Comments on non-degenerate staggered fermions, staggered-Wilson and Overlap fermions, and the application of Chiral Perturbation theory to lattice fermions

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Outline

- Using unrooted staggered fermions to simulate 2+2, 2+1+1 and 1+1+1+1 flavors: Is it practical?
- Staggered-Wilson fermions---Adams version
- Staggered-Wilson fermions---Hoelbling-like versions
- Constraining low energy coefficients in ChPT using Weingarten mass inequalities (Flash talk?)

Caveat

- Using unrooted staggered fermions to simulate 2+2, 2+1+1 and 1+1+1+1 flavors: Is it practical?
- Staggered-Wilson fermions---Adams version
- Staggered-Wilson fermions---Hoelbling-like versions
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Work very much in progress!

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Staggered fermions

- Very OLD type of fermion! [Susskind, 1976]
- Single component on each site, describing 4 continuum fermions (tastes)
- Computationally efficient
- In practice, each continuum flavor described by a rooted staggered fermion
- Why not instead use 4 tastes to describe u,d,s & c?
- Such non-degenerate staggered fermions discussed long ago by [Golterman & Smit (1984)]
- Take a NEW look at this possibility

Action & Bilinears

$$S_{\text{unimproved}} = \sum \bar{\chi}(D_{\text{st}} + m)\chi = \sum_n \bar{\chi}_n \left[\sum_\mu \eta_\mu(n)\nabla_\mu + m \right] \chi_n$$

- Covariant bilinears transform covariantly under lattice symmetries (unlike hypercube bilinears)

$$\mathcal{O}^{\text{cov}}_{S\otimes F} = \sum_n \frac{1}{N_\Delta} \sum_{|\Delta|=|S-F|} \bar{\chi}_n \overline{(\gamma_S \otimes \xi_F)}_{n,n+S-F} U_{n,n+\Delta} \chi_{n+\Delta}$$

$$\sim a^4 \int d^4x\, \bar{Q}(\gamma_S \otimes \xi_F) Q$$

S & F are "hypercube vectors": $S_\mu=\{0,1\}$
Δ includes forward & backward differences (gives "symmetric shifts"),
e.g. S-F=(1100)⇒Δ=(1100),(1,-1,00),(-1,1,00),(-1,-1,00)

$$\overline{(\gamma_S \otimes \xi_F)}_{AB} \equiv \frac{1}{4} \mathrm{Tr}\left[\gamma_A^\dagger \gamma_S \gamma_B \gamma_F^\dagger \right] \qquad \gamma_{12} = \gamma_1\gamma_2, \ldots$$

e.g normal mass term in this notation $\mathcal{O}^{\text{cov}}_{I\otimes I} = \sum_n \bar{\chi}_n \chi_n$

Breaking degeneracy

$$S_{\mathrm{non-degen}} = \sum \bar{\chi} D_{st} \chi + \underset{\text{0-link}}{\bar{m} \mathcal{O}^{\mathrm{cov}}_{I\otimes I}} + m_A \underbrace{\mathcal{O}^{\mathrm{cov}}_{I\otimes \xi_5}}_{\Gamma_{55}\Gamma_5} + \underset{\text{2-link}}{m_{12} i \mathcal{O}^{\mathrm{cov}}_{I\otimes \xi_{12}}} + \underset{\text{2-link}}{m_{34} i \mathcal{O}^{\mathrm{cov}}_{I\otimes \xi_{34}}}$$

4-link

- Many choices: use even # links so determinant is real, positive [de Forcrand]
- Use diagonal matrices (Weyl basis)

$$M = \bar{m} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + m_A \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + m_{12} \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} + m_{34} \begin{pmatrix} -\sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

- $m_{12}=m_{34}=0$ gives 2+2 (Adams-type)
- $m_{12}=\pm m_{34}$ gives 2+1+1 (modified Hoelbling)

- Here we take all lattice masses to be of O(a), i.e. physical
- Each mass is independently multiplicatively renormalized, with no mixing [G+S]
  - ✳ Follows because in different irreps of lattice symm. group
  - ✳ No fine tuning, only usual tuning

Lattice symmetries [G+S]

$$S = \sum \bar\chi D_{st}\chi + \bar m \mathcal{O}^{\rm cov}_{I\otimes I}$$

Symmetries: $\{C_0, \Xi_\mu, R_{\mu\nu}, I_s\} \times \underbrace{U_1^\epsilon}_{\text{if } \bar m=0}$

Lattice Charge Conjugation
Acts on spin & taste

Shifts (with e^{ip} removed)
$Q \to (1\otimes\xi_\mu)Q$
Form Γ_4 subgroup

Hypercubic rotations
Act on spin & taste

Spatial inversion
Acts on spin & taste

Only small discrete subgroup of SU(4) taste is preserved (and mixed up with spin)

Lattice symmetries (2+2 flavors)

$$S_A = \sum \bar{\chi} D_{st} \chi + \bar{m} \mathcal{O}^{\rm cov}_{I\otimes I} + m_A \mathcal{O}^{\rm cov}_{I\otimes \xi_5}$$

- Useful properties:

$$\Xi_\mu: \ (\gamma_S \otimes \xi_F) \rightarrow (\gamma_S \otimes \xi_\mu \xi_F \xi_\mu) = (\gamma_S \otimes \xi_F)(-)^{\sum_{\nu\neq\mu} F_\nu}$$

✳ Adams mass flips sign under all shifts

$$I_\mu: \ (\gamma_S \otimes \xi_F) \rightarrow (\gamma_{\mu 5} \gamma_S \gamma_{5\mu} \otimes \xi_{\mu 5} \xi_F \xi_{5\mu}) = (\gamma_S \otimes \xi_F)(-)^{S_\mu + F_\mu}$$

✳ Adams mass flips sign under all axis inversions (and $I_s = I_1 I_2 I_3$)

$$\Rightarrow \ \Xi'_\mu = \Xi_\mu I_\mu$$ are unbroken

Lattice symmetries (2+2 flavors)

$$S_A = \sum \bar{\chi} D_{st} \chi + \bar{m} \mathcal{O}^{\rm cov}_{I\otimes I} + m_A \mathcal{O}^{\rm cov}_{I\otimes \xi_5}$$

Symmetries:

$$\{C_0, \Xi'_\mu, R_{\mu\nu}\} \times \underbrace{U_1^\epsilon}_{\text{if } \bar{m}=m_A=0}$$

Discrete group is halved in size, with all transformations acting both on spin and taste. Rotation symmetry unchanged.

Lattice symmetries (2+1+1 flavors)

$$S_{2+1+1} = \sum \bar{\chi} D_{st} \chi + \bar{m} \mathcal{O}^{\rm cov}_{I\otimes I} + m_A \mathcal{O}^{\rm cov}_{I\otimes\xi_5} + m_{1234}\, \underbrace{i \left[\mathcal{O}^{\rm cov}_{I\otimes\xi_{12}} + \mathcal{O}^{\rm cov}_{I\otimes\xi_{34}}\right]}_{\begin{pmatrix} 0 & 0 \\ 0 & 2\sigma_3 \end{pmatrix}}$$

Symmetries:

$$\{C_T, \Xi'_\mu, R_{12}, R_{34}, R_{13}R_{24}\} \times \underbrace{U_1^\epsilon}_{\text{if } \bar{m}=m_A=m_{1234}=0}$$

Modified C [Misumi]

$$C_T = R_{21} R_{13}^2 C_0$$

Compared to S_A, rotational symmetry broken from 192 element SW_4 to 64 element subgroup

Lattice symmetries (2+1+1 flavors a la Hoelbling)

$$S_H = \sum \bar{\chi} D_{st} \chi + \bar{m} \mathcal{O}^{\rm cov}_{I\otimes I} + m_A \mathcal{O}^{\rm cov}_{I\otimes\xi_5} + m_H \mathcal{O}_H$$

$$\mathcal{O}_H = i \underbrace{\left[\mathcal{O}^{\rm cov}_{I\otimes\xi_{12}} + \mathcal{O}^{\rm cov}_{I\otimes\xi_{34}}\right]}_{\begin{pmatrix} 0 & 0 \\ 0 & 2\sigma_3 \end{pmatrix}} + i \underbrace{\left[\mathcal{O}^{\rm cov}_{I\otimes\xi_{13}} + \mathcal{O}^{\rm cov}_{I\otimes\xi_{42}}\right]}_{\begin{pmatrix} 0 & 0 \\ 0 & -2\sigma_2 \end{pmatrix}} + i \underbrace{\left[\mathcal{O}^{\rm cov}_{I\otimes\xi_{14}} + \mathcal{O}^{\rm cov}_{I\otimes\xi_{23}}\right]}_{\begin{pmatrix} 0 & 0 \\ 0 & 2\sigma_1 \end{pmatrix}}$$

Symmetries:

$$\{C_T, \Xi'_\mu, R_{12}R_{43}, R_{14}R_{32}\} \times \underbrace{U_1^\epsilon}_{\text{if } \bar{m}=m_A=m_H=0}$$

Compared to S_{2+1+1}, rotational symmetry group reduced from 64 to 8 elements

Lattice symmetries (1+1+1+1 flavors)

$$S_{1+1+1+1} = \sum \bar\chi D_{st}\chi + \bar m \mathcal{O}^{\rm cov}_{I\otimes I} + m_A \mathcal{O}^{\rm cov}_{I\otimes \xi_5} + m_{12} i\mathcal{O}^{\rm cov}_{I\otimes \xi_{12}} + m_{34} i\mathcal{O}^{\rm cov}_{I\otimes \xi_{34}}$$

Symmetries:

$$\{C_T, \Xi'_\mu, R_{12}, R_{34}\} \times \underbrace{U_1^\epsilon}_{\text{if } \bar m = m_A = m_{12} = m_{34} = 0}$$

Compared to S_{2+1+1}, rotational symmetry group reduced from 64 to 16 elements.
Compared to S_H, rotation group here is larger but is Abelian.

Pseudoscalar operators for 2+2 theory

$$S_A = \sum \bar\chi D_{st}\chi + \bar m \mathcal{O}^{\rm cov}_{I\otimes I} + m_A \mathcal{O}^{\rm cov}_{I\otimes\xi_5}$$

- Consider pseudoscalars at rest
  - ✳ In continuum, have light-light, heavy-light and heavy-heavy states
  - ✳ On lattice, classify operators by timeslice group
  - ✳ For standard staggered this is: $\{C_0, \Xi_j, R_{jk}, I_s\}$
  - ✳ For 2+2 theory it reduces to: $\{C_0, \Xi'_j, R_{jk}\}$
- Use (1 or 2) timeslice operators with spin-taste: $(\gamma_5 \otimes \xi_F)$
- Use methods of "toolkit" to classify operators under lattice symmetries [Kilcup & SS]

Pseudoscalar operators for 2+2 theory

- Pion irreps from original staggered theory mix in pairs

$(\gamma_5 \otimes I)$ and $(\gamma_5 \otimes \xi_5)$ mix   Create $\bar{\ell}\ell$ and $\bar{\mathbf{h}}\mathbf{h}$

$(\gamma_5 \otimes \xi_4)$ and $(\gamma_5 \otimes \xi_{45})$ mix   Create $\bar{\ell}\mathbf{h}$ and $\bar{\mathbf{h}}\ell$

$(\gamma_5 \otimes \xi_{j4})$ and $(\gamma_5 \otimes \xi_{j45})$ mix   Create $\bar{\ell}\sigma_{\mathbf{j}}\ell$ and $\bar{\mathbf{h}}\sigma_{\mathbf{j}}\mathbf{h}$ States in 3-d irrep

$(\gamma_5 \otimes \xi_j)$ and $(\gamma_5 \otimes \xi_{j5})$ mix   Create $\bar{\ell}\sigma_{\mathbf{j}}\mathbf{h}$ and $\bar{\mathbf{h}}\sigma_{\mathbf{j}}\ell$ States in 3-d irrep

Good news: discrete symmetries enough to have 3-d irreps as in continuum

Problems for 2+2 theory

$(\gamma_5 \otimes I)$ and $(\gamma_5 \otimes \xi_5)$ mix Create $\bar{\ell}\ell$ and $\bar{\mathbf{h}}\mathbf{h}$

- Both correlators have disconnected contractions
- Cannot separate heavy from light states

$(\gamma_5 \otimes \xi_4)$ and $(\gamma_5 \otimes \xi_{45})$ mix Create $\bar{\ell}\mathbf{h}$ and $\bar{\mathbf{h}}\ell$

- Cannot separate l-bar h and h-bar l (lattice induces FCNC!)

$(\gamma_5 \otimes \xi_{j4})$ and $(\gamma_5 \otimes \xi_{j45})$ mix Create $\bar{\ell}\sigma_{\mathbf{j}}\ell$ and $\bar{\mathbf{h}}\sigma_{\mathbf{j}}\mathbf{h}$ States in 3-d irrep

- Cannot separate heavy and light states

$(\gamma_5 \otimes \xi_j)$ and $(\gamma_5 \otimes \xi_{j5})$ mix Create $\bar{\ell}\sigma_{\mathbf{j}}\mathbf{h}$ and $\bar{\mathbf{h}}\sigma_{\mathbf{j}}\ell$ States in 3-d irrep

- Cannot separate l-bar h and h-bar l

Generalize to 1+1+1+1 theory

$$S_{1+1+1+1} = \sum \bar{\chi} D_{st} \chi + \bar{m} \mathcal{O}^{\rm cov}_{I\otimes I} + m_A \mathcal{O}^{\rm cov}_{I\otimes \xi_5} + m_{12} i \mathcal{O}^{\rm cov}_{I\otimes \xi_{12}} + m_{34} i \mathcal{O}^{\rm cov}_{I\otimes \xi_{34}}$$

- Timeslice group reduces to $\{C_T, \Xi'_j, R_{12}\}$

$(\gamma_5 \otimes I), (\gamma_5 \otimes \xi_5), (\gamma_5 \otimes \xi_{34})$ and $(\gamma_5 \otimes \xi_{12})$ mix (and have disconnected contractions)

create $\bar{u}u, \bar{d}d, \bar{s}s$ and $\bar{c}c$

$(\gamma_5 \otimes \xi_4), (\gamma_5 \otimes \xi_{45}), (\gamma_5 \otimes \xi_3)$ and $(\gamma_5 \otimes \xi_{35})$ mix

create $\bar{u}s, \bar{d}c, \bar{s}u$ and $\bar{c}d$

$(\gamma_5 \otimes \xi_1), (\gamma_5 \otimes \xi_2), (\gamma_5 \otimes \xi_{15})$ and $(\gamma_5 \otimes \xi_{25})$ mix

create $\bar{u}c, \bar{d}s, \bar{s}d$ and $\bar{c}u$

$(\gamma_5 \otimes \xi_{14}), (\gamma_5 \otimes \xi_{24}), (\gamma_5 \otimes \xi_{13})$ and $(\gamma_5 \otimes \xi_{23})$ mix

create $\bar{u}d, \bar{d}u, \bar{s}c$ and $\bar{c}s$

Outlook for non-degenerate staggered fermions

- Non-degenerate staggered fermions seem impractical
  - Only advantage over Wilson is lack of fine tuning
- Even in simplest case (pseudoscalars at rest) one cannot separately create states with different tastes
  - Similar problem with 2+1+1 twisted mass "solved" in practice using partial quenching
- States in motion, baryons, and weak operators would likely be a nightmare
- Bottom line: too few symmetries

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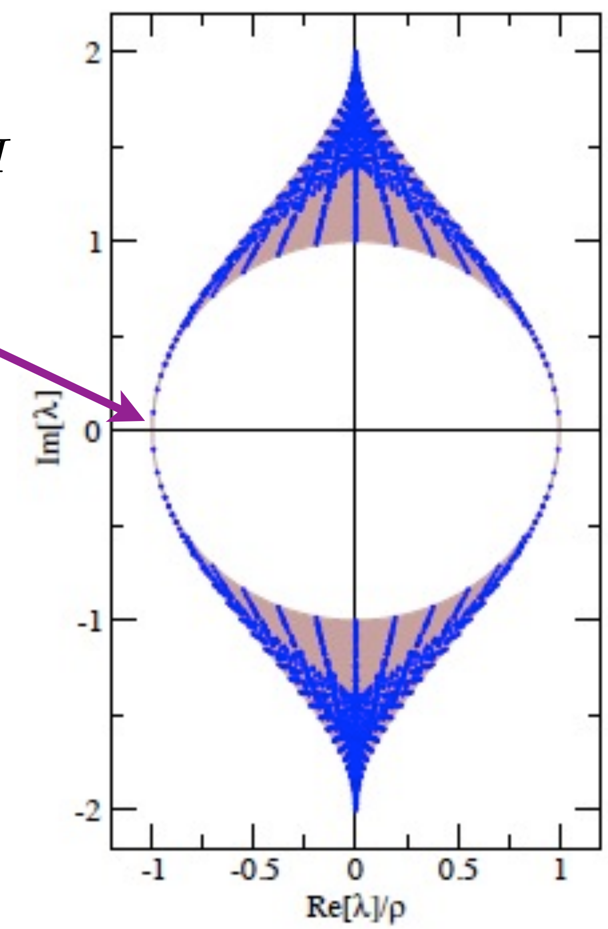
General set-up

- Use same actions as described above, but with lattice masses now of O(1) [physical masses ~1/a]
    - Requires fine-tuning to obtain light states just as with Wilson fermions
- 2 light-flavor example [Adams]

[de Forcrand et al, 2012]

$$S_A = \sum \bar{\chi} D_{st} \chi + r\left(\mathcal{O}^{\rm cov}_{I\otimes I} - \mathcal{O}^{\rm cov}_{I\otimes \xi_5}\right) + m\mathcal{O}^{\rm cov}_{I\otimes I}$$

Shift spectrum so one branch is light



- Symmetries as described above, but now cannot treat masses as small, so can have any number of mass insertions
- Only interested in light sector: ξ_5=+1

Questions and issues

- What additional terms are allowed in the action?

  [Adams] $\mathbf{Schematically:}\ \bar{\chi}\mathbf{D_{st}}(\mathbf{1}\otimes\xi_5)\chi$

  In physical sector, this simply renormalizes kinetic term

- What is the symmetry group in the light sector?
- What is the spectrum of light states?
- What are the impact of discretization errors in pion and vacuum sectors?
- Computational efficiency?
- Utility as kernel for overlap?

Some preliminary answers

- What is the symmetry group in the light sector?

Full group is: $\{C_0, \Xi'_\mu, R_{\mu\nu}\}$ (m~1/a implies loss of approximate U_1^ϵ)

Proposed method: keep only transformations which take ξ_5=1 subspace into itself, and drop "heavy part"

Results:

$$\Xi'_j \Xi'_4 R^2_{j4} = \Xi_j \Xi_4 \sim (1 \otimes \xi_{j4}) \to (1 \otimes \sigma_j)$$

$$\epsilon_{jkl} \Xi'_k \Xi'_l R^2_{kl} \sim (1 \otimes \sigma_j)$$

$$\Xi'_4 R^2_{34} R^2_{12} = \Xi_4 I_s \sim (\gamma_4 \otimes I) = P$$

$$C_0 \Xi'_2 \Xi'_4 R^2_{24} \sim C_{\rm cont}$$

Rotations $R_{\mu\nu}$: R_{ij} and R_{k4} act simultaneously in spin, space and flavor (as an SU(2) rotation about iso-axis k by π/2)

- Same result holds for overlap version

Some preliminary answers

- What is the spectrum of pions at rest?

Can use earlier analysis of 2+2 flavor staggered theory, keeping only light-light states

η: $(\gamma_5 \otimes I)$ and $(\gamma_5 \otimes \xi_5)$ mix Create $\bar{\ell}\ell$ and $\cancel{\bar{h}h}$ Has disconnected contractions

π: $(\gamma_5 \otimes \xi_{j4})$ and $(\gamma_5 \otimes \xi_{j45})$ mix Create $\bar{\ell}\sigma_j\ell$ and $\cancel{\bar{h}\sigma_j h}$ States in 3-d irrep

- Symmetries sufficient to have degenerate pion triplet
- Same holds for overlap version
- Expect symmetry breaking for pions in motion, other mesons and for baryons

Some preliminary answers

- What are the impact of discretization errors in pion and vacuum sectors?

Method adapted from those for Wilson and staggered ChPT [SS & Singleton, Lee & SS]

- Write down all dimension 5 & 6 terms allowed by lattice symmetries; these would be needed to improve the action, and so, without improvement, tell us the form of discretization errors in Symanzik's continuum effective action
- Project these terms into the physical subspace (new step)
- Map the projected terms into the continuum Symanzik effective action (here for 2 flavors)
- Match the operators into the chiral effective theory
- Analyze their effects (particularly those of SU(2) breaking operators) on the vacuum (e.g. is there an Aoki phase?) and pion spectrum

Very preliminary analysis

$$\textbf{Symmetries}: \{C_0, \Xi'_\mu, R_{\mu\nu}\}\ (\textbf{no}\ U_1^\epsilon)$$

Allowed operators of dimension 3 & 4

Mass and kinetic terms collapse to standard 2 flavor continuum forms on ξ_5=1 subspace

$$\text{e.g. } \bar{Q}(1 \otimes \xi_5)Q \longrightarrow \bar{\psi}\psi$$

Allowed operators of dimension 5

e.g. for $\bar{Q}(i\sigma_{\mu\nu}G_{\mu\nu} \otimes \xi_F)Q$ **Only** $\xi_F = I$ **and** ξ_5 **allowed**

⇒ only standard flavor singlet clover term in physical subspace

$$\bar{\psi}i\sigma_{\mu\nu}G_{\mu\nu}\psi$$

⇒ no flavor breaking at O(a)

Very preliminary analysis

Allowed operators of dimension 6

- Compared to normal staggered analysis, loss of U_I^ϵ increases operator count (24 to 35)
- Loss of I_s implies doubling of operators (so 70 in all)
- Examples of new operators:

$$\bar{Q}(I\otimes I)Q\bar{Q}(I\otimes\xi_5)Q, \quad \sum_{\mu\neq\mu}\bar{Q}(\gamma_{\mu\nu}\otimes\xi_{\mu\nu})Q\bar{Q}(\gamma_{\mu\nu}\otimes\xi_{\mu\nu})Q$$

In physical subspace:

$$(\bar\psi\psi)^2 \qquad \sum_{j\neq k\neq \ell\neq j}(\bar\psi\gamma_{jk}\otimes\sigma_\ell\psi)^2 \qquad \sum_j(\bar\psi\gamma_{j4}\otimes\sigma_j\psi)^2$$

- Also get $\displaystyle\sum_j\bar\psi\sigma_j\psi\bar\psi\sigma_j\psi\propto(\bar\psi\psi)^2$ (by Fierz)
- Only flavor-breaking occurs in operators with correlated spin and flavor indices
- These do not contribute to potential in ChPT, since require derivatives
- Conclusion: vacuum structure analysis identical to that for Wilson fermions at $O(a^2)$

Many open questions

- Can one understand from ChPT analysis (to all orders) why there is no isospin breaking in the pion states at rest, as deduced from the lattice symmetries?
- Do the symmetries of the resulting chiral Lagrangian match those determined above? (A cross check on the methodology)
- What are the symmetries of the chiral Lagrangian describing the overlap version? Chiral symmetry will add additional constraints, but flavor-breaking should still enter.
- Can one understand how the pion spectrum evolves as m_A and m vary from 0 (usual staggered) to ~1/a (Adams)? Must have level crossings since Goldstone pion becomes the η in Adams theory. Can analyze using ChPT for $m_A < \Lambda_{QCD}$
- ....

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Perils of fewer symmetries

$$S_{12} = \sum \bar{\chi} D_{\rm st} \chi + r \underbrace{\left(\mathcal{O}^{\rm cov}_{I\otimes I} - i\mathcal{O}^{\rm cov}_{I\otimes \xi_{12}}\right)}_{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}} + m\mathcal{O}^{\rm cov}_{I\otimes I}$$

- Gives 2+2 theory, but with different physical subspace from Adams theory
- Has advantage of using 2-link mass term (instead of 4-link)
- Disadvantage is breaking of rotation symmetries

$$\{C_0, \Xi'_\mu, R_{\mu\nu}\} \longrightarrow \{C_T, \Xi'_\mu, I_3, R_{12}, R_{34}\}$$

Perils of fewer symmetries

$$\{C_0, \Xi'_\mu, R_{\mu\nu}\} \longrightarrow \{C_T, \Xi'_\mu, I_3, R_{12}, R_{34}\}$$

- Smaller group allows new kinetic terms (with independent coefficients)

$$(p_1\gamma_1 + p_2\gamma_2 \otimes I),\ (p_1\gamma_1 + p_2\gamma_2 \otimes i\xi_{12}),$$
$$(p_3\gamma_3 + p_4\gamma_4 \otimes I),\ \text{and}\ (p_4\gamma_3 + p_4\gamma_4 \otimes i\xi_{12})$$

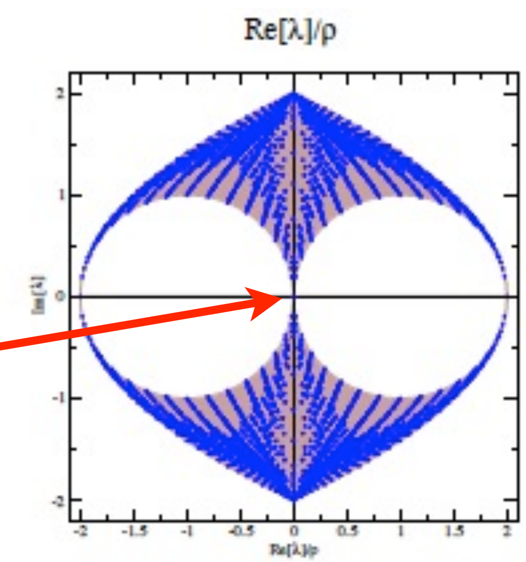
- Projecting onto physical 2 flavor subspace ($i\ \xi_{12}$=+1) gives 2 independent terms:

$$\bar\psi(p_1\gamma_1 + p_2\gamma_2)\psi \text{ and } \bar\psi(p_3\gamma_3 + p_4\gamma_4)\psi$$

$\boxed{\Rightarrow \text{O(1) breaking of rotation invariance in the continuum limit!}}$

N_f=2 Hoelbling-like theory

$$S = \sum \bar{\chi} D_{\rm st} \chi + r\, i \underbrace{\left[\mathcal{O}^{\rm cov}_{I\otimes\xi_{12}} + \mathcal{O}^{\rm cov}_{I\otimes\xi_{34}}\right]}_{\begin{pmatrix} 0 & 0 \\ 0 & 2\sigma_3 \end{pmatrix}}$$



- Gives 1+2+1 flavor theory, with 2 flavor subspace at origin
- Variant of proposal of [de Forcrand, Kurkela and Panero]
- Symmetries include new charge-conjugation C'$_T$ [Misumi]
- C'$_T$ forbids mixing with other mass terms ➡No tuning!
- Spectrum is left-right symmetric if average {U, U*}
- Symmetry group: $\{C'_T, \Xi'_\mu, R_{12}, R_{34}, R_{24}R_{31}\}$
- Only kinetic terms consistent with symmetries are: $(p_\mu\gamma_\mu \otimes I)$ and $(p_\mu\gamma_\mu \otimes \xi_5)$
- Both give standard kinetic term when projected onto 2 flavor subspace

➡ Rotational invariance recovered in the continuum limit?

- Same holds for theory with Hoelbling mass, despite smaller symmetry group

Examples from analysis

Symmetry group: $\{C'_T, \Xi'_\mu, R_{12}, R_{34}, R_{24}R_{31}\}$

Ξ'_μ and rotations allow : $([p_1\gamma_1 + p_2\gamma_2] \otimes \xi_{12}) + ([p_3\gamma_3 + p_4\gamma_4] \otimes \xi_{34})$

Forbidden by C'_T

Ξ'_μ and C'_T allow : $(p_1\gamma_{15} \otimes \xi_{15})$

Forbidden by $(R_{12})^2$

Gluonic counterterms?

"12+34" Symmetry group: $\{C'_T, \Xi'_\mu, R_{12}, R_{34}, R_{24}R_{31}\}$

$G_{12}^2 + G_{34}^2$ and $G_{13}^2 + G_{23}^2 + G_{14}^2 + G_{24}^2$ can appear with different coefficients

➡ Need one gluonic counterterm to restore rotational invariance

"Hoelbling" Symmetry group: $\{C'_T, \Xi'_\mu, R_{12}R_{43}, R_{14}R_{32}\}$

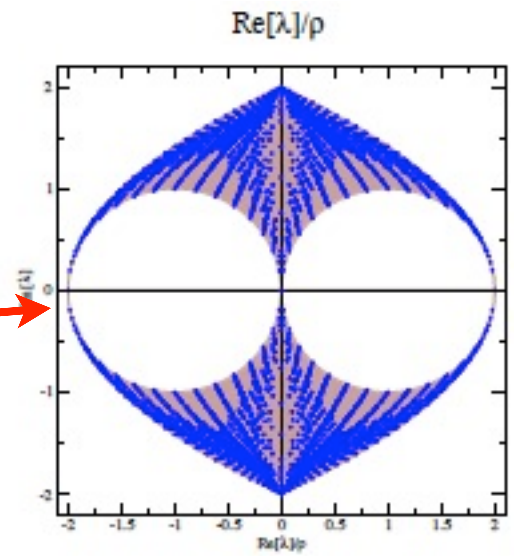
$G_{12}^2 + G_{34}^2$ and $G_{13}^2 + G_{24}^2$ and $G_{14}^2 + G_{23}^2$ can appear with different coefficients

➡ Need two gluonic counterterms to restore rotational invariance

David was right!

N_f=1 Hoelbling-like theory

$$S = \sum \bar\chi D_{st}\chi + r\Big[\underbrace{i\mathcal{O}^{\rm cov}_{I\otimes\xi_{12}} + i\mathcal{O}^{\rm cov}_{I\otimes\xi_{34}}}_{\text{or }\ \mathcal{O}_H/\sqrt{3}} - 2\mathcal{O}^{\rm cov}_{I\otimes I}\Big] + m\mathcal{O}^{\rm cov}_{I\otimes I}$$



[de Forcrand, Kurkela and Panero]

- Shift so that left-hand branch is near origin ⇒ fine tuning
- Symmetries allow mixing with $\mathcal{O}^{\rm cov}_{I\otimes\xi_5}$

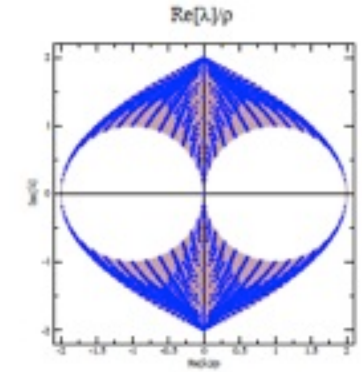
  ➡ spectrum not symmetric, but this is not important
- Symmetries allow mixing with rotationally non-invariant kinetic terms, e.g.

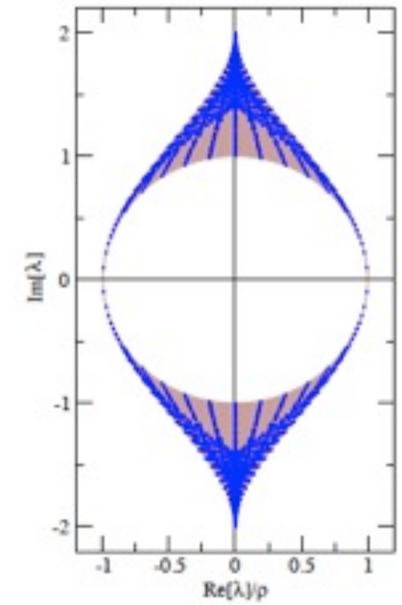
$$([p_1\gamma_1 + p_2\gamma_2]\otimes\xi_{12}) + ([p_3\gamma_3 + p_4\gamma_4]\otimes\xi_{34})$$

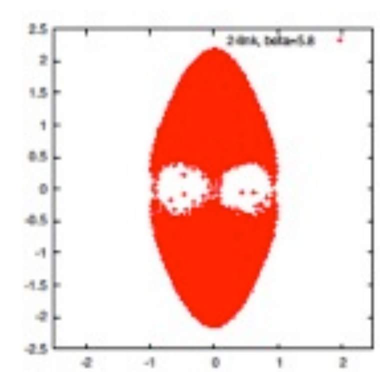
- All such terms reduce to standard kinetic term when projected onto 1-flavor subspace
- Holds for both "12+34" and Hoelbling mass terms (even though latter has smaller symmetry group so more intermediate terms arise)
- Gluonic counterms will be needed as for N_f=2 branches
- Will hold also for overlap versions, since inherit symmetries and subspace

Outlook for new fermions

- Adams N_f=2 theory passes some basic tests
  - Flavor-symmetry breaking enters beyond NNLO in ChPT
- N_f=1 and N_f=2 Hoelbling-like fermions require gluonic tuning due to breaking of rotation symmetry
  - Counterbalances attractive features (e.g. no mass tuning for N_f=2)
- To use in practice, need to understand complications of constructing operators (e.g. for weak matrix elements)?
  - In cases where have mixing with lower-dimension operators, reduced symmetry group may lead to problems
- Is the computational gain sufficient?







Outline

- Using unrooted staggered fermions to simulate 2+2, 2+1+1 and 1+1+1+1 flavors: Is it practical?
- Staggered-Wilson fermions---Adams version
- Staggered-Wilson fermions---Hoelbling-like versions
- Constraining low energy coefficients in ChPT using Weingarten mass inequalities (flash talk?)

Partially Quenched Wilson ChPT

- $SU(2)_L \times SU(2)_R \rightarrow SU(2+N_V|N_V)_L \times SU(2+N_V|N_V)_R$
- Construct L_χ including a^2 effects [SS & Singleton; Bar, Rupak & Shoresh; Aoki]

$$\mathcal{L}_0 = \frac{f^2}{4}\langle \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \rangle - \frac{f^2}{4} 2B_0 \langle M^\dagger \Sigma + \Sigma^\dagger M \rangle$$
$$- \hat{a}^2 W_6' \langle \Sigma + \Sigma^\dagger \rangle^2 - \hat{a}^2 W_7' \langle (\Sigma - \Sigma^\dagger)^2 \rangle - \hat{a}^2 W_8' \langle \Sigma^2 + (\Sigma^\dagger)^2 \rangle$$

$\Sigma \in SU(2+N_V|N_V)$

Supertrace

- Phase structure (Aoki vs. first-order) determined by

$$c_2 = -8\hat{a}^2(2W_6' + W_8')$$

- Can one constrain the signs of the low-energy coefficients (LECs)?

Can signs of LECs be predicted?

- General issue in effective field theories
- Sometimes can use causality [Pham & Truong, A. Adams et al.]
  - Doesn't apply here
- Hermiticity argument from ε-regime study in WChPT implies $W_8'<0$ [Akemann, Damgaard, Splittorff & Verbaarschot]
  - Important question: Is this argument correct?
- Another recent method is to use QCD mass inequalities to constrain LECs [Bar, Golterman & Shamir]
- In [Hansen & SS, arxiv:1111.2404] we derived $W_8'<0$, by calculating a PQ pion mass in ChPT and comparing to constraint from Weingarten-like mass inequalities