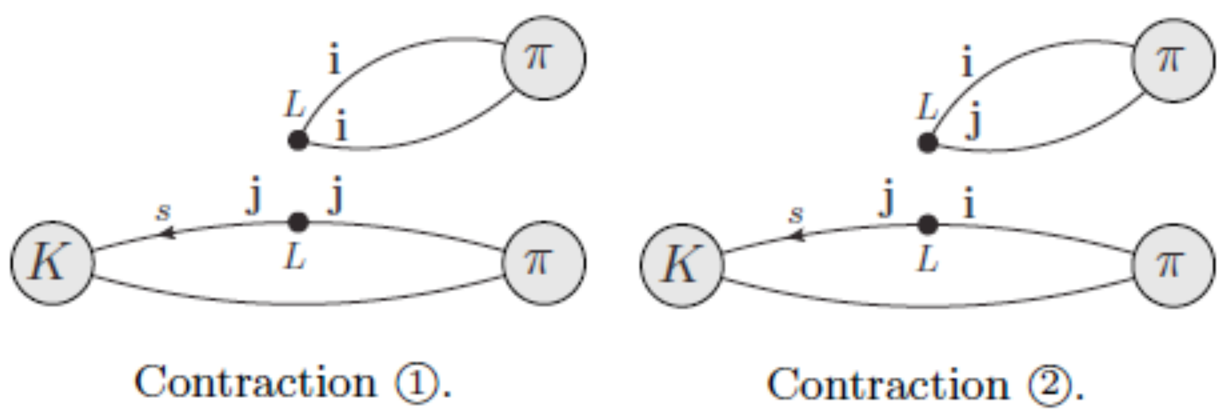
[RBC/UKQCD arXiv:1310.0434 & Lattice 2013 contributions by C.Kelly & by D.Zhang]

- Calculation at physical kinematics and a=0.15 fm underway
  - Requires G-parity BC, all-to-all propagators, improved pion sources.
- Auxiliary calculation of I=0 ππ scattering amplitude has clear signal
- Results with errors of ~20% in $\mathrm{Re}(A_0)$ and $\mathrm{Im}(A_0)$ expected in 1-2 years
- Finally will be able to use experimental result for ε' to constrain SM!

Anatomy of ΔI=1/2 rule

[RBC/UKQCD arXiv:1212.1474 & 1311.3844]

- Emerging understanding of $\Delta I=1/2$ rule
  - Re A_0~experiment
  - Re A_2/Re A_0 suppressed due to cancellation between color contractions
  - Penguins unimportant at $\mu\approx 2$GeV

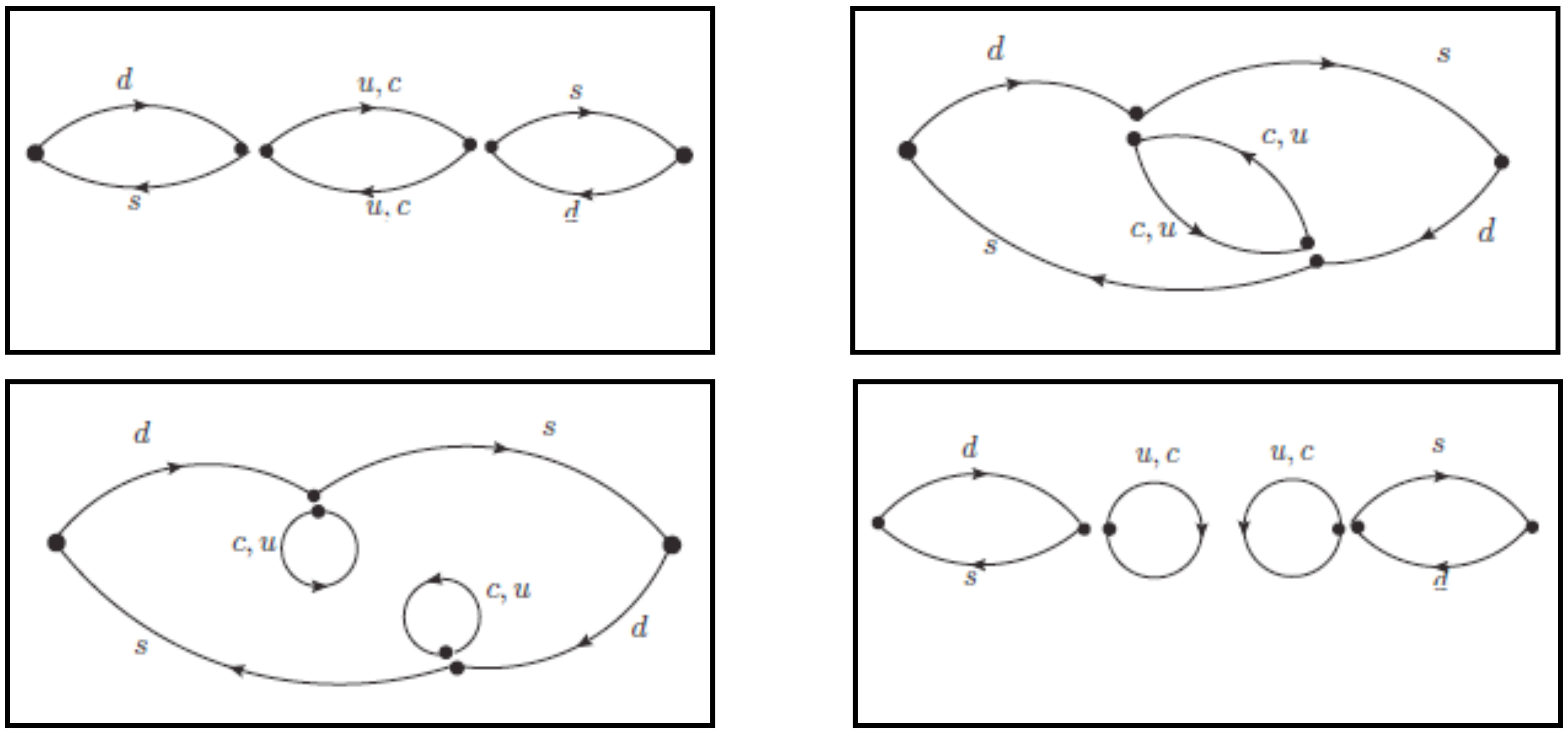


Naively expect $C_2=C_1/3$
In fact $C_2\sim -C_1$

Long-distance part of ΔM_K

[RBC/UKQCD: Christ (Lattice 2010), arXiv:1111.6953, 1201.2065, 1212.5931, 1312.0306]

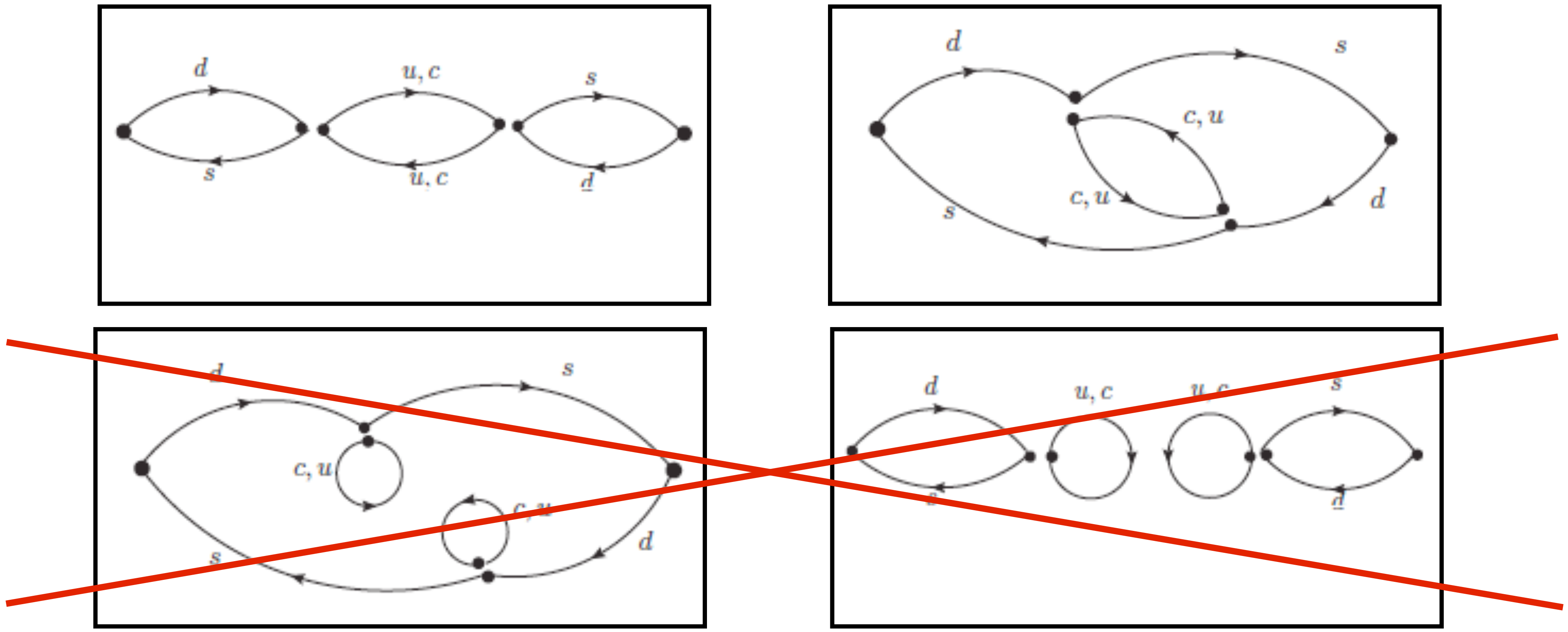
- Very challenging since two insertions of H_W & quark-disconnected diagrams
  - Requires new theoretical & numerical/algorithmic developments
  - Calculations first consider dominant up+charm contribution
  - Charm enforces crucial GIM cancellations



Long-distance part of ΔM_K

[RBC/UKQCD: Christ (Lattice 2010), arXiv:1111.6953, **1201.2065**, **1212.5931**, 1312.0306]

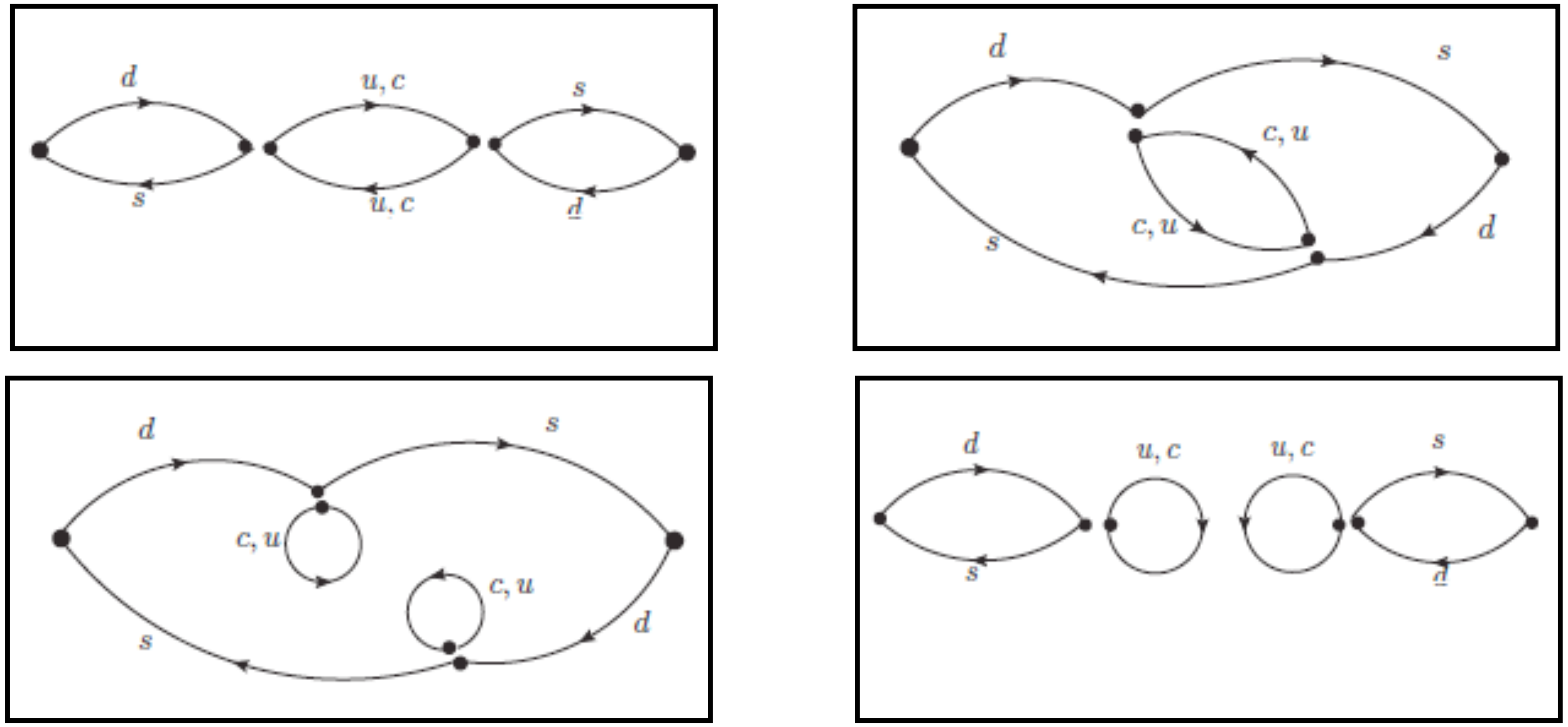
- 2012: Pilot calculation keeping non-disconnected contractions at unphysical masses and with valence (but not dynamical) charm
  - Proof of principle giving results with correct order of magnitude
  - a=0.12 fm, M_π=420 MeV, m_c=860 MeV



Long-distance part of ΔM_K

[RBC/UKQCD: Christ (Lattice 2010), arXiv:1111.6953, 1201.2065, 1212.5931, **1312.0306**]

- 2013: Keep **all** contractions, with lighter pion (m_π=330MeV) and m_c=950MeV
    - Achieve 10% statistical errors (highly non-trivial)
    - Fully disconnected diagram NOT Zweig suppressed
    - Result similar to experimental value (do not expect exact agreement given unphysical quark masses)



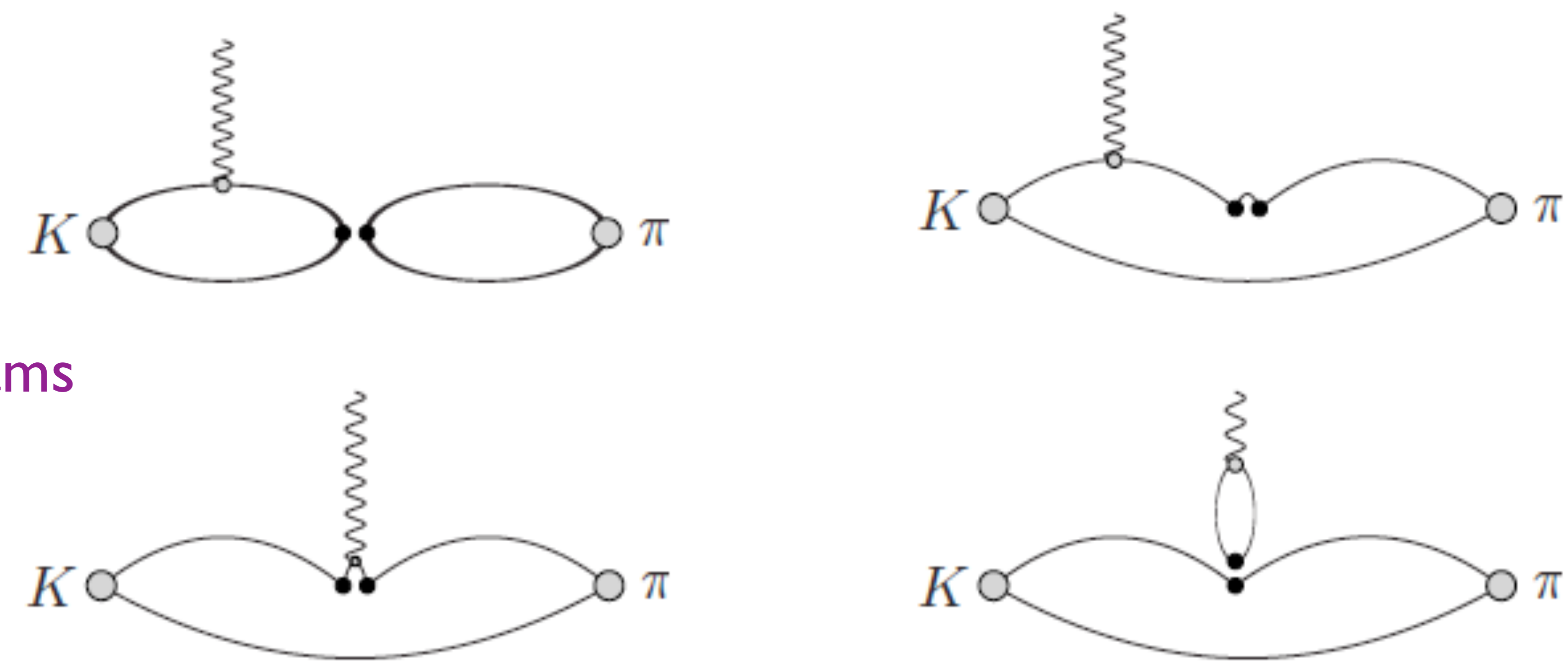
Prospects for long-distance kaon mixing

- RBC/UKQCD has further calculations of ΔM_K underway
  - m_π=170 MeV, larger box (L=4.6 fm), coarser lattice (a=0.15fm): closer to realistic kinematics
  - Physical pion mass, dynamical charm, a=0.07fm, very large lattice
  - Result with physical kinematics & 10-20% errors in 1-2 years?
- Calculation of long-distance contribution to ε_K (~4%) more challenging; methods under development

Other applications with double insertions

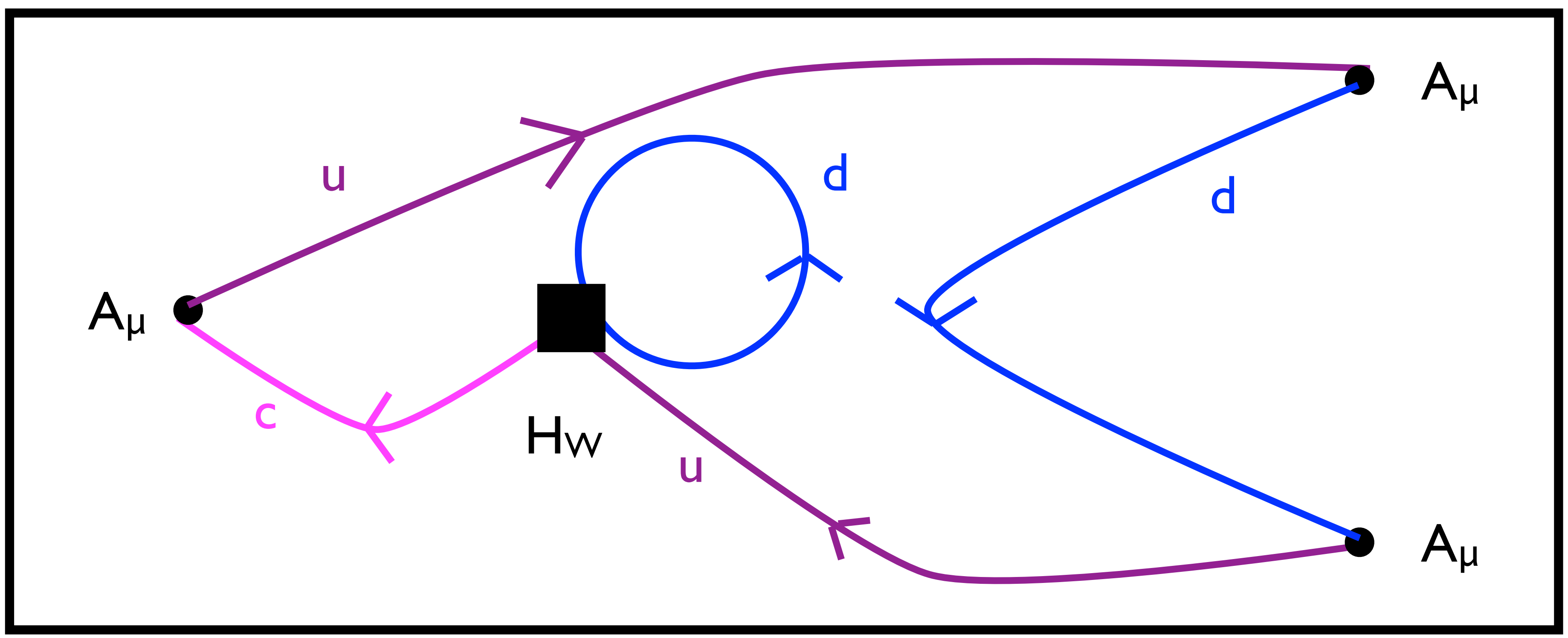
- Can LQCD help solidify SM predictions for $K\to\pi l^+l^-$ & $K\to\pi\nu\nu$?
  - Would calculating the sign of $K_S\to\pi\gamma^* \to\pi\ e^+e^-$ useful (since that seems to be the main uncertainty in predicting $K_L\to\pi e^+e^-$)?
  - Are present estimates of charm and long-distance contributions to $K\to\pi\nu\nu$ accurate enough?
  - Under active consideration [RBC/UKQCD]

Examples of diagrams for $K_S\to\pi\gamma^*$



CP violation in D→ππ,KK

- Evidence for CP-violation puts us in the same situation as we've been in with ε' for decades: can we reliably predict the SM contribution?
  - Many challenges, both computational and theoretical
  - Hardest (still unsolved) is that, at energy M_D, 2π & 2K states mix in a finite box with 4π, 6π, etc. and need to disentangle



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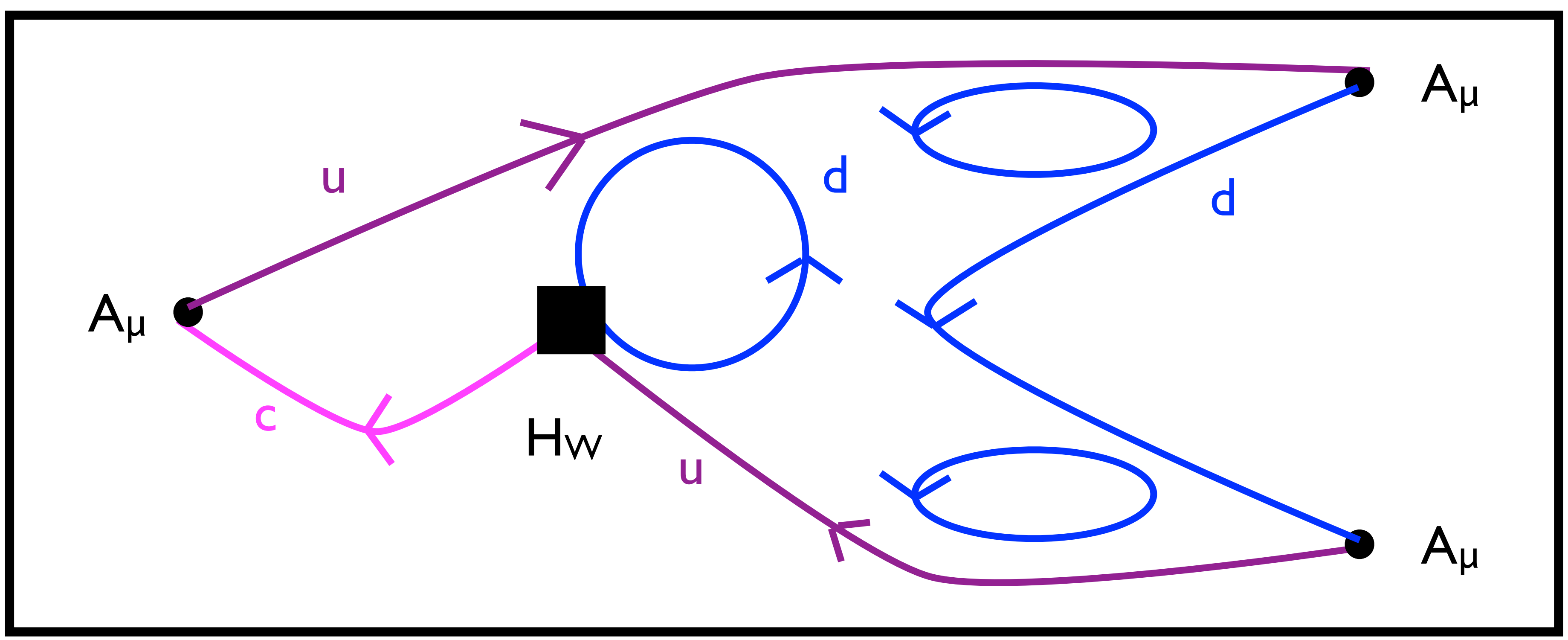

CP violation in D→ππ,KK

- Evidence for CP-violation puts us in the same situation as we've been in with ε' for decades: can we reliably predict the SM contribution to D→ππ, KK?
    - Many challenges, both computational and theoretical
    - Hardest (still unsolved) is that, at energy M_D, 2π & 2K states mix in a finite box with 4π, 6π, etc. and need to disentangle
    - Some progress with 3π case (generalization of Luscher formalism, but very complicated formulae) [Polejaeva & Rusetsky, Briceno & Davoudi, Hansen & SS]
- Progress on D→ππ, KK on a 5 year timescale?
- D-Dbar mixing is more challenging

Summary & Outlook

- LQCD calculations of standard kaon matrix elements now well controlled
  - Expect steady improvement, soon/now requiring inclusion of isospin breaking & EM effects
  - Additional standard quantities will be added (e.g. BSM kaon mixing)
- Next 5 years will see controlled calculations of non-standard matrix elements requiring qualitatively different approaches and improved numerical methods
  - We will then, finally, be able to understand the implications of ε'/ε, ΔM_K
  - Are there other quantities that these methods can be used for?
- Lattice QCD is powerful but also limited
  - E.g. long distance contribution to ΔM_D presently inaccessible