Comments on non-degenerate staggered fermions, staggered-Wilson and Overlap fermions, and the application of Chiral Perturbation theory to lattice fermions

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Outline

- Using unrooted staggered fermions to simulate 2+2, 2+1+1 and 1+1+1+1 flavors: Is it practical?
- Staggered-Wilson fermions---Adams version
- Staggered-Wilson fermions---Hoelbling-like versions
- Constraining low energy coefficients in ChPT using Weingarten mass inequalities (Flash talk?)
Caveat

• Using unrooted staggered fermions to simulate 2+2, 2+1+1 and 1+1+1+1 flavors: Is it practical?

• Staggered-Wilson fermions---Adams version

• Staggered-Wilson fermions---Hoelbling-like versions

• Constraining low energy coefficients in ChPT using Weingarten mass inequalities

Work very much in progress!
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Staggered fermions

- Very OLD type of fermion! [Susskind, 1976]
- Single component on each site, describing 4 continuum fermions (tastes)
- Computationally efficient
- In practice, each continuum flavor described by a rooted staggered fermion
- Why not instead use 4 tastes to describe u, d, s & c?
- Such non-degenerate staggered fermions discussed long ago by [Golterman & Smit (1984)]
- Take a NEW look at this possibility
Action & Bilinears

\[ S_{\text{unimproved}} = \sum \bar{\chi} (D_{st} + m) \chi = \sum_n \bar{\chi}_n \left[ \sum_{\mu} \eta_\mu (n) \nabla_\mu + m \right] \chi_n \]

- Covariant bilinears transform covariantly under lattice symmetries (unlike hypercube bilinears)

\[ \mathcal{O}^{\text{cov}}_{S \otimes F} = \sum_n \frac{1}{N_\Delta} \sum_{|\Delta|=|S-F|} \bar{\chi}_n (\gamma_S \otimes \xi_F)_{n,n+S-F} U_{n,n+\Delta} \chi_{n+\Delta} \]

\[ \sim a^4 \int d^4 x \quad \bar{Q} (\gamma_S \otimes \xi_F) Q \]

\( S \) \& \( F \) are “hypercube vectors”: \( S_\mu = \{0,1\} \)
\( \Delta \) includes forward \& backward differences (gives “symmetric shifts”),
e.g. \( S-F=(1100) \Rightarrow \Delta=(1100),(1,-1,00),(-1,1,00),(-1,-1,00) \)

\[ (\gamma_S \otimes \xi_F)_{AB} \equiv \frac{1}{4} \text{Tr} \left[ \gamma_A^\dagger \gamma_S \gamma_B \gamma_F^\dagger \right] \quad \gamma_{12} = \gamma_1 \gamma_2, \ldots \]

e.g normal mass term in this notation \( \mathcal{O}^{\text{cov}}_{I \otimes I} = \sum_n \bar{\chi}_n \chi_n \)
Breaking degeneracy

\[ S_{\text{non-degen}} = \sum \bar{\chi} D_{st} \chi + m_D^{\text{ov}} I_{I \otimes \xi_5} + m_A^{\text{ov}} I_{I \otimes \xi_{12}} + m_{12}^{\text{ov}} I_{I \otimes \xi_{34}} \]

- Many choices: use even # links so determinant is real, positive [de Forcrand]
- Use diagonal matrices (Weyl basis)

\[
M = \begin{pmatrix}
1 & 0 \\
0 & 1 \\
\end{pmatrix} + m_A \begin{pmatrix}
-1 & 0 \\
0 & 1 \\
\end{pmatrix} + m_{12} \begin{pmatrix}
\sigma_3 & 0 \\
0 & \sigma_3 \\
\end{pmatrix} + m_{34} \begin{pmatrix}
-\sigma_3 & 0 \\
0 & \sigma_3 \\
\end{pmatrix}
\]

- \( m_{12} = m_{34} = 0 \) gives 2+2 (Adams-type)
- \( m_{12} = \pm m_{34} \) gives 2+1+1 (modified Hoelbling)

- Here we take all lattice masses to be of \( O(a) \), i.e. physical
- Each mass is independently multiplicatively renormalized, with no mixing \([G+S]\)
  - \( \ast \) Follows because in different irreps of lattice symm. group
  - \( \ast \) No fine tuning, only usual tuning
Lattice symmetries \([G+S]\)

\[
S = \sum \bar{\chi} D_{st} \chi + \bar{m} \mathcal{O}_{I \otimes I}^{\text{cov}}
\]

Symmetries: \(\{C_0, \Xi_\mu, R_{\mu\nu}, I_s\} \times U_1^\epsilon\) if \(\bar{m} = 0\)

- Lattice Charge Conjugation
  Acts on spin & taste

- Spatial inversion
  Acts on spin & taste

- Hypercubic rotations
  Act on spin & taste

- Shifts (with e\(^i\pi\) removed)
  \(Q \rightarrow (1 \otimes \xi_\mu)Q\)
  Form \(\Gamma_4\) subgroup

Only small discrete subgroup of SU(4) taste is preserved (and mixed up with spin)
Lattice symmetries (2+2 flavors)

\[ S_A = \sum \bar{\chi} D_{st} \chi + \bar{m} O_{I \otimes I}^{\text{cov}} + m_A O_{I \otimes \xi_5}^{\text{cov}} \]

- Useful properties:

\[ \Xi_\mu : (\gamma_S \otimes \xi_F) \rightarrow (\gamma_S \otimes \xi_\mu \xi_F \xi_\mu) = (\gamma_S \otimes \xi_F)(-)^{\sum_{\nu \neq \mu} F_\nu} \]

\[ \star \text{Adams mass flips sign under all shifts} \]

\[ I_\mu : (\gamma_S \otimes \xi_F) \rightarrow (\gamma_5 \gamma_S \gamma_5 \mu \otimes \xi_\mu 5 \xi_F \xi_5 \mu) = (\gamma_S \otimes \xi_F)(-)^{S_\mu + F_\mu} \]

\[ \star \text{Adams mass flips sign under all axis inversions (and } I_s = I_1 I_2 I_3) \]

\[ \Rightarrow \quad \Xi'_\mu = \Xi_\mu I_\mu \quad \text{are unbroken} \]
Lattice symmetries (2+2 flavors)

\[ S_A = \sum \bar{\chi} D_{st} \chi + \bar{m} O_{I \otimes I}^{\text{cov}} + m_A O_{I \otimes \xi_5}^{\text{cov}} \]

Symmetries:

\[ \{ C_0, \Xi'_\mu, R_{\mu\nu} \} \times U_1^e \]

if \( \bar{m} = m_A = 0 \)

Discrete group is halved in size, with all transformations acting both on spin and taste. Rotation symmetry unchanged.
Lattice symmetries (2+1+1 flavors)

\[ S_{2+1+1} = \sum \bar{\chi} D_{st} \chi + \bar{m} O_{I \otimes I}^{\text{cov}} + m A O_{I \otimes \xi}^{\text{cov}} + m_{1234} i \left[ O_{I \otimes \xi_{12}}^{\text{cov}} + O_{I \otimes \xi_{34}}^{\text{cov}} \right] \]

Symmetries:

\[ \{ C_T, \Xi'_\mu, R_{12}, R_{34}, R_{13} R_{24} \} \times \left( \begin{array}{cc} 0 & 0 \\ 0 & 2\sigma_3 \end{array} \right) \]

if \( \bar{m} = m_A = m_{1234} = 0 \)

Modified C [Misumi]

\[ C_T = R_{21} R_{13}^2 C_0 \]

Compared to \( S_A \), rotational symmetry broken from 192 element \( \text{SW}_4 \) to 64 element subgroup
Lattice symmetries (2+1+1 flavors a la Hoelbling)

\[ S_H = \sum \bar{\chi} D_{st} \chi + \bar{m} O_{I \otimes I}^{\text{cov}} + m_A O_{I \otimes \xi_5}^{\text{cov}} + m_H O_H \]

\[ O_H = i \left[ O_{I \otimes \xi_{12}}^{\text{cov}} + O_{I \otimes \xi_{34}}^{\text{cov}} \right] + i \left[ O_{I \otimes \xi_{13}}^{\text{cov}} + O_{I \otimes \xi_{42}}^{\text{cov}} \right] + i \left[ O_{I \otimes \xi_{14}}^{\text{cov}} + O_{I \otimes \xi_{23}}^{\text{cov}} \right] \]

\[
\begin{pmatrix}
0 & 0 \\
0 & 2\sigma_3
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & -2\sigma_2
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 2\sigma_1
\end{pmatrix}
\]

Symmetries:

\[ \{ C_T, \Xi'_\mu, R_{12}R_{43}, R_{14}R_{32} \} \times U_1^\epsilon \]

if \( \bar{m} = m_A = m_H = 0 \)

Compared to \( S_{2+1+1} \), rotational symmetry group reduced from 64 to 8 elements
Lattice symmetries (1+1+1+1 flavors)

\[ S_{1+1+1+1} = \sum \bar{\chi} D_{st} \chi + \bar{m} \mathcal{O}_{I \otimes I}^{\text{cov}} + m_A \mathcal{O}_{I \otimes \xi_5}^{\text{cov}} + m_{12} i \mathcal{O}_{I \otimes \xi_{12}}^{\text{cov}} + m_{34} i \mathcal{O}_{I \otimes \xi_{34}}^{\text{cov}} \]

Symmetries:

\[ \{ C_T, \Xi'_\mu, R_{12}, R_{34} \} \times U_1^\epsilon \]

if \( \bar{m} = m_A = m_{12} = m_{34} = 0 \)

Compared to \( S_{2+1+1} \), rotational symmetry group reduced from 64 to 16 elements.

Compared to \( S_H \), rotation group here is larger but is Abelian.
Pseudoscalar operators for 2+2 theory

\[ S_A = \sum \bar{\chi} D_{st} \chi + \bar{m}\mathcal{O}_{I \otimes I}^{\text{cov}} + m_A \mathcal{O}_{I \otimes \xi_5}^{\text{cov}} \]

- Consider pseudoscalars at rest
  - In continuum, have light-light, heavy-light and heavy-heavy states
  - On lattice, classify operators by timeslice group
  - For standard staggered this is: \{C_0, \Xi_j, R_{jk}, I_s\}
  - For 2+2 theory it reduces to: \{C_0, \Xi'_j, R_{jk}\}

- Use (1 or 2) timeslice operators with spin-taste: \((\gamma_5 \otimes \xi_F)\)

- Use methods of “toolkit” to classify operators under lattice symmetries [Kilcup & SS]
Pseudoscalar operators for 2+2 theory

- Pion irreps from original staggered theory mix in pairs

\[(\gamma_5 \otimes I) \text{ and } (\gamma_5 \otimes \xi_5) \text{ mix} \quad \text{Create } \bar{l}l \text{ and } \bar{h}h\]

\[(\gamma_5 \otimes \xi_4) \text{ and } (\gamma_5 \otimes \xi_{45}) \text{ mix} \quad \text{Create } \bar{l}h \text{ and } \bar{h}l\]

\[(\gamma_5 \otimes \xi_{j4}) \text{ and } (\gamma_5 \otimes \xi_{j45}) \text{ mix} \quad \text{Create } \bar{\ell}\sigma_j l \text{ and } \bar{h}\sigma_j h \quad \text{States in 3-d irrep}\]

\[(\gamma_5 \otimes \xi_j) \text{ and } (\gamma_5 \otimes \xi_{j5}) \text{ mix} \quad \text{Create } \bar{\ell}\sigma_j h \text{ and } \bar{h}\sigma_j l \quad \text{States in 3-d irrep}\]

Good news: discrete symmetries enough to have 3-d irreps as in continuum
Problems for $2+2$ theory

$(\gamma_5 \otimes I)$ and $(\gamma_5 \otimes \xi_5)$ mix Create $\bar{\ell}\ell$ and $\bar{h}h$

- Both correlators have disconnected contractions
- Cannot separate heavy from light states

$(\gamma_5 \otimes \xi_4)$ and $(\gamma_5 \otimes \xi_{45})$ mix Create $\bar{\ell}h$ and $\bar{h}\ell$

- Cannot separate $l$-bar $h$ and $h$-bar $l$ (lattice induces FCNC!)

$(\gamma_5 \otimes \xi_{j4})$ and $(\gamma_5 \otimes \xi_{j45})$ mix Create $\bar{\ell}\sigma_j\ell$ and $\bar{h}\sigma_jh$ States in 3-d irrep

- Cannot separate heavy and light states

$(\gamma_5 \otimes \xi_j)$ and $(\gamma_5 \otimes \xi_{j5})$ mix Create $\bar{\ell}\sigma_jh$ and $\bar{h}\sigma_j\ell$ States in 3-d irrep

- Cannot separate $l$-bar $h$ and $h$-bar $l$
Generalize to $1+1+1+1$ theory

$$S_{1+1+1+1} = \sum \bar{\chi} D_{st} \chi + m_O^{\text{cov}} + m_A O^{\text{cov}}_I + m_{12} i O^{\text{cov}}_I \otimes \xi_{12} + m_{34} i O^{\text{cov}}_I \otimes \xi_{34}$$

- Timeslice group reduces to $\{C_T, \Xi'_j, R_{12}\}$

$(\gamma_5 \otimes I), (\gamma_5 \otimes \xi_5), (\gamma_5 \otimes \xi_{34})$ and $(\gamma_5 \otimes \xi_{12})$ mix (and have disconnected contractions)

create $\bar{u} u, \bar{d} d, \bar{s} s$ and $\bar{c} c$

$(\gamma_5 \otimes \xi_4), (\gamma_5 \otimes \xi_{45}), (\gamma_5 \otimes \xi_3)$ and $(\gamma_5 \otimes \xi_{35})$ mix

create $\bar{u} s, \bar{d} c, \bar{s} u$ and $\bar{c} d$

$(\gamma_5 \otimes \xi_1), (\gamma_5 \otimes \xi_2), (\gamma_5 \otimes \xi_{15})$ and $(\gamma_5 \otimes \xi_{25})$ mix

create $\bar{u} c, \bar{d} s, \bar{s} d$ and $\bar{c} u$

$(\gamma_5 \otimes \xi_{14}), (\gamma_5 \otimes \xi_{24}), (\gamma_5 \otimes \xi_{13})$ and $(\gamma_5 \otimes \xi_{23})$ mix

create $\bar{u} d, \bar{d} u, \bar{s} c$ and $\bar{c} s$
Outlook for non-degenerate staggered fermions

- Non-degenerate staggered fermions seem impractical
  - Only advantage over Wilson is lack of fine tuning
- Even in simplest case (pseudoscalars at rest) one cannot separately create states with different tastes
  - Similar problem with 2+1+1 twisted mass “solved” in practice using partial quenching
- States in motion, baryons, and weak operators would likely be a nightmare
- Bottom line: too few symmetries
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**General set-up**

- Use same actions as described above, but with lattice masses now of $O(1)$ [physical masses $\sim 1/a$]
  - Requires fine-tuning to obtain light states just as with Wilson fermions

- 2 light-flavor example [Adams]

\[ S_A = \sum \bar{\chi} D_{st} \chi + r (O^\text{cov}_{I \otimes I} - O^\text{cov}_{I \otimes \xi_5}) + m O^\text{cov}_{I \otimes I} \]

- Shift spectrum so one branch is light

- Symmetries as described above, but now cannot treat masses as small, so can have any number of mass insertions

- Only interested in light sector: $\xi_5 = +1$

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S. Sharpe, “Comments on new fermions” 2/17/12 @ Kyoto workshop “New types of fermions on the lattice”
Questions and issues

- What additional terms are allowed in the action?
  
  [Adams] Schematically: \( \bar{\chi} D_{st} (1 \otimes \xi_5) \chi \)
  
  In physical sector, this simply renormalizes kinetic term

- What is the symmetry group in the light sector?

- What is the spectrum of light states?

- What are the impact of discretization errors in pion and vacuum sectors?

- Computational efficiency?

- Utility as kernel for overlap?
Some preliminary answers

• What is the symmetry group in the light sector?

Full group is: \( \{ C_0, \Xi'_\mu, R_{\mu\nu} \} \) (m~1/a implies loss of approximate U_1^\varepsilon)

Proposed method: keep only transformations which take \( \xi_5=1 \) subspace into itself, and drop “heavy part”

Results:

\[ \Xi'_j \Xi'_4 R^2_{j4} = \Xi_j \Xi_4 \sim (1 \otimes \xi_{j4}) \rightarrow (1 \otimes \sigma_j) \]
\[ \epsilon_{jkl} \Xi'_k \Xi'_l R^2_{kl} \sim (1 \otimes \sigma_j) \]
\[ \Xi'_4 R^2_{34} R^2_{12} = \Xi_4 I_s \sim (\gamma_4 \otimes I) = P \]
\[ C_0 \Xi'_2 \Xi'_4 R^2_{24} \sim C_{cont} \]

Rotations \( R_{\mu\nu} \): \( R_{ij} \) and \( R_{k4} \) act simultaneously in spin, space and flavor (as an SU(2) rotation about iso-axis \( k \) by \( \pi/2 \))

• Same result holds for overlap version
Some preliminary answers

- What is the spectrum of pions at rest?

Can use earlier analysis of 2+2 flavor staggered theory, keeping only light-light states

\[ \eta: (\gamma_5 \otimes I) \text{ and } (\gamma_5 \otimes \xi_5) \text{ mix} \quad \text{Create } \bar{\ell} \ell \text{ and } \bar{\ell} \ell^\prime \quad \text{Has disconnected contractions} \]

\[ \pi: (\gamma_5 \otimes \xi_{j4}) \text{ and } (\gamma_5 \otimes \xi_{j45}) \text{ mix} \quad \text{Create } \bar{\ell} \sigma_j \ell \text{ and } h_{j4h} \quad \text{States in 3-d irrep} \]

- Symmetries sufficient to have degenerate pion triplet
- Same holds for overlap version
- Expect symmetry breaking for pions in motion, other mesons and for baryons
Some preliminary answers

• What are the impact of discretization errors in pion and vacuum sectors?

Method adapted from those for Wilson and staggered ChPT [SS & Singleton, Lee & SS]

• Write down all dimension 5 & 6 terms allowed by lattice symmetries; these would be needed to improve the action, and so, without improvement, tell us the form of discretization errors in Symanzik’s continuum effective action

• Project these terms into the physical subspace (new step)

• Map the projected terms into the continuum Symanzik effective action (here for 2 flavors)

• Match the operators into the chiral effective theory

• Analyze their effects (particularly those of SU(2) breaking operators) on the vacuum (e.g. is there an Aoki phase?) and pion spectrum
Very preliminary analysis

Symmetries: \(\{C_0, \Xi'_\mu, R_{\mu\nu}\}\) (no \(U^c_1\))

Allowed operators of dimension 3 & 4

Mass and kinetic terms collapse to standard 2 flavor continuum forms

\[\bar{\psi} \sigma_{\mu\nu} G_{\mu\nu} \psi \]

\(e.g.\) \(\bar{Q}(1 \otimes \xi_5)Q \rightarrow \bar{\psi}\psi\)

Allowed operators of dimension 5

\(e.g.\) for \(\bar{Q}(i\sigma_{\mu\nu} G_{\mu\nu} \otimes \xi_F)Q\)

Only \(\xi_F = I\) and \(\xi_5\) allowed

\[\Rightarrow\) only standard flavor singlet clover term in physical subspace

\[\bar{\psi}i\sigma_{\mu\nu} G_{\mu\nu}\psi\]

\[\Rightarrow\) no flavor breaking at \(O(a)\)
Very preliminary analysis

Allowed operators of dimension 6

• Compared to normal staggered analysis, loss of $U_1^\epsilon$ increases operator count (24 to 35)
• Loss of $I_s$ implies doubling of operators (so 70 in all)
• Examples of new operators:

\[ Q(I \otimes I)Q\bar{Q}(I \otimes \xi_5)Q, \sum_{\mu \neq \mu} \bar{Q}(\gamma_{\mu\nu} \otimes \xi_{\mu\nu})Q\bar{Q}(\gamma_{\mu\nu} \otimes \xi_{\mu\nu})Q \]

In physical subspace:

\[ (\bar{\psi}\psi)^2, \sum_{j \neq k \neq \ell \neq j} (\bar{\psi}\gamma_{jk} \otimes \sigma_\ell \psi)^2, \sum_j (\bar{\psi}\gamma_{4} \otimes \sigma_j \psi)^2 \]

• Also get

\[ \sum_j \bar{\psi}\sigma_j \psi \bar{\psi}\sigma_j \psi \propto (\bar{\psi}\psi)^2 \] (by Fierz)

• Only flavor-breaking occurs in operators with correlated spin and flavor indices
• These do not contribute to potential in ChPT, since require derivatives
• Conclusion: vacuum structure analysis identical to that for Wilson fermions at $O(a^2)$
Many open questions

• Can one understand from ChPT analysis (to all orders) why there is no isospin breaking in the pion states at rest, as deduced from the lattice symmetries?

• Do the symmetries of the resulting chiral Lagrangian match those determined above? (A cross check on the methodology)

• What are the symmetries of the chiral Lagrangian describing the overlap version? Chiral symmetry will add additional constraints, but flavor-breaking should still enter.

• Can one understand how the pion spectrum evolves as $m_A$ and $m$ vary from 0 (usual staggered) to $\sim 1/a$ (Adams)? Must have level crossings since Goldstone pion becomes the $\eta$ in Adams theory. Can analyze using ChPT for $m_A < \Lambda_{QCD}$

• ....
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Perils of fewer symmetries

\[ S_{12} = \sum \bar{\chi} D_{st} \chi + r \left( \mathcal{O}_{I \otimes I}^{\text{cov}} - i \mathcal{O}_{I \otimes \xi_{12}}^{\text{cov}} \right) + m \mathcal{O}_{I \otimes I}^{\text{cov}} \]

\[
\begin{pmatrix}
1 & 0 \\
0 & 1 \\
\end{pmatrix}
- \begin{pmatrix}
\sigma_3 & 0 \\
0 & \sigma_3 \\
\end{pmatrix}
\]

- Gives 2+2 theory, but with different physical subspace from Adams theory
- Has advantage of using 2-link mass term (instead of 4-link)
- Disadvantage is breaking of rotation symmetries

\[ \{ C_0, \Xi'_\mu, R_{\mu \nu} \} \rightarrow \{ C_T, \Xi'_\mu, I_3, R_{12}, R_{34} \} \]
Perils of fewer symmetries

\[ \{C_0, \Xi'_\mu, R_{\mu\nu}\} \longrightarrow \{C_T, \Xi'_\mu, I_3, R_{12}, R_{34}\} \]

- Smaller group allows new kinetic terms (with independent coefficients)
  
  \[(p_1 \gamma_1 + p_2 \gamma_2 \otimes I), \ (p_1 \gamma_1 + p_2 \gamma_2 \otimes i \xi_{12}), \]
  
  \[(p_3 \gamma_3 + p_4 \gamma_4 \otimes I), \text{ and } (p_4 \gamma_3 + p_4 \gamma_4 \otimes i \xi_{12}) \]

- Projecting onto physical 2 flavor subspace (i \( \xi_{12} = +1 \)) gives 2 independent terms:
  
  \[\bar{\psi}(p_1 \gamma_1 + p_2 \gamma_2)\psi \text{ and } \bar{\psi}(p_3 \gamma_3 + p_4 \gamma_4)\psi\]

\[\Rightarrow \mathcal{O}(1) \text{ breaking of rotation invariance in the continuum limit!}\]
$N_f=2$ Hoelbling-like theory

$$S' = \sum \bar{\chi} D_{st} \chi + r i \left[ O^{\text{cov}}_{I \otimes \xi_{12}} + O^{\text{cov}}_{I \otimes \xi_{34}} \right]$$

$$\begin{pmatrix}
0 & 0 \\
0 & 2\sigma_3
\end{pmatrix}$$

- Gives 1+2+1 flavor theory, with 2 flavor subspace at origin
- Variant of proposal of [de Forcrand, Kurkela and Panero]
- Symmetries include new charge-conjugation $C'_T$ [Misumi]
- $C'_T$ forbids mixing with other mass terms $\Rightarrow$ No tuning!
- Spectrum is left-right symmetric if average $\{U, U^*\}$
- Symmetry group: $\{C'_T, \Xi'_\mu, R_{12}, R_{34}, R_{24}R_{31}\}$
- Only kinetic terms consistent with symmetries are: $(p_\mu \gamma_\mu \otimes I)$ and $(p_\mu \gamma_\mu \otimes \xi_5)$
- Both give standard kinetic term when projected onto 2 flavor subspace $\Rightarrow$ Rotational invariance recovered in the continuum limit?
- Same holds for theory with Hoelbling mass, despite smaller symmetry group
Examples from analysis

Symmetry group: \{C'_T, \Xi'_\mu, R_{12}, R_{34}, R_{24}R_{31}\}

\Xi'_\mu \text{ and rotations allow: } ([p_1\gamma_1 + p_2\gamma_2] \otimes \xi_{12}) + ([p_3\gamma_3 + p_4\gamma_4] \otimes \xi_{34})

Forbidden by $C'_T$

\Xi'_\mu \text{ and } C'_T \text{ allow: } (p_1\gamma_{15} \otimes \xi_{15})

Forbidden by $(R_{12})^2$
Gluonic counterterms?

“12+34” Symmetry group: \( \{ C'_T, \Xi'_\mu, R_{12}, R_{34}, R_{24}R_{31} \} \)

\[ G_{12}^2 + G_{34}^2 \text{ and } G_{13}^2 + G_{23}^2 + G_{14}^2 + G_{24}^2 \]

can appear with different coefficients

⇒ Need one gluonic counterterm to restore rotational invariance

“Hoelbling” Symmetry group: \( \{ C'_T, \Xi'_\mu, R_{12}R_{43}, R_{14}R_{32} \} \)

\[ G_{12}^2 + G_{34}^2 \text{ and } G_{13}^2 + G_{24}^2 \text{ and } G_{14}^2 + G_{23}^2 \]

can appear with different coefficients

⇒ Need two gluonic counterterms to restore rotational invariance

David was right!
\( N_f = 1 \) Hoelbling-like theory

\[ S = \sum \bar{\chi} D_{st} \chi + r \left[ i O_{I \otimes \xi_{12}}^{\text{cov}} + i O_{I \otimes \xi_{34}}^{\text{cov}} - 2 O_{I \otimes I}^{\text{cov}} \right] + m O_{I \otimes I}^{\text{cov}} \]

or \( O_H / \sqrt{3} \)

- Shift so that left-hand branch is near origin \( \Rightarrow \) fine tuning
- Symmetries allow mixing with \( O_{I \otimes \xi_5}^{\text{cov}} \)
  \( \Rightarrow \) spectrum not symmetric, but this is not important
- Symmetries allow mixing with rotationally non-invariant kinetic terms, e.g.
  \[ ([p_1 \gamma_1 + p_2 \gamma_2] \otimes \xi_{12}) + ([p_3 \gamma_3 + p_4 \gamma_4] \otimes \xi_{34}) \]
- All such terms reduce to standard kinetic term when projected onto 1-flavor subspace
- Holds for both “12+34” and Hoelbling mass terms (even though latter has smaller symmetry group so more intermediate terms arise)
- Gluonic counterterms will be needed as for \( N_f = 2 \) branches
- Will hold also for overlap versions, since inherit symmetries and subspace

[de Forcrand, Kurkela and Panero]
Outlook for new fermions

- Adams $N_f=2$ theory passes some basic tests
  - Flavor-symmetry breaking enters beyond NNLO in ChPT

- $N_f=1$ and $N_f=2$ Hoelbling-like fermions require gluonic tuning due to breaking of rotation symmetry
  - Counterbalances attractive features (e.g. no mass tuning for $N_f=2$)

- To use in practice, need to understand complications of constructing operators (e.g. for weak matrix elements)?
  - In cases where have mixing with lower-dimension operators, reduced symmetry group may lead to problems

- Is the computational gain sufficient?
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Partially Quenched Wilson ChPT

- $\text{SU}(2)_L \times \text{SU}(2)_R \rightarrow \text{SU}(2+N_V|N_V)_L \times \text{SU}(2+N_V|N_V)_R$

- Construct $L_X$ including $a^2$ effects $[\text{SS & Singleton; Bar, Rupak & Shoresh; Aoki}]

\[
L_0 = \frac{f^2}{4} \langle \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \rangle - \frac{f^2}{4} 2B_0 \langle M^\dagger \Sigma + \Sigma^\dagger M \rangle - \hat{a}^2 W'_6 \langle \Sigma + \Sigma^\dagger \rangle^2 - \hat{a}^2 W'_7 \langle (\Sigma - \Sigma^\dagger)^2 \rangle - \hat{a}^2 W'_8 \langle \Sigma^2 + (\Sigma^\dagger)^2 \rangle \\
\Sigma \in \text{SU}(2+N_V|N_V)
\]

- Phase structure (Aoki vs. first-order) determined by

\[
c_2 = -8 \hat{a}^2 (2W'_6 + W'_8)
\]

- Can one constrain the signs of the low-energy coefficients (LECs)?
Can signs of LECs be predicted?

- General issue in effective field theories
- Sometimes can use causality [Pham & Truong, A. Adams et al.]
  - Doesn’t apply here
- Hermiticity argument from $\varepsilon$-regime study in $W$ChPT implies $W_8'<0$ [Akemann, Damgaard, Splittorff & Verbaarschot]
  - Important question: Is this argument correct?
- Another recent method is to use QCD mass inequalities to constrain LECs [Bar, Golterman & Shamir]
- In [Hansen & SS, arxiv:1111.2404] we derived $W_8'<0$, by calculating a PQ pion mass in ChPT and comparing to constraint from Weingarten-like mass inequalities