Resolving the left-hand-cut problem in lattice studies of the doubly-charmed tetraquark

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Based on work in preparation with
Zack Draper, Max Hansen,
& Fernando Romero-López

$T_{cc}(3875)$

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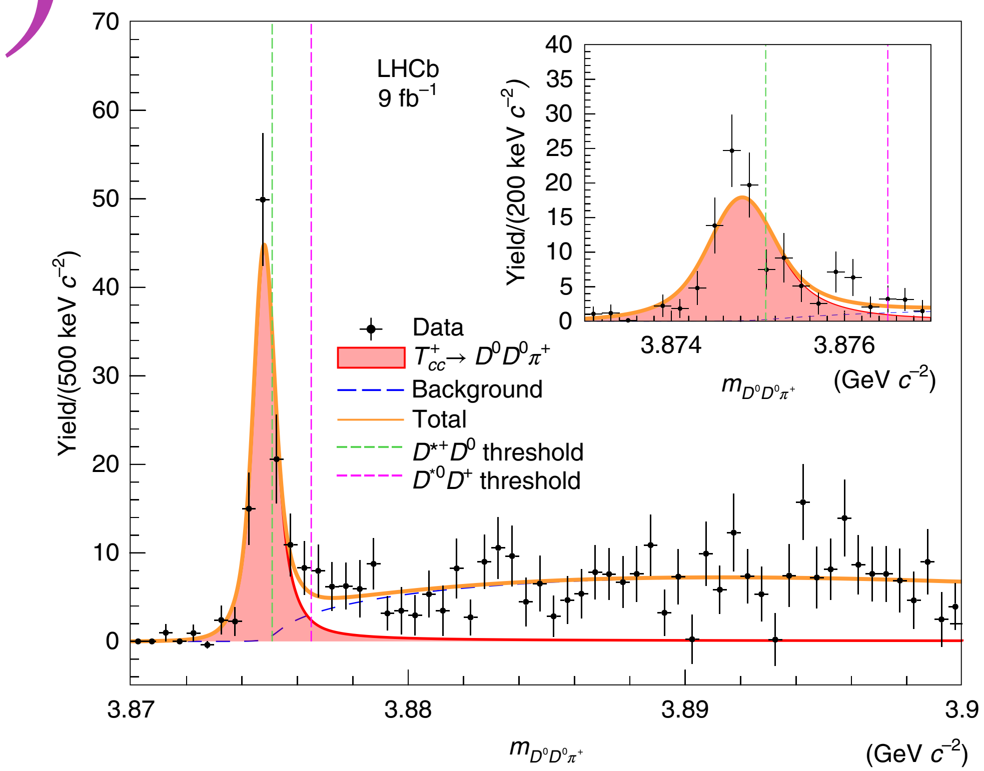
Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration*



- $T_{cc} \to D^0 D^{*+} \to D^0 D^0 \pi^+$
- $I = 0,\ J^P = 1^+$
- $M = 3875\,\mathrm{MeV}, \Gamma \approx 400\,\mathrm{KeV}$
- Lies slightly below the DD^* threshold
- $cc\bar{u}\bar{d}$ $\Rightarrow$ Tetraquark, and thus exotic
- Strong candidate for DD^* molecule

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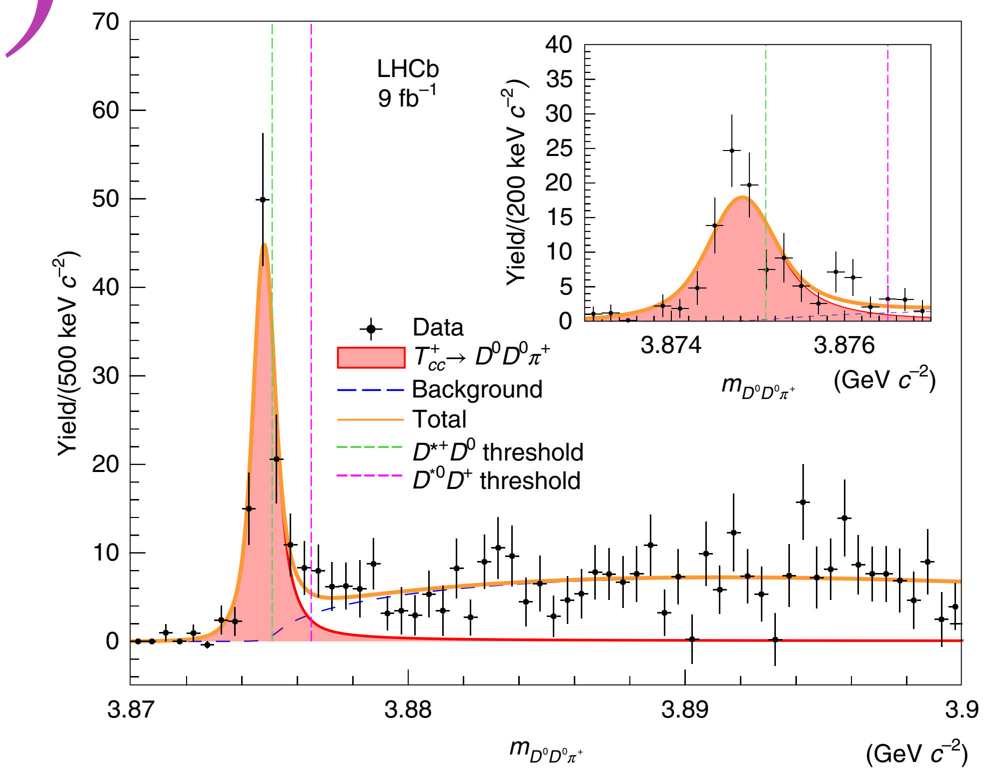

Natural candidate for study using LQCD

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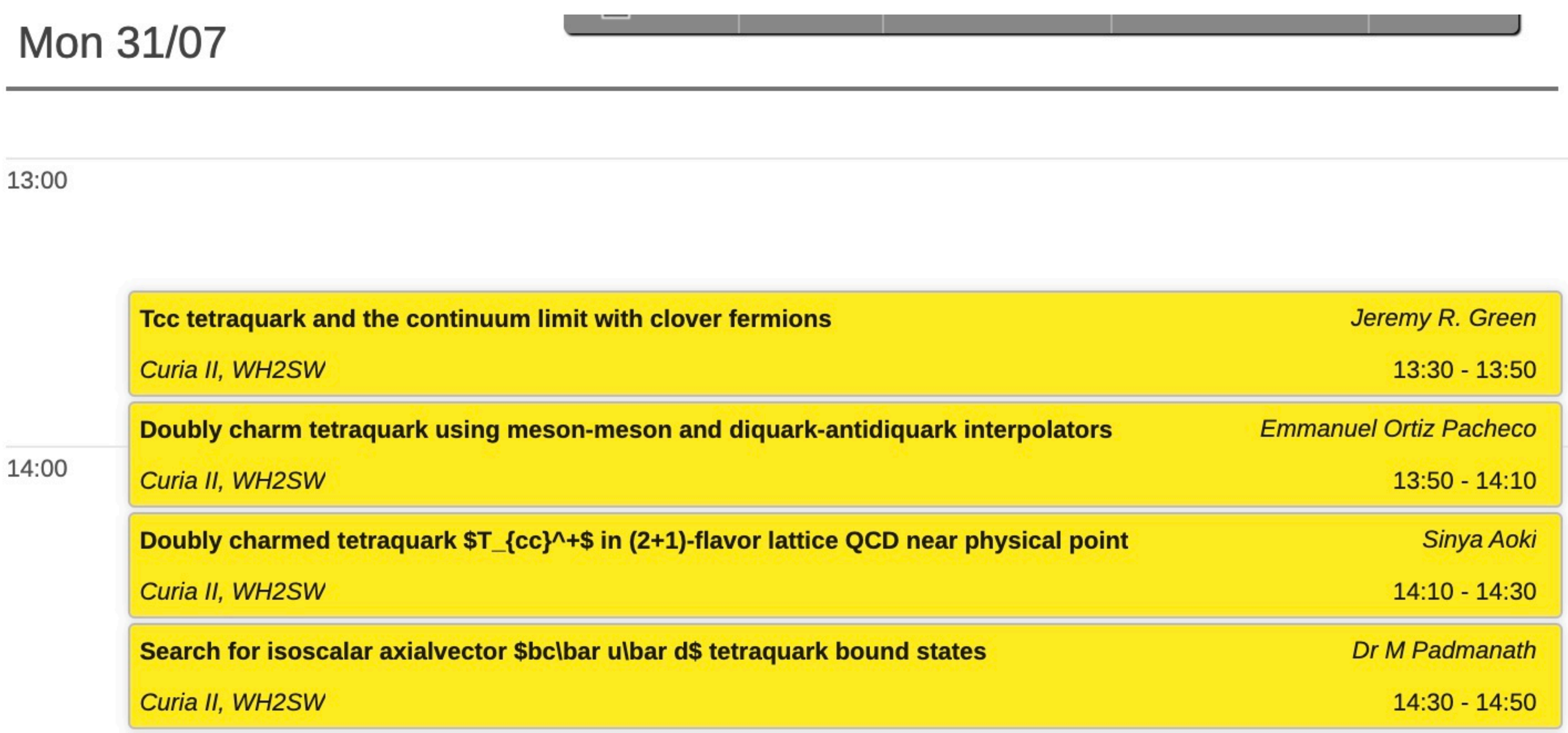
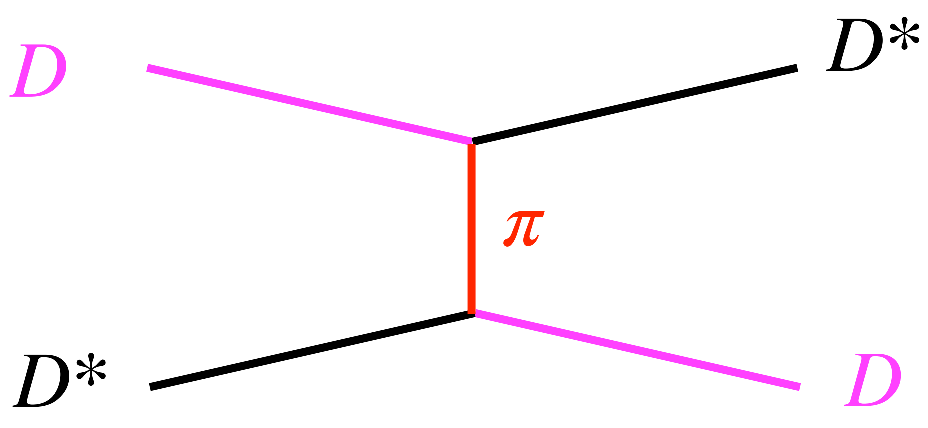
Lattice studies of $T_{cc}(3875)$

- [Padmanath & Prelovsec, 2202.10110] $M_\pi \approx 280\,\mathrm{MeV}$ (so D^* is stable) Lüscher method
- [Chen et al., 2206.06185] $M_\pi \approx 350\,\mathrm{MeV}$ (D^* is stable) Lüscher method
- [Lyu et al. (HALQCD), 2302.04505] $M_\pi \approx 146\,\mathrm{MeV}$ (D^* still stable!) Determine D^*D potential using HALQCD method
- Active area of research!



Left-hand cut in D^*D amplitude



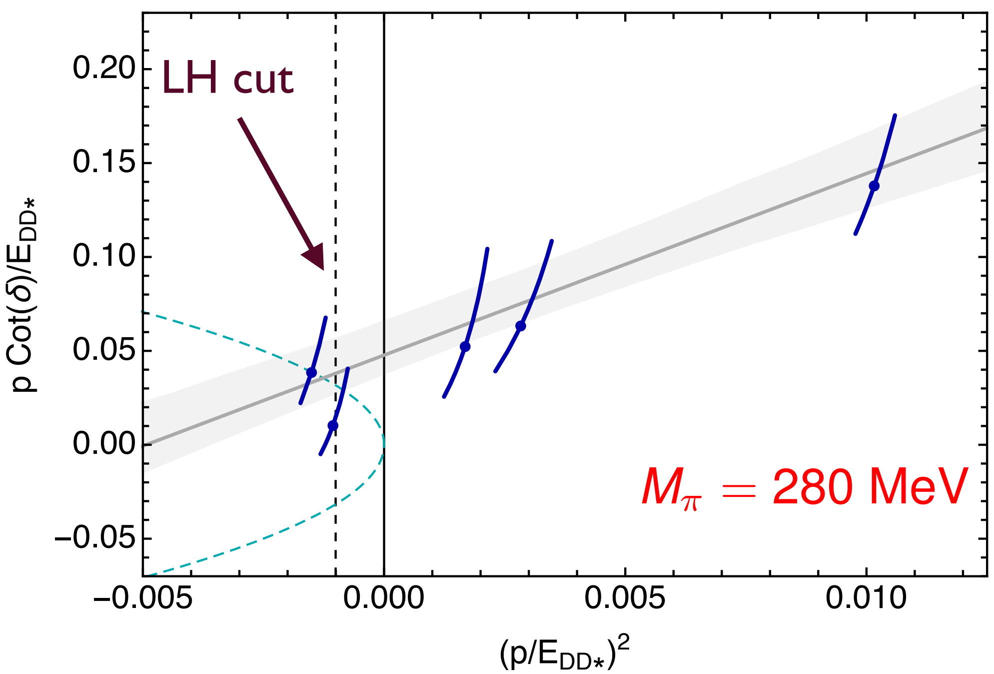
$$u = M_\pi^2,\ t = 0$$

$$s = s_{\rm thr} - (M_\pi^2 - [M_{D^*} - M_D]^2)$$

- Two-particle (Lüscher) quantization condition (QC2) fails at and below left-hand cut
  - Nonanalyticity in $\mathcal{M}_2$ and $\mathcal{K}_2$ leads to additional finite-volume effects [Raposo & Hansen, 23]
  - $\mathcal{K}_2 \propto 1/(k \cot \delta)$ becomes complex, ERE fails
- In LQCD studies, left-hand cut lies in vicinity of putative virtual bound state, invalidating the analysis using QC2

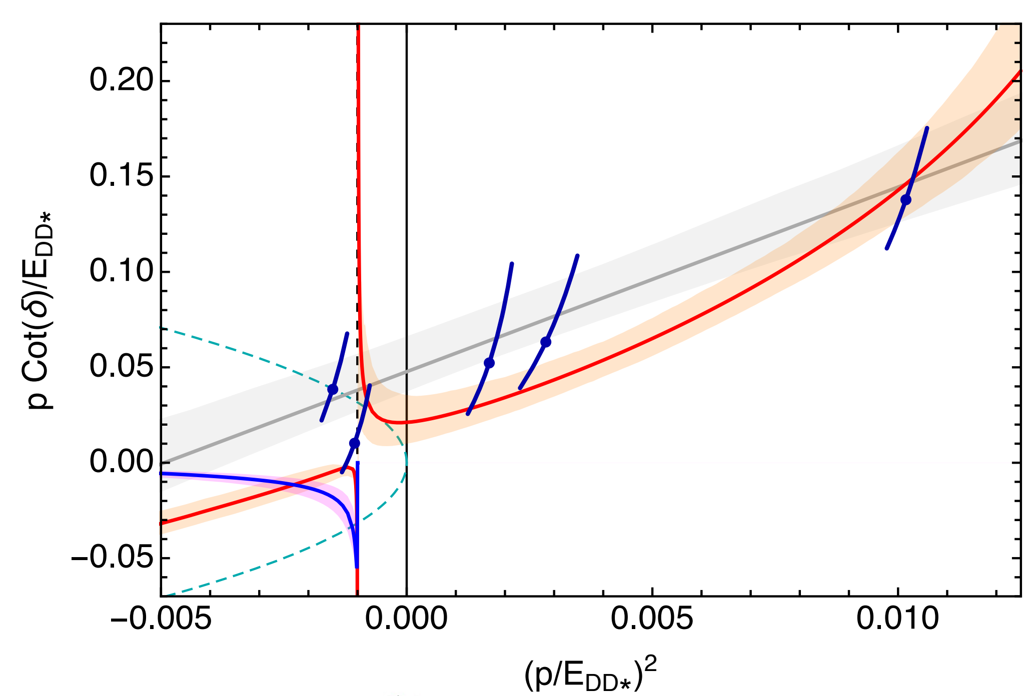
Left-hand cut in D^*D amplitude

[Padmanath & Prelovsec, 2202.10110]



- Ignoring LH cut suggests a virtual bound state
- Authors argue that this transitions to a real bound state for physical case

[Du et al., 2303.09441]



- Need to include pion exchange
- Example using model finds 0 or 2 virtual bound states
- See also talk by Md Habib E Islam

Including LH cut in LQCD analyses

- Extend QC2 by explicit inclusion of LH cut [Raposo & Hansen, talk by Raposo]
- Use HALQCD method to obtain D^*D potential? [Lyu et al, talk by Aoki]
- Use three-particle quantization condition (QC3) applied to $DD\pi$ system [present proposal]
  - D^* included as bound state in p-wave $D\pi$ channel (so don't need to use QC2, or QC2+3 [Briceño, Hansen, SRS 17])
  - Pion exchange automatically incorporated into formalism
  - Applies also for physical case with unstable D^*
- Analogous to use of QC3 for three identical particles in which two form a dimer, and study dimer-particle interactions and trimer formation [Blanton et al., 1908.02411; David et al., 2303.04394; talks by Md Habib E Islam & Dawid]

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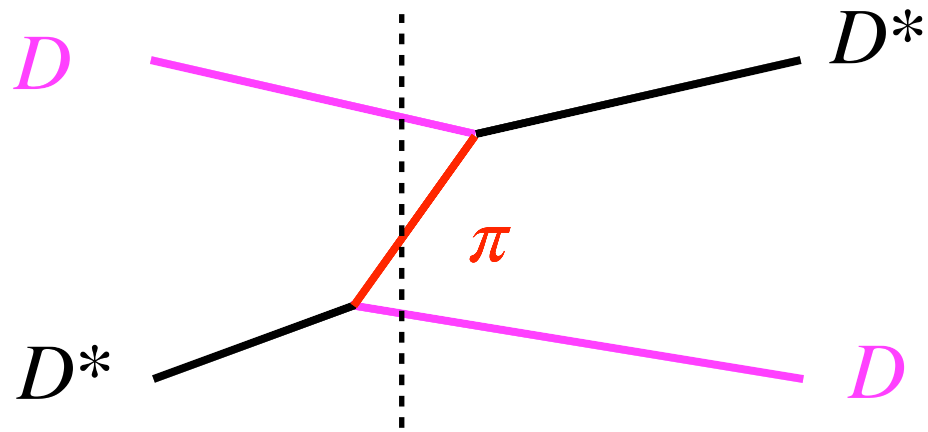
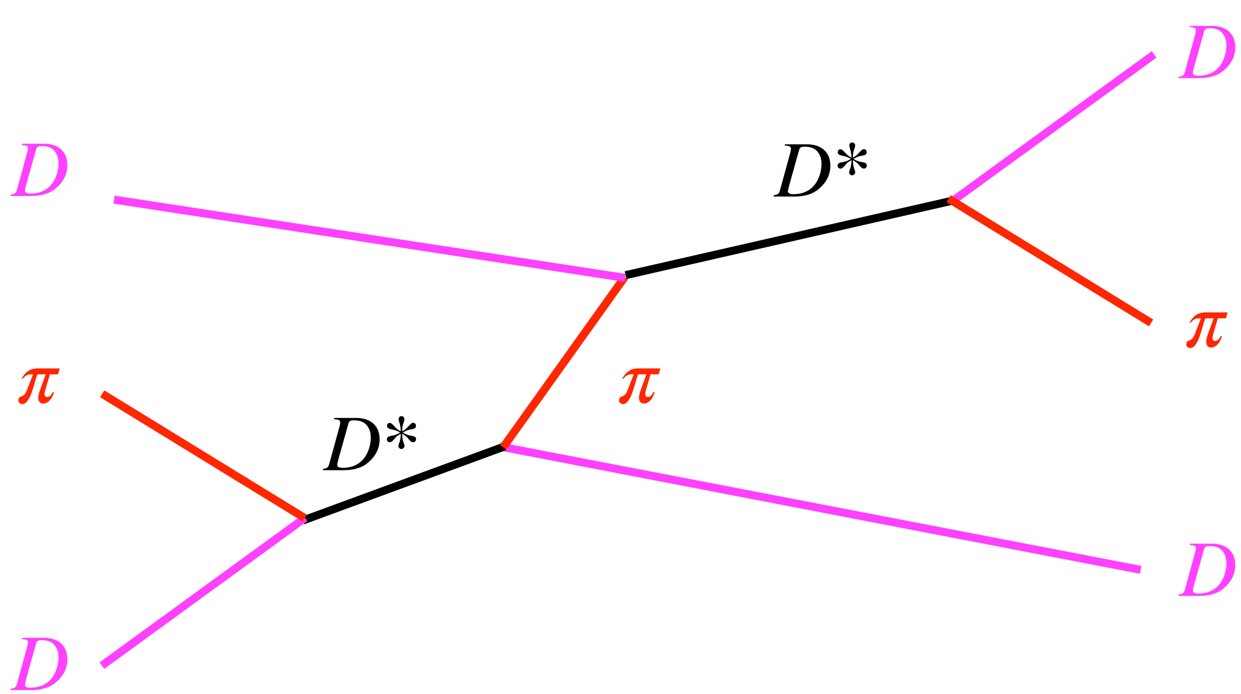
Here we present only the formalism; applications are for future work

Essential idea

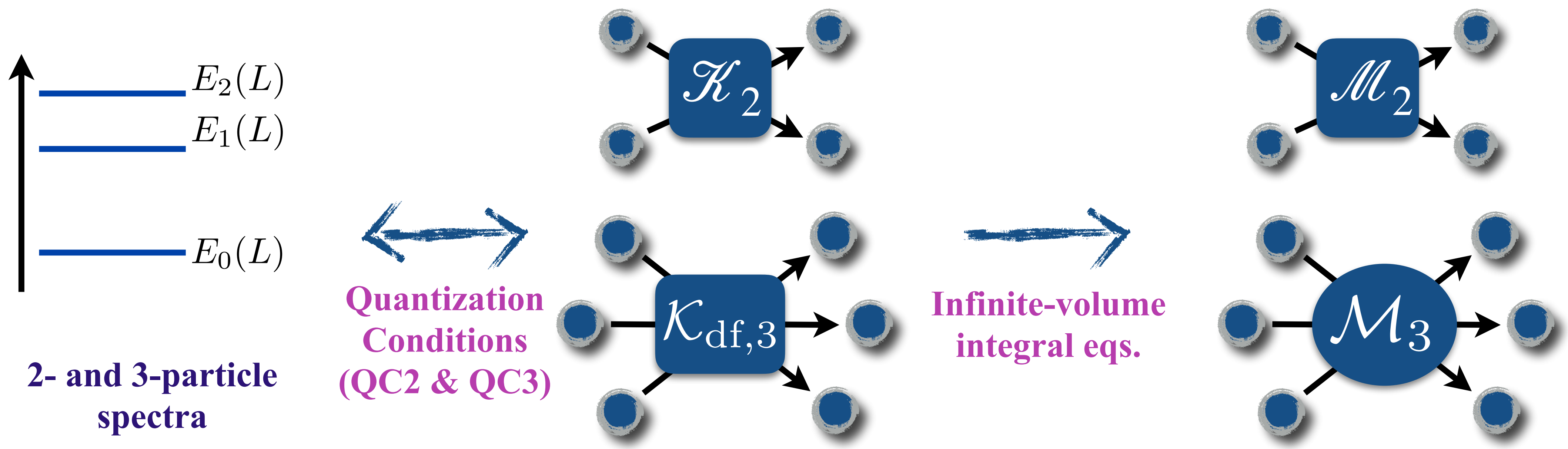
- In TOPT, LH cut arises from intermediate $DD\pi$ state, which is not included in standard analysis



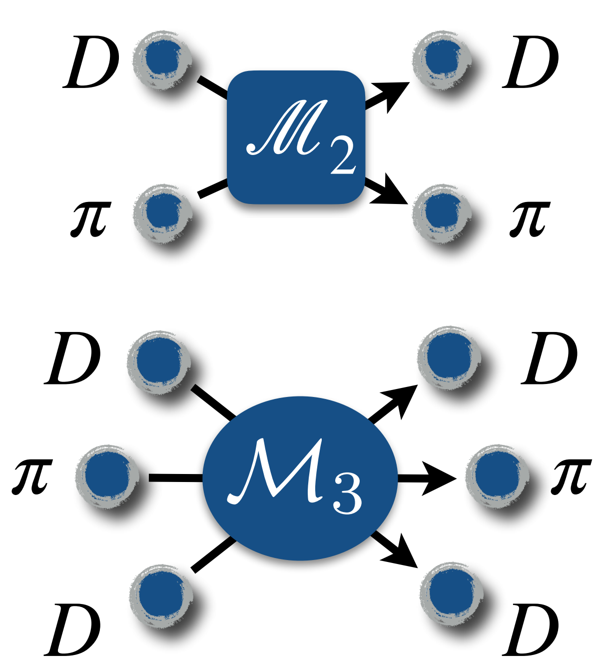
- 3 particle $DD\pi$ formalism does include such a state, and D^* included as $D\pi$ bound state



Workflow of QC3

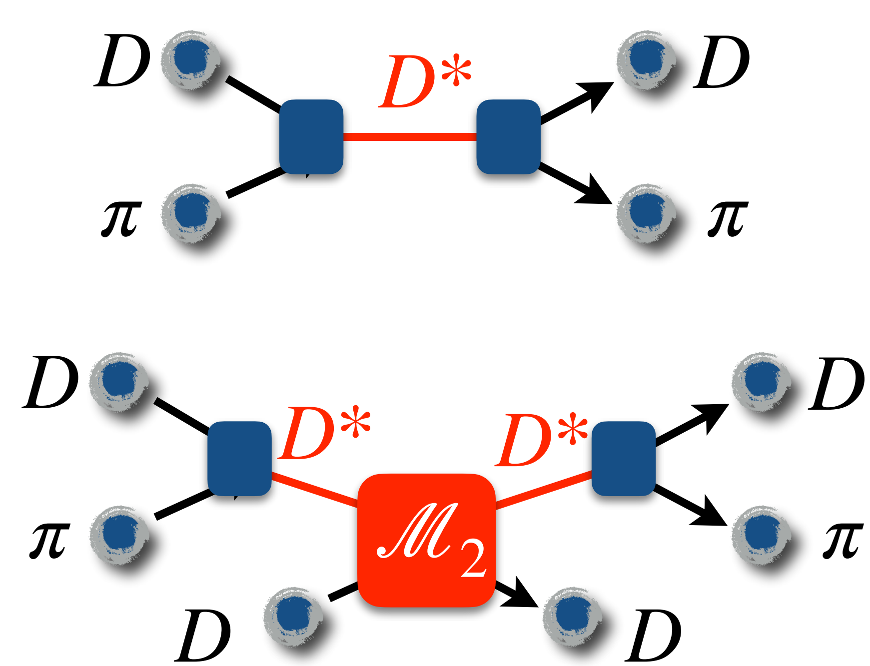


Special to this application:



includes D^* bound state

includes D^*D scattering



Generalization required

- We work in isosymmetric QCD
- Focus on charge 1 states: $D^0D^0\pi^+, D^+D^+\pi^-, D^+D^0\pi^0$, all of which mix
  - Need a combination of "$2+1$" and fully nonidentical QC3s [Blanton & SRS]
    - ✳ 2+1 QC3 has two flavor channels (two choices of spectator)
    - ✳ Fully nonidentical QC3 has three flavor channels
  - Thus might expect that full QC3 requires $2+2+3=7$ channels
  - In fact, our preferred approach has 8 channels, corresponding to symmetric ($I=1$) and antisymmetric ($I=0$) combinations of D^+D^0 in $D^+D^0\pi^0$ state
- Block diagonalizes into different total isospins: $\frac{1}{2}\otimes\frac{1}{2}\otimes 1 = 0\oplus 1\oplus 1\oplus 2$
  - $I=0$ (case of interest for T_{cc}), and $I=2$, have 2-d flavor structure
  - $I=1$ has 4-d flavor structure

Methods of derivation

- Use both TOPT-based method of [Blanton & SRS] and intuitive method based on nontrivial generalization of derivation for 3π of all allowed isospins [Hansen, Romero-López, SRS]
- Former leads to both asymmetric and symmetric forms of QC3, latter only to symmetric form

Results: QC3

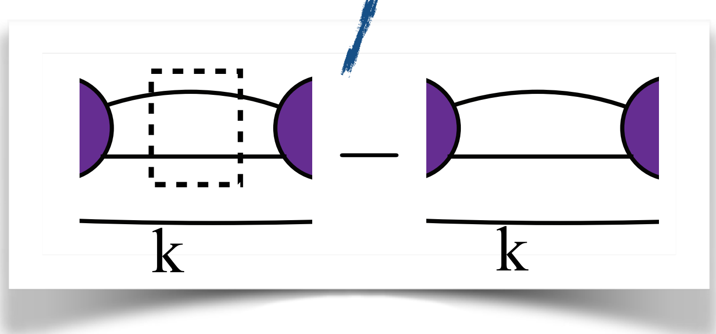
- Symmetric form of QC3 takes the by-now familiar form

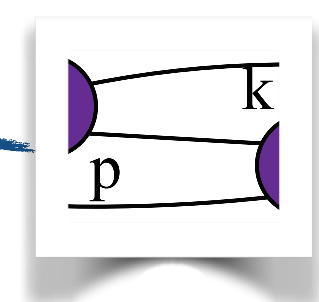
$$\prod_{I\in\{0,1,2\}} \det_{i,\boldsymbol{k},\ell,m}\left[1+\widehat{\mathcal{K}}_{\mathrm{df},3}^{[I]}\;\widehat{F}_3^{[I]}\right]=0$$

$$\widehat{F}_3^{[I]} \equiv \frac{\widehat{F}^{[I]}}{3} - \widehat{F}^{[I]}\frac{1}{1+\widehat{\mathcal{M}}_{2,L}^{[I]}\widehat{G}^{[I]}}\widehat{\mathcal{M}}_{2,L}^{[I]}\widehat{F}^{[I]}, \qquad \widehat{\mathcal{M}}_{2,L}^{[I]} \equiv \frac{1}{\widehat{\mathcal{K}}_{2,L}^{[I]-1}+\widehat{F}^{[I]}}$$

- Focus here on most relevant case: $I = 0$
  - 2-d flavor structure corresponding to $[(D\pi)_{I=1/2}D]_{I=0}$ and $[(DD)_{I=1}\pi]_{I=0}$

$$\widehat{F}^{[I=0]} = \mathrm{diag}\left(\widetilde{F}^D, \widetilde{F}^\pi\right) \qquad \widehat{G}^{[I=0]} = \begin{pmatrix} G^{DD} & \sqrt{2}P_\ell G^{D\pi} \\ \sqrt{2}G^{\pi D}P_\ell & 0 \end{pmatrix}$$





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$$\widehat{F}^{[I]}_3 \equiv \frac{\widehat{F}^{[I]}}{3} - \widehat{F}^{[I]}\frac{1}{1+\widehat{\mathcal{M}}^{[I]}_{2,L}\widehat{G}^{[I]}}\widehat{\mathcal{M}}^{[I]}_{2,L}\widehat{F}^{[I]}, \qquad \widehat{\mathcal{M}}^{[I]}_{2,L} \equiv \frac{1}{\widehat{\mathcal{K}}^{[I]-1}_{2,L}+\widehat{F}^{[I]}}$$

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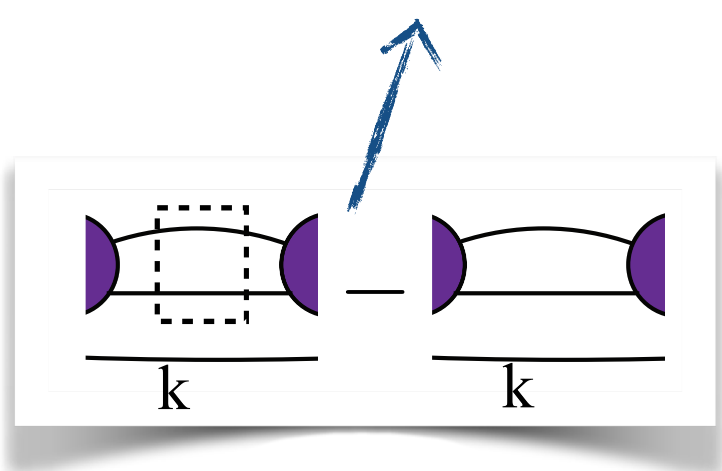

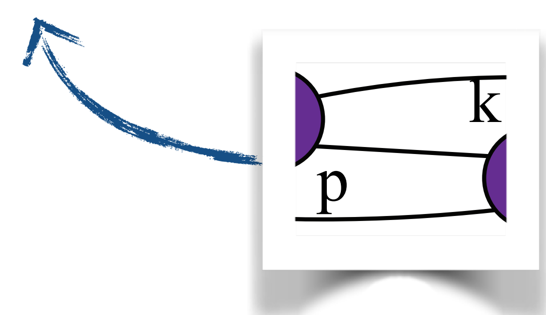

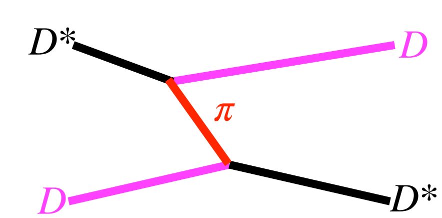

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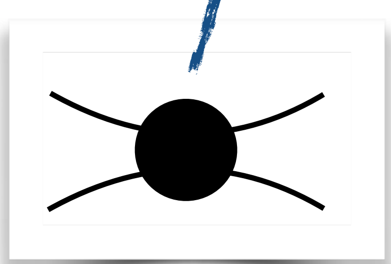
  - 2-d flavor structure corresponding to $[(D\pi)_{I=1/2}D]_{I=0}$ and $[(DD)_{I=1}\pi]_{I=0}$

$$\widehat{\mathcal{K}}_{2,L}^{[I=0]} = \mathrm{diag}\left(\overline{\mathcal{K}}_{2,L}^{D\pi,I=1/2}, \frac{1}{2}\overline{\mathcal{K}}_{2,L}^{DD,I=1}\right)$$



$$\widehat{\mathcal{K}}_{\mathrm{df},3}^{[I=2,0]} = \begin{pmatrix} \left[\mathcal{K}_{\mathrm{df},3}^{[I=2,0]}\right]_{DD} & \frac{1}{\sqrt{2}}\left[\mathcal{K}_{\mathrm{df},3}^{[I=2,0]}\right]_{D\pi} \\ \frac{1}{\sqrt{2}}\left[\mathcal{K}_{\mathrm{df},3}^{[I=2,0]}\right]_{\pi D} & \frac{1}{2}\left[\mathcal{K}_{\mathrm{df},3}^{[I=2,0]}\right]_{\pi\pi} \end{pmatrix}$$

Same 3-particle K matrix expressed in different coordinates

Results: Integral equations

- Obtained by taking infinite-volume limit of a "finite-volume scattering amplitude"

$$\widehat{\mathcal{M}}_3^{[I]} = \lim_{\epsilon\to 0^+} \lim_{L\to\infty} \widehat{\mathcal{M}}_{3,L}^{[I]} \qquad \widehat{\mathcal{M}}_{3,L}^{[I=0,\,2]} = \langle \alpha_{\mathcal{S}} |\, \widehat{\mathcal{M}}_{3,L}^{(u,u),[I=0,\,2]} \,| \alpha_{\mathcal{S}} \rangle \qquad |\alpha_{\mathcal{S}}\rangle = \begin{pmatrix} 2 \\ \sqrt{2} \end{pmatrix}$$

The unsymmetrized finite-volume amplitude is

$$\widehat{\mathcal{M}}_{3,L}^{(u,u),[I]} = \widehat{\mathcal{D}}_{L}^{(u,u),[I]} + \widehat{\mathcal{M}}_{\mathrm{df},3,L}^{(u,u),[I]} \,,$$

and it is composed by the ladder amplitude, which contains pairwise rescattering,

$$\widehat{\mathcal{D}}_{L}^{(u,u),[I]} = -\widehat{\mathcal{M}}_{2,L}^{[I]} \widehat{G}^{[I]} \widehat{\mathcal{M}}_{2,L}^{[I]} \frac{1}{1 + \widehat{G}^{[I]} \widehat{\mathcal{M}}_{2,L}^{[I]}} \,,$$

and a short-distance piece that depends on the three-particle K matrix.

$$\widehat{\mathcal{M}}_{\mathrm{df},3,L}^{(u,u),[I]} = \left[\frac{1}{3} - \widehat{\mathcal{D}}_{23,L}^{(u,u),[I]} \widehat{F}^{[I]} \right] \widehat{\mathcal{K}}_{\mathrm{df},3}^{[I]} \frac{1}{1 + \widehat{F}_3^{[I]} \widehat{\mathcal{K}}_{\mathrm{df},3}^{[I]}} \left[\frac{1}{3} - \widehat{F}^{[I]} \widehat{\mathcal{D}}_{23,L}^{(u,u),[I]} \right] ,$$

where

$$\widehat{\mathcal{D}}_{23,L}^{(u,u),[I]} = \widehat{\mathcal{M}}_{2,L}^{[I]} + \widehat{\mathcal{D}}_{L}^{(u,u),[I]} \,.$$

Implementation for $I = 0$

- QC3 for $[DD\pi]_{I=0}$ is essentially the same as for $K^+K^+\pi^+$, which has been implemented
  - Input needed is spectrum of $(D\pi)_{I=1/2}$ (including D^* if stable), $(DD)_{I=1}$, & $[DD\pi]_{I=0}$ (including D^*D if D^* is stable) states
  - Minimal choice for $\mathscr{K}_2$ is s- and p-waves in $(D\pi)_{I=1/2}$ and s-wave in $(DD)_{I=1}$
  - Use effective-range expansion for $\mathscr{K}_2$ up to terms linear in q^2
    - ✳ Choice for p-wave $(D\pi)_{I=1/2}$ interaction must lead to bound-state pole in $\mathscr{M}_2(D\pi, I = 1/2)^{(\ell=1)}$ at the position found in LQCD simulations
  - Projections onto lattice irreps can be carried over from $K^+K^+\pi^+$
  - Form to use for $\mathscr{K}_{\text{df},3}^{[I=0]}$ is unclear; may require a pole in the $J^P = 1^+$ channel, in which case a form consistent with the symmetries is

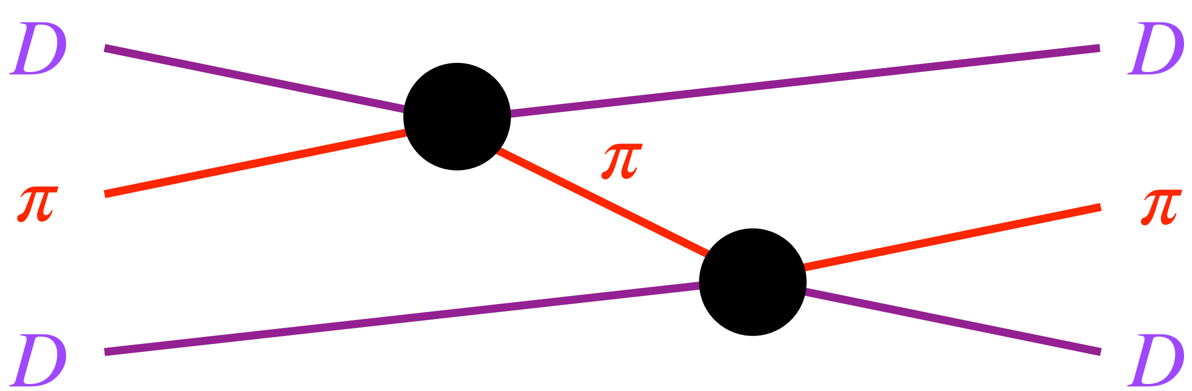
$$\mathcal{K}_{\text{df},3}^{[I=0]} \supset \mathcal{K}_{T_{cc}} \frac{1}{P^2 - M_{T_{cc}}^2} (p_1 + p_2)^\mu (k_1 + k_2)^\nu \left(g_{\mu\nu} - \frac{P_\nu P_\mu}{P^2} \right)$$

- Integral equations are a multichannel generalization of previous work, and also involve a projection onto overall $J^P = 1^+$
  - Under study in collaboration with Sebastian Dawid

Summary & Outlook

- Three-particle formalism generalized to $DD\pi$ for $I = 0,1,2$
    - Allows study of $I = 1$ $T_{cc}(3875)$ including the physics of the LH cut
    - Valid for both physical and heavier-than-physical light-quark masses
- Analysis of LQCD results more complicated than applying two-particle (Lüscher) quantization condition
    - Requires 1-, 2- and 3-particle spectra if D^* is stable
- To extract D^*D amplitude need to solve multichannel integral equations
- Same formalism applies to $BB\pi$ tetraquarks [Bicudo et al., 1505.00613; Francis et al., 1607.05214; Hudspith & Mohler, 2303.17295; Aoki et al., 2306.03565], and to $KK\pi$ systems of general isospin

Thanks
Any questions?



References

RFT 3-particle papers

Max Hansen & SRS:

"Relativistic, model-independent, three-particle quantization condition,"

arXiv:1408.5933 (PRD) [HS14]

"Expressing the 3-particle finite-volume spectrum in terms of the 3-to-3 scattering amplitude,"

arXiv:1504.04028 (PRD) [HS15]

"Perturbative results for 2- & 3-particle threshold energies in finite volume,"

arXiv:1509.07929 (PRD) [HSPT15]

"Threshold expansion of the 3-particle quantization condition,"

arXiv:1602.00324 (PRD) [HSTH15]

"Applying the relativistic quantization condition to a 3-particle bound state in a periodic box,"

arXiv: 1609.04317 (PRD) [HSBS16]

"Lattice QCD and three-particle decays of Resonances,"

arXiv: 1901.00483 (Ann. Rev. Nucl. Part. Science) [HSREV19]

Raúl Briceño, Max Hansen & SRS:

"Relating the finite-volume spectrum and the 2-and-3-particle S-matrix for relativistic systems of identical scalar particles," arXiv:1701.07465 (PRD) [BHS17]

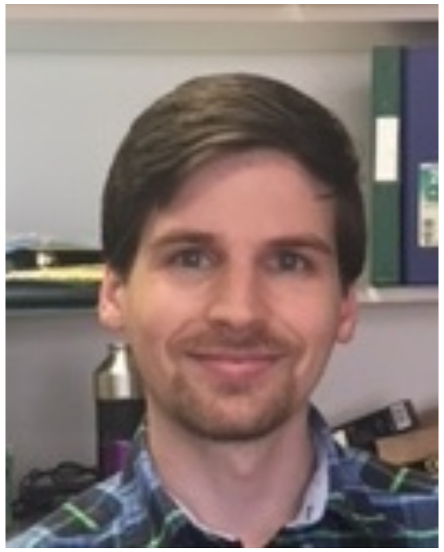
"Numerical study of the relativistic three-body quantization condition in the isotropic approximation," arXiv:1803.04169 (PRD) [BHS18]

"Three-particle systems with resonant sub-processes in a finite volume," arXiv:1810.01429 (PRD 19) [BHS19]

SRS

"Testing the threshold expansion for three-particle energies at fourth order in φ^4 theory," arXiv:1707.04279 (PRD) [SPT17]

Tyler Blanton, Fernando Romero-López & SRS:



"Implementing the three-particle quantization condition including higher partial waves," arXiv:1901.07095 (JHEP) [BRS19]

"I=3 three-pion scattering amplitude from lattice QCD," arXiv:1909.02973 (PRL) [BRS-PRL19]

"Implementing the three-particle quantization condition for $\pi^+\pi^+K^+$ and related systems" 2111.12734 (JHEP)

Tyler Blanton, Raúl Briceño, Max Hansen, Fernando Romero-López, SRS:

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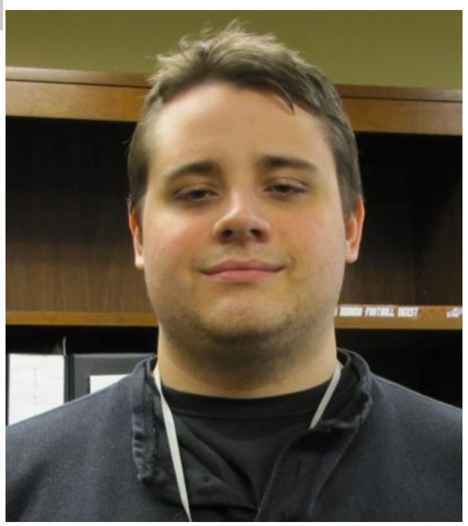
Raúl Briceño, Max Hansen, SRS & Adam Szczepaniak:

"Unitarity of the infinite-volume three-particle scattering amplitude arising from a finite-volume formalism," arXiv:1905.11188 (PRD)



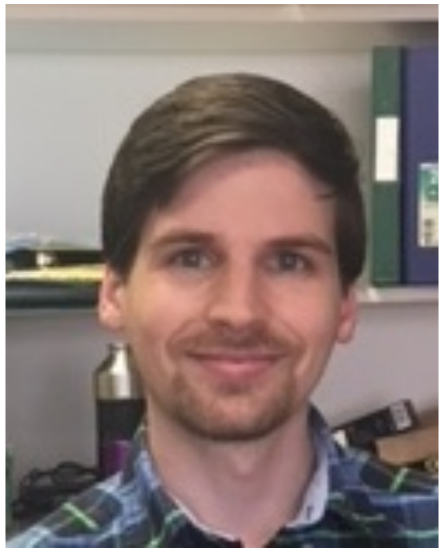
Andrew Jackura, S. Dawid, C. Fernández-Ramírez, V. Mathieu, M. Mikhasenko, A. Pilloni, SRS & A. Szczepaniak:

"On the Equivalence of Three-Particle Scattering Formalisms," arXiv:1905.12007 (PRD)

Max Hansen, Fernando Romero-López, SRS:

"Generalizing the relativistic quantization condition to include all three-pion isospin channels", arXiv:2003.10974 (JHEP) [HRS20]

"Decay amplitudes to three particles from finite-volume matrix elements," arXiv: 2101.10246 (JHEP)



Tyler Blanton & SRS:

"Alternative derivation of the relativistic three-particle quantization condition," arXiv:2007.16188 (PRD) [BS20a]

"Equivalence of relativistic three-particle quantization conditions," arXiv:2007.16190 (PRD) [BS20b]

"Relativistic three-particle quantization condition for nondegenerate scalars," arXiv:2011.05520 (PRD)

"Three-particle finite-volume formalism for $\pi^+\pi^+K^+$ & related systems," arXiv:2105.12904 (PRD)

Tyler Blanton, Drew Hanlon, Ben Hörz, Colin Morningstar, Fernando Romero-López & SRS

"$3\pi^+$ & $3K^+$ interactions beyond leading order from lattice QCD," arXiv:2106.05590 (JHEP)

Zack Draper, Drew Hanlon, Ben Hörz, Colin Morningstar, Fernando Romero-López & SRS

"Interactions of πK, $\pi\pi K$ and $KK\pi$ systems at maximal isospin from lattice QCD," arXiv:2302.13587



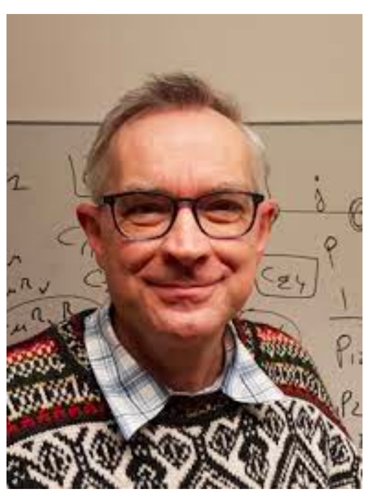


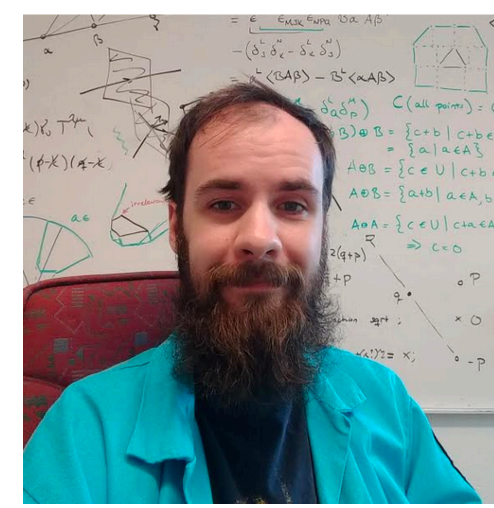
Zach Draper, Max Hansen, Fernando Romero-López & SRS:

“Three relativistic neutrons in a finite volume,”

arXiv:2303.10219

Jorge Baeza-Ballesteros, Johan Bijnens, Tomas Husek, Fernando Romero-López, SRS & Mattias Sjö: **“The isospin-3 three-particle K-matrix at NLO in ChPT,” arXiv:2303.13206**





Other work

★ Implementing RFT integral equations

- M.T. Hansen et al. (HADSPEC), 2009.04931, PRL [Calculating $3\pi^+$ spectrum and using to determine three-particle scattering amplitude]
- A. Jackura et al., 2010.09820, PRD [Solving s-wave RFT integral equations in presence of bound states]
- S. Dawid, Md. Islam and R. Briceño, 2303.04394 [Analytic continuation of 3-particle amplitues]

★ Reviews

- A. Rusetsky, 1911.01253 [LATTICE 2019 plenary]
- M. Mai, M. Döring and A. Rusetsky, 2103.00577 [Review of formalisms and chiral extrapolations]
- F. Romero-López, 2112.05170, [Three-particle scattering amplitudes from lattice QCD]

★ Other numerical simulations

- F. Romero-López, A. Rusetsky, C. Urbach, 1806.02367, JHEP [2- & 3-body interactions in φ^4 theory]
- M. Fischer et al., 2008.03035, Eur.Phys.J.C [$2\pi^+$ & $3\pi^+$ at physical masses]
- M. Garofolo et al., 2211.05605, JHEP [3-body resonances in φ^4 theory]

Other work

★ **NREFT approach**

- H.-W. Hammer, J.-Y. Pang & A. Rusetsky, 1706.07700, JHEP & 1707.02176, JHEP [Formalism & examples]
- M. Döring et al., 1802.03362, PRD [Numerical implementation]
- J.-Y. Pang et al., 1902.01111, PRD [large volume expansion for excited levels]
- F. Müller, T. Yu & A. Rusetsky, 2011.14178, PRD [large volume expansion for I=1 three pion ground state]
- F. Romero-López, A. Rusetsky, N. Schlage & C. Urbach, 2010.11715, JHEP [generalized large-volume exps]
- F. Müller & A. Rusetsky, 2012.13957, JHEP [Three-particle analog of Lellouch-Lüscher formula]
- J-Y. Pang, M. Ebert, H-W. Hammer, F. Müller, A. Rusetsky, 2204.04807, JHEP, [Spurious poles in a finite volume]
- F. Müller, J-Y. Pang, A. Rusetsky, J-J. Wu, 2110.09351, JHEP [Relativistic-invariant formulation of the NREFT three-particle quantization condition]
- J. Lozano, U. Meißner, F. Romero-López, A. Rusetsky & G. Schierholz, 2205.11316, JHEP [Resonance form factors from finite-volume correlation functions with the external field method]
- F. Müller, J-Y. Pang, A. Rusetsky, J-J. Wu, 2211.10126, JHEP [3-particle Lellouch-Lüscher formalism in moving frames
- R. Bubna, F. Müller, A. Rusetsky, 2304.13635 [Finite-volume energy shift of the three-nucleon ground state]

Alternate 3-particle approaches

★ Finite-volume unitarity (FVU) approach

- M. Mai & M. Döring, 1709.08222, EPJA [formalism]
- M. Mai et al., 1706.06118, EPJA [unitary parametrization of M_3 involving R matrix; used in FVU approach]
- A. Jackura et al., 1809.10523, EPJC [further analysis of R matrix parametrization]
- M. Mai & M. Döring, 1807.04746, PRL [3 pion spectrum at finite-volume from FVU]
- M. Mai et al., 1909.05749 ,PRD [applying FVU approach to $3\pi^+$spectrum from Hanlon & Hörz]
- C. Culver et al., 1911.09047, PRD [calculating $3\pi^+$ spectrum and comparing with FVU predictions]
- A. Alexandru et al., 2009.12358, PRD [calculating $3K^-$ spectrum and comparing with FVU predictions]
- R. Brett et al., 2101.06144, PRD [determining $3\pi^+$ interaction from LQCD spectrum]
- M. Mai et al., 2107.03973, PRL [three-body dynamics of the $a_1(1260)$ from LQCD]
- D. Dasadivan et al., 2112.03355, PRD [pole position of $a_1(1260)$ in a unitary framework]

★ HALQCD approach

- T. Doi et al. (HALQCD collab.), 1106.2276, Prog.Theor.Phys. [3 nucleon potentials in NR regime]

Backup slides

Forms of F and G

- Symmetric form of QC3 takes the by-now familiar form

$$\prod_{I\in\{0,1,2\}} \det_{i,\boldsymbol{k},\ell,m} \left[1+\widehat{\mathcal{K}}_{\mathrm{df},3}^{[I]}\ \widehat{F}_3^{[I]}\right]=0$$

$$\widehat{F}_3^{[I]} \equiv \frac{\widehat{F}^{[I]}}{3} - \widehat{F}^{[I]} \frac{1}{1+\widehat{\mathcal{M}}_{2,L}^{[I]}\widehat{G}^{[I]}} \widehat{\mathcal{M}}_{2,L}^{[I]} \widehat{F}^{[I]}, \qquad \widehat{\mathcal{M}}_{2,L}^{[I]} \equiv \frac{1}{\widehat{\mathcal{K}}_{2,L}^{[I]-1}+\widehat{F}^{[I]}}$$

$$\widehat{F}^{[I=0]} = \mathrm{diag}\ \left(\widetilde{F}^{D}, \widetilde{F}^{\pi}\right) \qquad \widehat{G}^{[I=0]} = \begin{pmatrix} G^{DD} & \sqrt{2}P_\ell G^{D\pi} \\ \sqrt{2}G^{\pi D}P_\ell & 0 \end{pmatrix}$$

$$\left[\widetilde{F}^{(i)}\right]_{p'\ell'm';p\ell m} = \delta_{\boldsymbol{p}'\boldsymbol{p}} \frac{H^{(i)}(\boldsymbol{p})}{2\omega_p^{(i)}L^3} \left[\frac{1}{L^3}\sum_{\boldsymbol{a}}^{\mathrm{UV}} - \mathrm{PV}\int^{\mathrm{UV}} \frac{d^3a}{(2\pi)^3}\right] \left[\frac{\mathcal{Y}_{\ell'm'}(\boldsymbol{a}^{*(i,j,p)})}{(q_{2,p'}^{*(i)})^{\ell'}} \frac{1}{4\omega_a^{(j)}\omega_b^{(k)}(E-\omega_p^{(i)}-\omega_a^{(j)}-\omega_b^{(k)})} \frac{\mathcal{Y}_{\ell m}(\boldsymbol{a}^{*(i,j,p)})}{(q_{2,p}^{*(i)})^{\ell}}\right]$$

$$\left[\widetilde{G}^{(ij)}\right]_{p\ell'm';r\ell m} = \frac{1}{2\omega_p^{(i)}L^3} \frac{\mathcal{Y}_{\ell'm'}(\boldsymbol{r}^{*(i,j,p)})}{(q_{2,p}^{*(i)})^{\ell'}} \frac{H^{(i)}(\boldsymbol{p})H^{(j)}(\boldsymbol{r})}{b_{ij}^2-m_k^2} \frac{\mathcal{Y}_{\ell m}(\boldsymbol{p}^{*(j,i,r)})}{(q_{2,r}^{*(j)})^{\ell}} \frac{1}{2\omega_r^{(j)}L^3},$$

where $b_{ij} = (E-\omega_p^{(i)}-\omega_r^{(j)}, \boldsymbol{P}-\boldsymbol{p}-\boldsymbol{r})$.