Using chiral perturbation theory to study Wilson(-like) fermions: Introduction, a small proposal & open questions

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Outline

- Brief introduction to Wilson fermions & associated discretization errors
- Determining the low-energy effective theory
- Predictions for phase diagram
- Summary of application to eigenvalue spectrum
- A proposal for determination of all three $O(a^2)$ partially quenched low-energy coefficients
- Some open questions

Wilson fermions

- [Wilson, 1974] resolved fermion doubling problem by adding irrelevant term to lattice action

$$S_W = \sum_n \bar\psi_n \left[\underbrace{m_0}_{am} + \gamma_\mu \underbrace{\frac{\nabla_\mu + \nabla^*_\mu}{2}}_{a\partial_\mu} - \underbrace{\frac{\nabla^*_\mu \nabla_\mu}{2}}_{a^2\Box/2} \right] \psi_n$$

$$\sim \int_x \bar\psi(m + \gamma_\mu\partial_\mu - a\Box/2)\psi$$

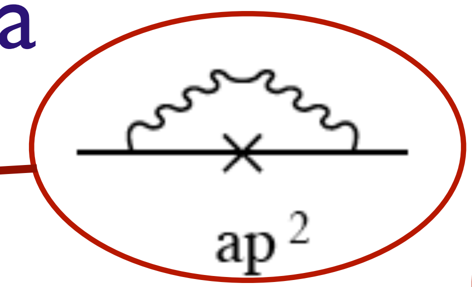
- Violates chiral symmetry
  - ➡ Additive mass renormalization, challenge to simulate at low masses, ...
- ✦ Maintains flavor symmetry
  - ➡ 1 continuum fermion for each lattice fermion (cf. staggered fermions)

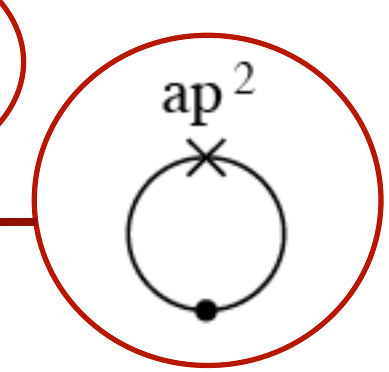
Improved Wilson fermions

- Wilson fermions have O(a) discretization errors
- Improved Wilson action, with additional term having coefficient determined non-perturbatively, has O(a^2) errors [Symanzik, Alpha Collaboration]
- Twisted-mass Wilson fermions also have O(a^2) errors & some advantages [Frezzotti & Rossi, ETM Collab.]
- All large-scale simulations use such improved fermions, but I won't show you the actions because
- Most of what I discuss today is insensitive to the choice of action

Implications of χSB

- Irrelevant term mixes with lower-dimension operators in presence of cut-off 1/a
  - ➡ Additive mass renormalization: $\delta m \sim \alpha_s/a$
  - ➡ Condensate diverges: $\bar{\psi}\psi \sim 1/a^3$
  - ➡ Chiral Ward-Takahashi identities violated





- In most cases, problems overcome by clever methods, allowing Wilson-like fermions to remain highly competitive for phenomenological applications

- Nevertheless, residual $O(a)$ or $O(a^2)$ errors need to be understood and minimized

Impact of mass renormalization (1)

- Need to fine-tune lattice quark mass

$$\delta m \sim \frac{\alpha_s}{a} \gg m_{\rm phys} \quad \Leftrightarrow \quad \delta m_0 = \delta(am) \sim \alpha_s \gg am_{\rm phys}$$

- Can do so non-perturbatively

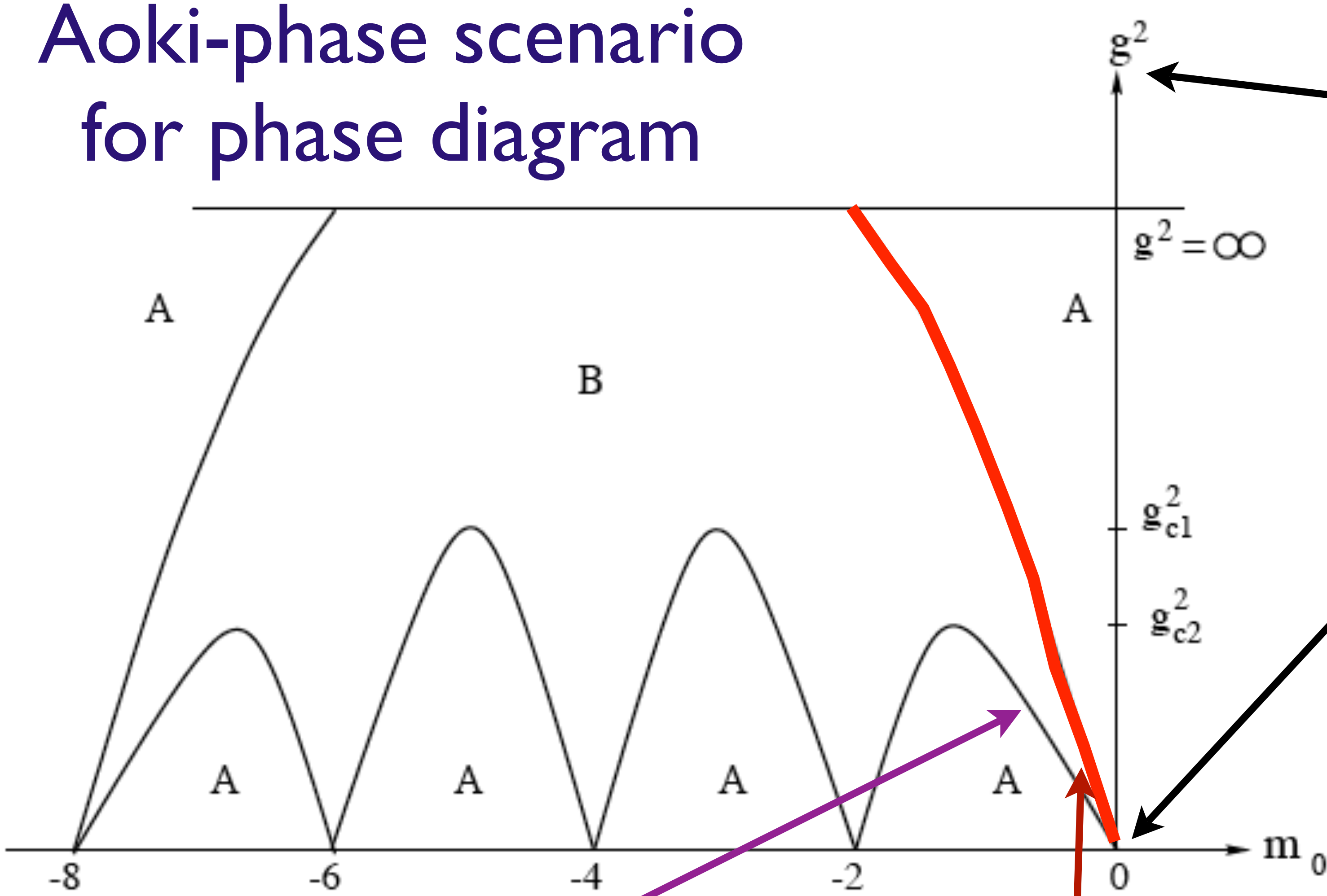
➡ Determine $m_{0,c}$ for which $M_\pi \to 0$ (or, more technically, $m_{PCAC} \to 0$)

- Renormalized quark mass given by:

$$m = Z_S^{-1} \frac{m_0 - m_{0,c}}{a}$$

Impact of mass renormalization (2)

Aoki-phase scenario for phase diagram



gauge coupling

continuum limit

lattice mass

Phase structure caused by discretization effects: discussed later

Critical line along which M_π=0

Size of discretization errors

- Compare physical quark masses

$$\frac{m_u+m_d}{2} \approx 3.5 \text{ MeV}$$

- to discretization errors with an unimproved action (Λ_{QCD}=300 MeV, a=0.07fm)

$$a\Lambda_{\rm QCD}^2 \approx 30 \text{ MeV}$$

- and to discretization errors with an improved action

$$a^2\Lambda_{\rm QCD}^3 \approx 3 \text{ MeV}$$

Need improved actions, and must assume that χSB due to mass & due to discretization errors are comparable: m~a^2

Strategy

- Although Wilson-term is an irrelevant operator, it can impact IR physics because it explicitly breaks a spontaneously broken symmetry
- Can thus impact vacuum alignment and properties of psuedo-Goldstone bosons (PGBs)
- Need to incorporate discretization errors into description of IR physics of QCD

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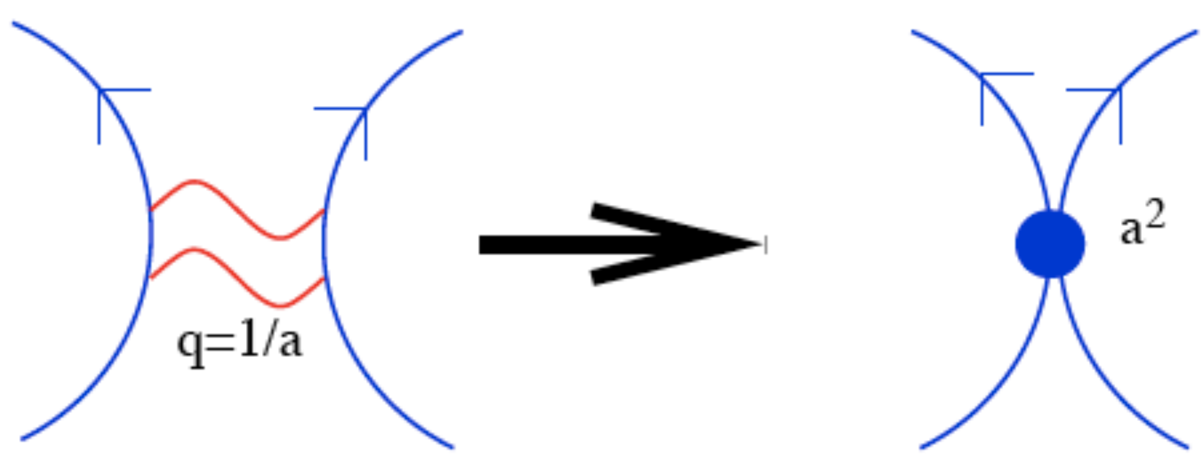
Symanzik effective theory

[Symanzik, 1975, 1983]

- Low energy effective theory for lattice QCD
- Describes quarks & gluons with $\Lambda_{QCD} << p << 1/a$
- Integrate out d.o.f. with p~1/a
- Obtain continuum action with corrections having explicit powers of a [& implicit factors of log(a)]



Symanzik effective theory

$$\mathcal{L}_{\rm Sym} = \mathcal{L}_{\rm QCD} + a\mathcal{L}^{(5)} + a^2\mathcal{L}^{(6)} + \dots$$

- After field redefinitions: [Luscher et al., 96]

$$\mathcal{L}^{(5)} = b_1 \bar\psi i\sigma_{\mu\nu}F_{\mu\nu}\psi + b_4 m \mathcal{L}_{\rm glue} + b_5 m^2 \bar\psi\psi$$

- And, showing only examples of important terms:

[Luscher & Weisz, 85; Sheikholeslami & Wohlert, 85; Bar, Rupak & Shoresh, 04]

$$\mathcal{L}^{(6)} \sim \mathcal{L}^{(6)}_{\rm glue} + (\bar\psi\psi)^2 + (\bar\psi\gamma_\mu\psi)(\bar\psi\gamma_\mu\psi) + \dots$$

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Absent if use NP O(a) improved action, but this has little effect on subsequent analysis

- And, showing only examples of important terms:

[Luscher & Weisz, 85; Sheikholeslami & Wohlert, 85; Bar, Rupak & Shoresh, 04]

$$\mathcal{L}^{(6)} \sim \mathcal{L}^{(6)}_{\rm glue} + (\bar\psi\psi)^2 + (\bar\psi\gamma_\mu\psi)(\bar\psi\gamma_\mu\psi) + \dots$$

Power counting

- Second step is to map quark operators into the chiral effective theory
- Work in "large cut-off effects" (LCE) regime a.k.a. "Aoki regime": $m \sim a^2$

LO: m, p^2, a^2

NLO: am, ap^2, a^3

NNLO: $m^2, mp^2, p^4, a^2m, a^2p^2, a^4$

<u>Lattice artifacts</u>
No term linear in a
even if action unimproved
We will see why!

These are NLO in usual continuum
chiral power counting

Power counting

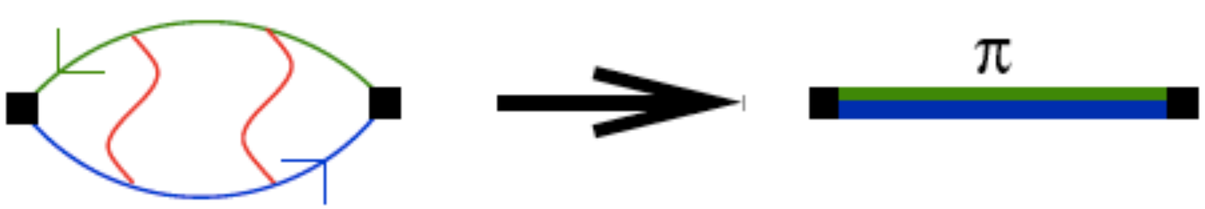
$$\mathcal{L}_{\rm Sym} = \underbrace{\mathcal{L}_{\rm QCD}}_{\text{LO }(m,p^2)} + a\mathcal{L}^{(5)} + a^2\mathcal{L}^{(6)} + \dots$$

$$a\mathcal{L}^{(5)} = \underbrace{ab_1\bar\psi i\sigma_{\mu\nu}F_{\mu\nu}\psi}_{O(a)\,\to\,\text{LO}(m)} + \underbrace{ab_4 m\mathcal{L}_{\rm glue} + b_5 a m^2\bar\psi\psi}_{\text{NNNLO}}$$

$$a^2\mathcal{L}^{(6)} \sim \underbrace{a^2\mathcal{L}^{(6)}_{\rm glue}}_{\text{NNLO}} + \underbrace{a^2(\bar\psi\psi)^2 + a^2(\bar\psi\gamma_\mu\psi)(\bar\psi\gamma_\mu\psi)}_{\text{LO }(a^2)} + \dots$$

Chiral effective theory

- Due to spontaneous chiral symmetry breaking, long distance d.o.f. are pseudo-Goldstone bosons
- Can describe vacuum, PGB interactions, and correlators involving currents and densities using chiral effective theory: ChPT [Weinberg, Gasser&Leutwyler]
- SU(2) ChPT very successful (SU(3) ChPT less so)
  - Consider only two degenerate flavors here
- Mapping from QCD to ChPT is non-perturbative



“Wilson ChPT” (WChPT)

[SS & Singleton]

- Mapping to ChPT works as for QCD, except there are now additional quark-level operators

$$\bar\psi\gamma_\mu D_\mu\psi \longrightarrow \frac{f^2}{4}\langle\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\rangle \qquad \boxed{\mathrm{LO}(p^2)}$$

$$\bar\psi_L M\psi_R + \bar\psi_R M^\dagger\psi_L \longrightarrow -\frac{f^2}{4}2B_0\langle M^\dagger\Sigma + \Sigma^\dagger M\rangle \qquad \boxed{\mathrm{LO}(m)}$$

$$ab_1\bar\psi i\sigma_{\mu\nu}F_{\mu\nu}\psi \longrightarrow -\frac{f^2}{4}\underbrace{2W_0a}_{\hat a}\langle\Sigma+\Sigma^\dagger\rangle \qquad \boxed{\mathrm{O}(a)?}$$

$$(b_1\ \text{term})^2 + a^2(\bar\psi\psi)^2 + \ldots \longrightarrow \frac{c_2}{16}\langle\Sigma+\Sigma^\dagger\rangle^2 \qquad \boxed{\mathrm{LO}(a^2)}$$

f, B_0, W_0 & c_2 are unknown low energy constants (LECs)

$$\Sigma = \exp(\sqrt{2}i\pi/f) \to L\Sigma R^\dagger \quad \text{with} \quad \Sigma, L, R \in \mathrm{SU}(2)$$

WChPT at LO

- O(a) term can be absorbed by shift in M

$$M \to M + \frac{\hat{a}}{2B_0} \;\Rightarrow\; 2B_0\langle M^\dagger\Sigma + \Sigma^\dagger M\rangle + \hat{a}\langle\Sigma + \Sigma^\dagger\rangle \to 2B_0\langle M^\dagger\Sigma + \Sigma^\dagger M\rangle$$

- LO chiral Lagrangian in WChPT is thus ($2B_0 M = \chi\mathbf{1}$)

$$\mathcal{L}_\chi^{(LO)} = \frac{f^2}{4}\langle\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\rangle - \frac{f^2}{4}\chi\langle\Sigma + \Sigma^\dagger\rangle + \frac{c_2}{16}\langle\Sigma + \Sigma^\dagger\rangle^2$$

- Discretization errors introduce a single new LEC at LO ⇒ O(a^2) effects in different quantities related

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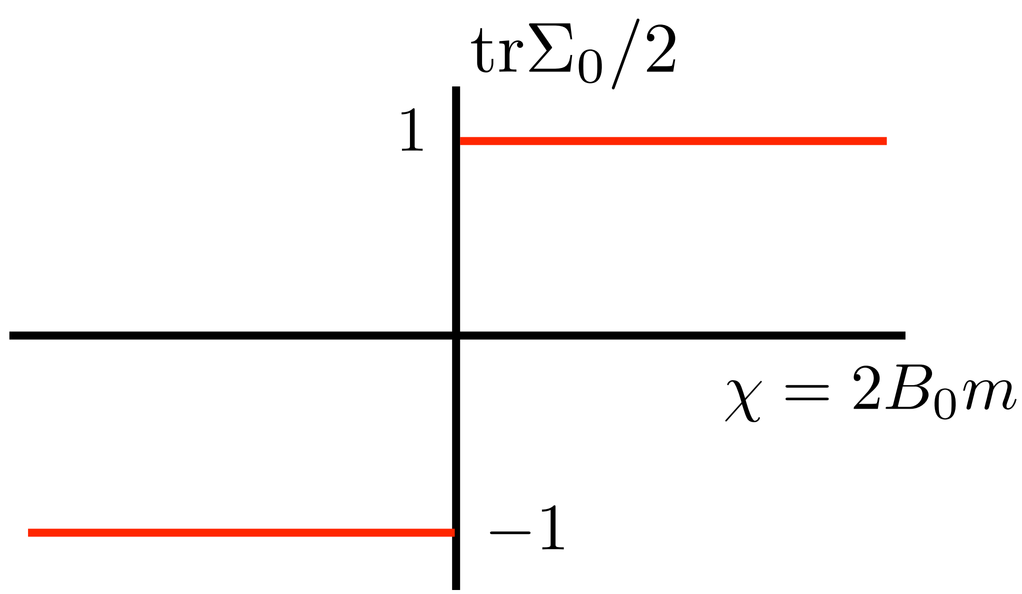
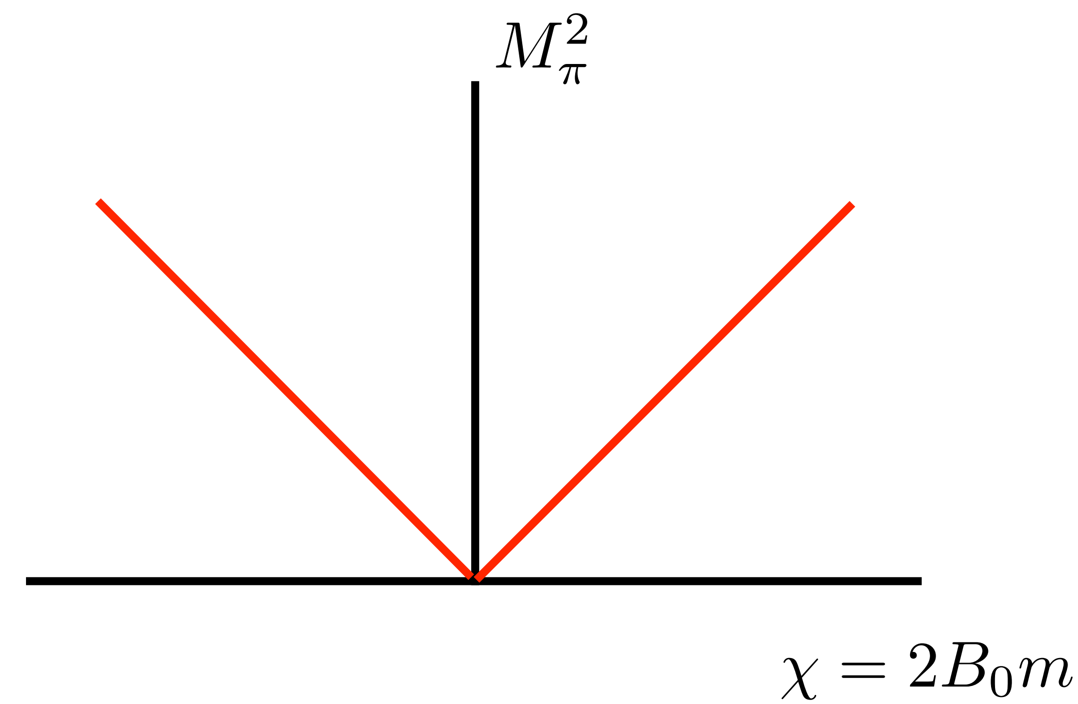
Phase structure: continuum

- In continuum, have first-order transition when m passes through zero, though the two sides are related by non-singlet axial SU(2) transformation

$$\mathcal{V} \propto -m\langle \Sigma + \Sigma^\dagger \rangle \quad \Rightarrow \Sigma_0 = \langle 0|\Sigma|0\rangle = \text{sign}(m)\mathbf{1}$$

$$\Rightarrow M_\pi^2 = 2B_0|m|$$





Phase structure: lattice

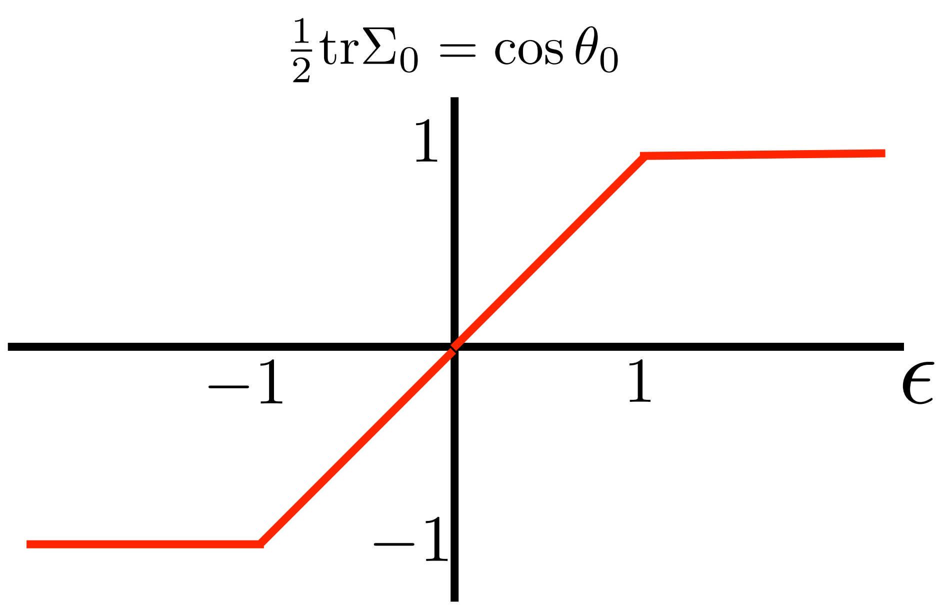
[Creutz 96, SS & Singleton 98]

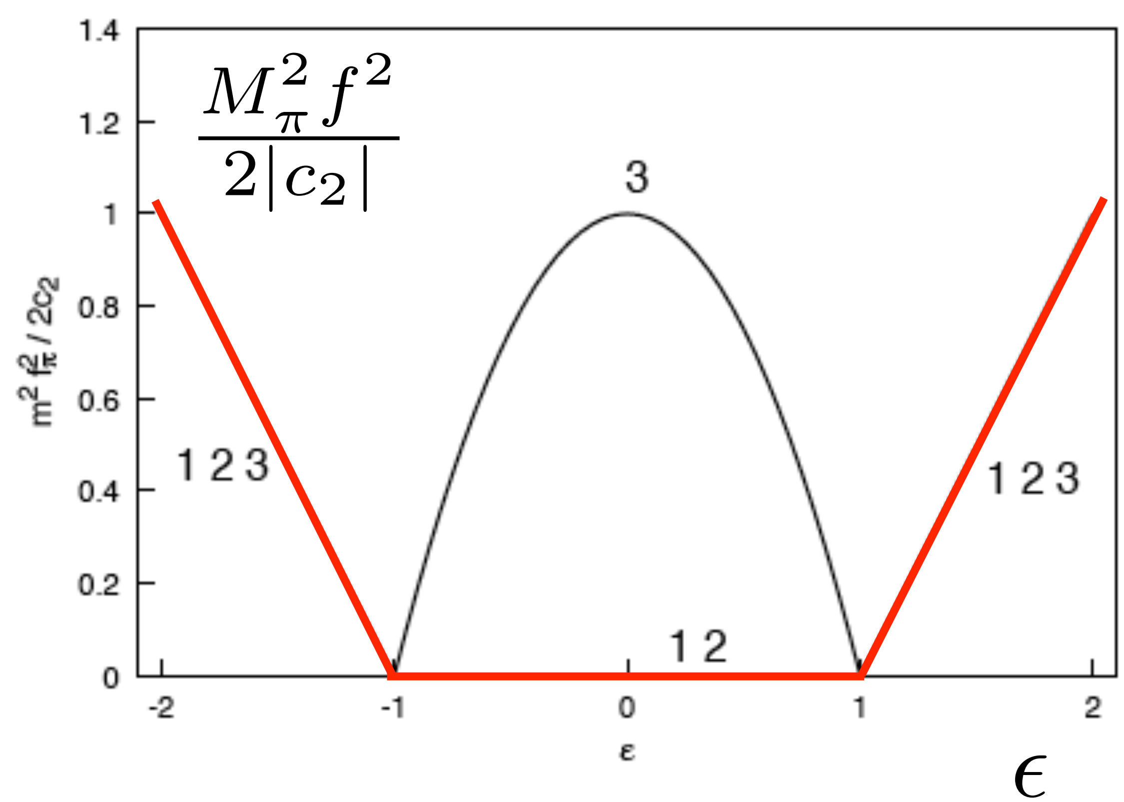
- Competition between two terms when m~a^2

$$\mathcal{V} = -\frac{f^2}{4}\chi\langle\Sigma+\Sigma^\dagger\rangle + \frac{c_2}{16}\langle\Sigma+\Sigma^\dagger\rangle^2 \propto -\epsilon\cos\theta_0 + \frac{1}{2}\frac{|c_2|}{c_2}\cos\theta_0^2$$

$$\epsilon = \frac{2mB_0 f^2}{2|c_2|} \qquad \Sigma_0 = \cos(\theta_0) + i\sin(\theta_0)\vec{n}_0\cdot\sigma$$

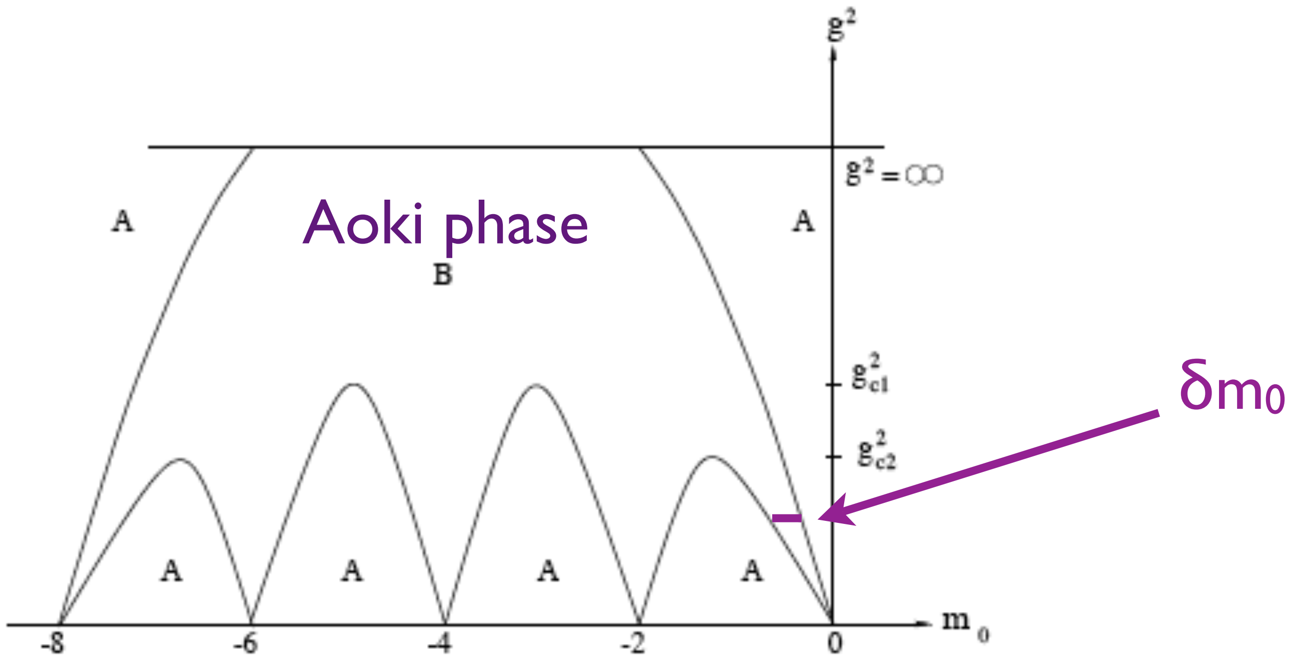
- If $c_2 > 0$, then get Aoki phase, flavor spont. broken:





Aoki phase [Aoki 84]

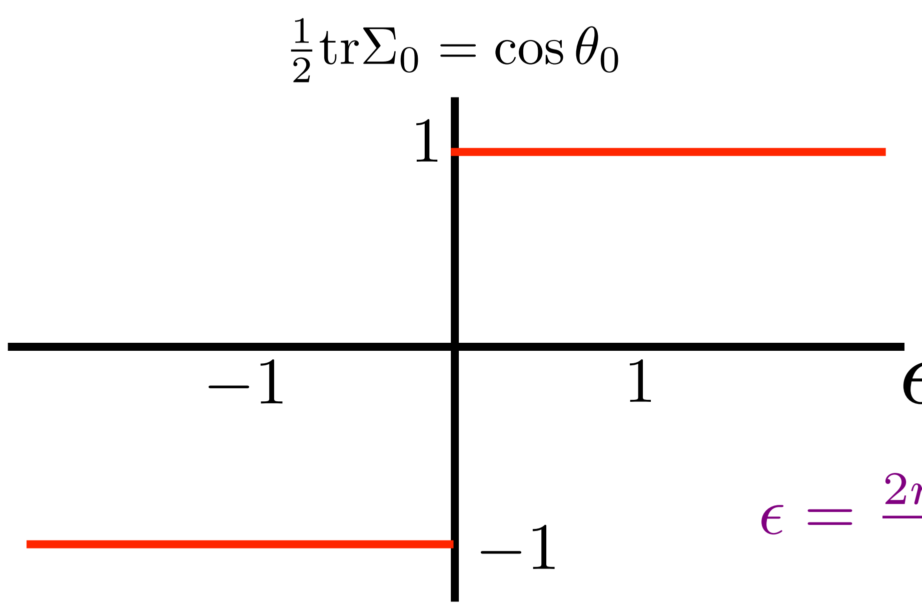
- Explains why $M_\pi=0$ on lattice, even though have no chiral symmetry!
  - ✓ (two) pions are PGBs of flavor breaking: $SU(2)_f \rightarrow U(1)_f$
- Parity is also broken (but not in the continuum)
- Width of phase is $\delta m \sim a^2 \Rightarrow \delta m_0 \sim a^3$



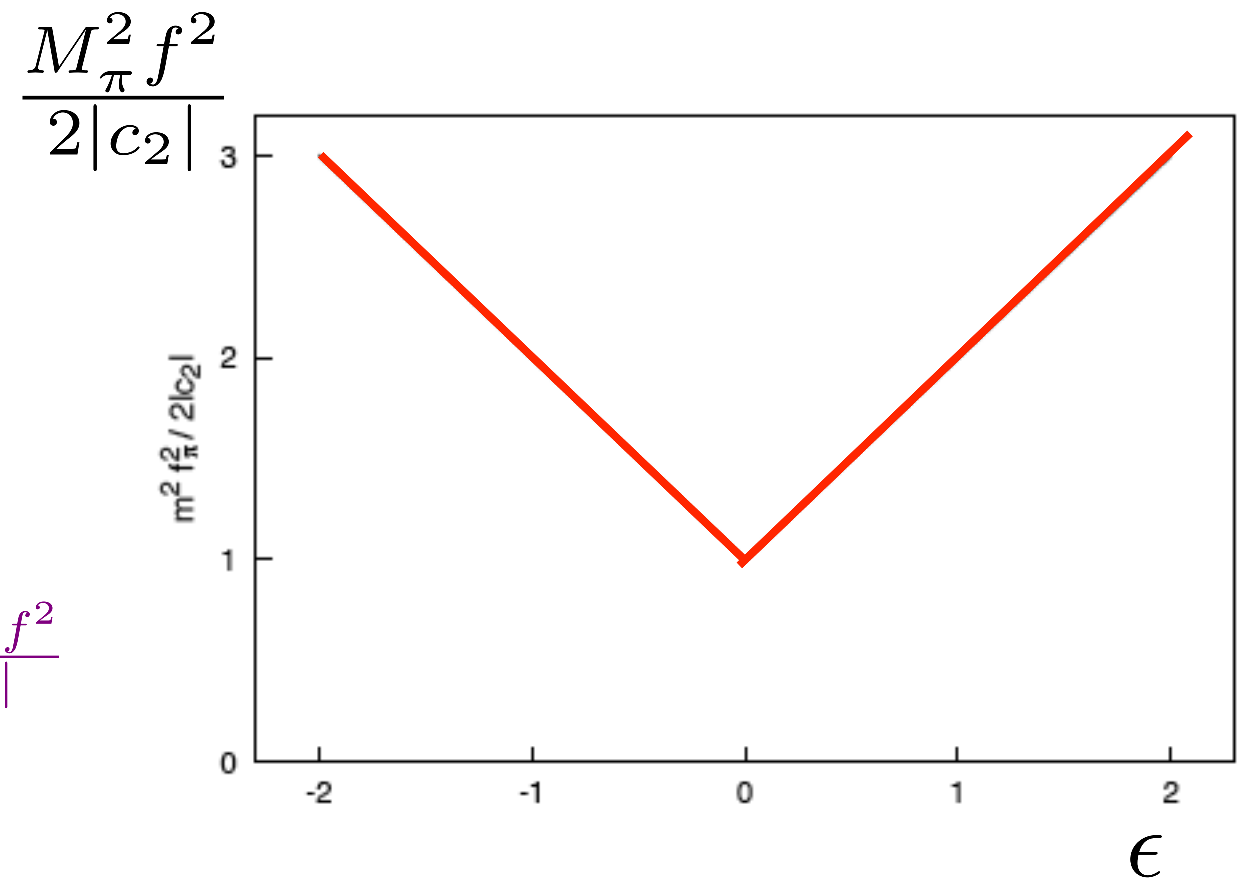
First-order scenario

$$\mathcal{V} = -\frac{f^2}{4}\chi\langle\Sigma+\Sigma^\dagger\rangle + \frac{c_2}{16}\langle\Sigma+\Sigma^\dagger\rangle^2 \propto -\epsilon\cos\theta_0 + \frac{1}{2}\frac{|c_2|}{c_2}\cos\theta_0^2$$

- If $c_2 < 0$, get first-order transition, with minimum pion mass M_π(min)~a
- Explicit chiral symmetry breaking ⇒ No GB



$$\epsilon = \frac{2mB_0f^2}{2|c_2|}$$



Can sign of c_2 be predicted?

- c_2 is non-universal (depends on gauge/fermion action)
- Prediction seems very difficult from first principles

Lattice → Symanzik EFT → (W)ChPT

Perhaps can estimate using perturbation theory

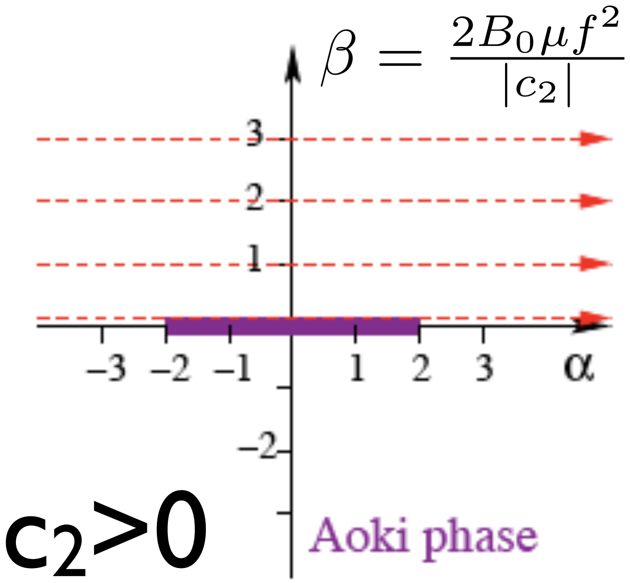
Non-perturbative, with multiple operators in L_{Sym} contributing

- Sometimes can use causality [Pham & Truong, A. Adams et al.] or mass inequalities to constrain LECs [Bar, Golterman & Shamir]
  - Neither approach applies here
- Hermiticity argument from ε-regime study + large N_c suggests that $c_2 > 0$ [Akemann, Damgaard, Splittorff & Verbaarschot]
  - Important question: Is this argument correct?

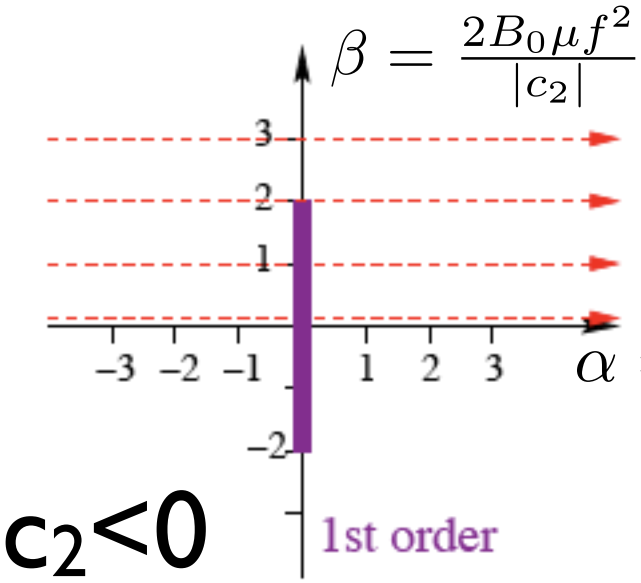
Determining sign of c_2

- Useful to add twisted mass: $\mu\bar{\psi}i\gamma_5\tau_3\psi$
- Two scenarios generalize in m,μ plane to:

[Munster; SS&Wu; Scorzato]



$c_2>0$



$c_2<0$

$\alpha = 2\epsilon = \frac{2B_0 m f^2}{|c_2|}$

$$M_{\pi^0} \geq M_{\pi^\pm} \qquad\qquad M_{\pi^0} \leq M_{\pi^\pm}$$

Equality only for μ=0 & outside Aoki phase

Infact, can use mass difference to determine value of c_2 [Scorzato]

Example with $c_2 < 0$

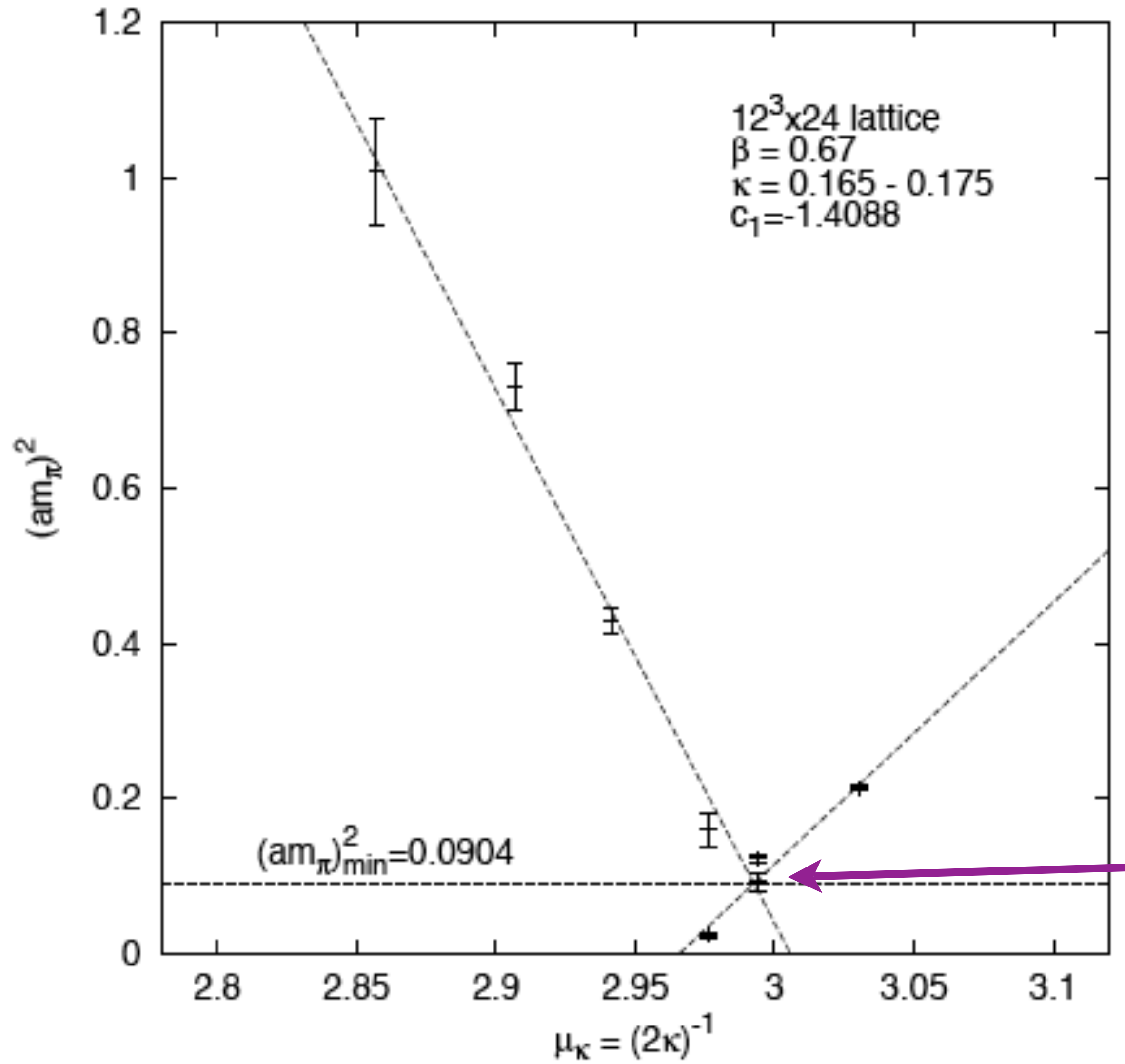


Figure 9. Unquenched results for $(am_\pi)^2$ as a function of $(2\kappa)^{-1} = m_0 + 4$ for $\mu = 0$ and with $a^{-1} \approx 0.2$ fm[60]. Straight lines are to guide the eye.

[Farchioni et al., 05]

$a \approx 0.2$fm

First-order scenario with minimum pion mass

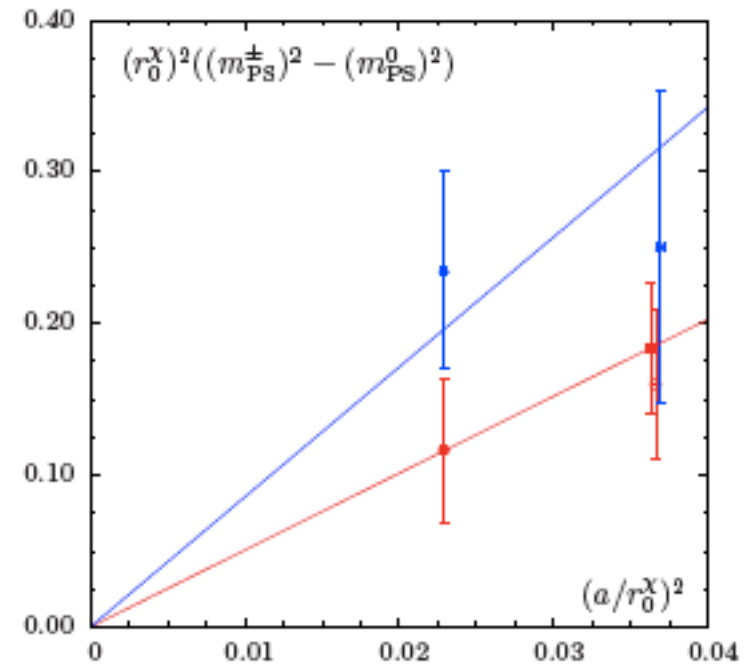
Caveat: LO WChPT may not apply for such a coarse lattice

Other results for c_2

- Comparing $M_{\pi+}$ & $M_{\pi0}$

  [ETM Collab., Baron et al., 2009] find

  $M_{\pi+} > M_{\pi0}$, indicating first-order scenario



- Calculate $\pi\pi$ scattering lengths [Aoki, Bar & Biedermann]

$$a_0^2 = -\frac{1}{16\pi}\left(\frac{M_\pi^2}{f^2}+\frac{c_2}{f^4}\right)+\mathrm{NLO}+\mathrm{NNLO}$$

  ✓ Simulations underway by [Bernardoni, Sommer, et al.]

- Sign (and value) of c_2 unclear for many actions used in production runs

Why we care about c_2

- Since $m_{phys} \approx a^2 \Lambda^3$, ultimately we will reach the LCE regime
  - BMW collaboration (working at m_{phys}) sees no indication of phase structure; perhaps gluon smearing reduces c_2
- Presence of nearby second-order endpoints distorts physical quantities in their vicinity [Aoki]
  - Chiral logarithms distorted: $M^2\log(M) \rightarrow (M^2+a^2)\log(M)$
- Reduction in gap in spectrum of Hermitian Wilson-Dirac operator can slow simulations

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Predictions in continuum

- ChPT predicts spectrum of Dirac operator [Banks & Casher; Smilga & Stern; Osborne, Toublan & Verbaarschot]

$$\rho_{\not D}(\lambda) = -\frac{\langle \bar{q}_S q_S \rangle}{\pi}\left[1 + O(|\lambda|/\Lambda_{\rm QCD})\right]$$

- ChPT in ε-regime ⇒ low e'values (λ~1/V) described by random matrix theory (RMT) [Osborne, Toublan & Verbaarschot; Damgaard et al., ...]
  - Detailed predictions for distributions of individual e'values,...
  - Provides method to determine LECs (f & B_0) from simulations
- Derivations require use of partial quenching (PQ)
  - Need $m_{val} \neq m_{sea}$ to access spectrum for fixed sea quark masses
  - Thus need PQ extension of ChPT [Bernard & Golterman]

$$\langle \bar{q}_V q_V \rangle(m_V) = -\int d\lambda \frac{\rho_{\not D}(\lambda)}{i\lambda + m_V} \qquad \Rightarrow \qquad {\rm Disc}[\langle \bar{q}_V q_V \rangle]\Big|_{m_V = -i\lambda} = -2\pi \rho_{\not D}(\lambda)$$

E'value spectrum at $O(a^2)$

- Requires PQWChPT, and calculate most naturally spectral properties of <u>Hermitian</u> Wilson-Dirac op
  - ✓ Infinite volume spectrum distorted by a^2 effects [SS]
  - ✓ Large volume spectrum obtained for $m \sim a >> a^2$ by [Necco & Shindler]
  - ✓ Extended to ε-regime, RMTs determined, detailed predictions for eigenvalues obtained [Damgaard et al., Akemann et al.,...]
- Gives method to determine additional LECs due to discretization errors [Damgaard, Heller & Splittorff]
- Theoretical puzzles & inconsistencies remain

PQWChPT

- $SU(2)_L \times SU(2)_R \to SU(2+N_V|N_V)_L \times SU(2+N_V|N_V)_R$
- Construct L_χ in 2 steps as before [Bar, Rupak & Shoresh; Aoki]

$$\mathcal{L}_0 = \frac{f^2}{4}\langle \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \rangle - \frac{f^2}{4} 2B_0 \langle M^\dagger \Sigma + \Sigma^\dagger M \rangle$$
$$- \hat{a}^2 W_6' \langle \Sigma + \Sigma^\dagger \rangle^2 - \hat{a}^2 W_7' \langle (\Sigma - \Sigma^\dagger)^2 \rangle - \hat{a}^2 W_8' \langle \Sigma^2 + (\Sigma^\dagger)^2 \rangle$$

$\Sigma \in SU(2+N_V|N_V)$

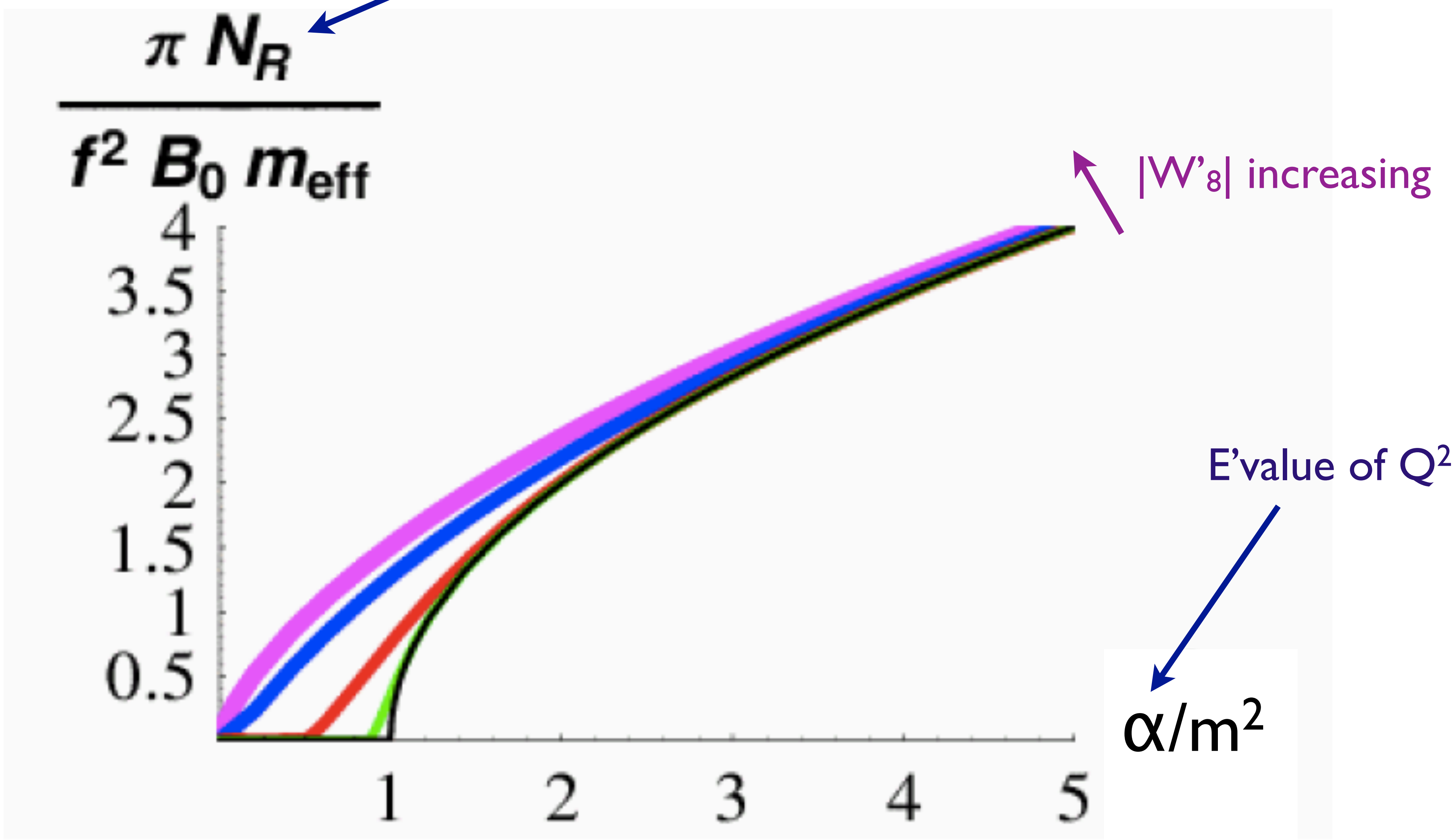
Supertrace

- 3 $O(a^2)$ terms, compared to 1 in unquenched WChPT
  - If restrict Σ to SU(2) subspace, W'_7 term vanishes, and $W'_{6,8}$ terms combine
  - Recover WChPT Lagrangian, with $c_2 = -8\hat{a}^2(2W_6' + W_8')$
- At large N_c, W'_8 dominates: $\frac{W_6'}{W_8'} \sim \frac{W_7'}{W_8'} \sim \frac{1}{N_c}$

Example of results (1)

[SS]

Integrated spectral density of $Q^2 = D_W^\dagger D_W$



$$W_8' < 0, W_6' = W_7' = 0$$

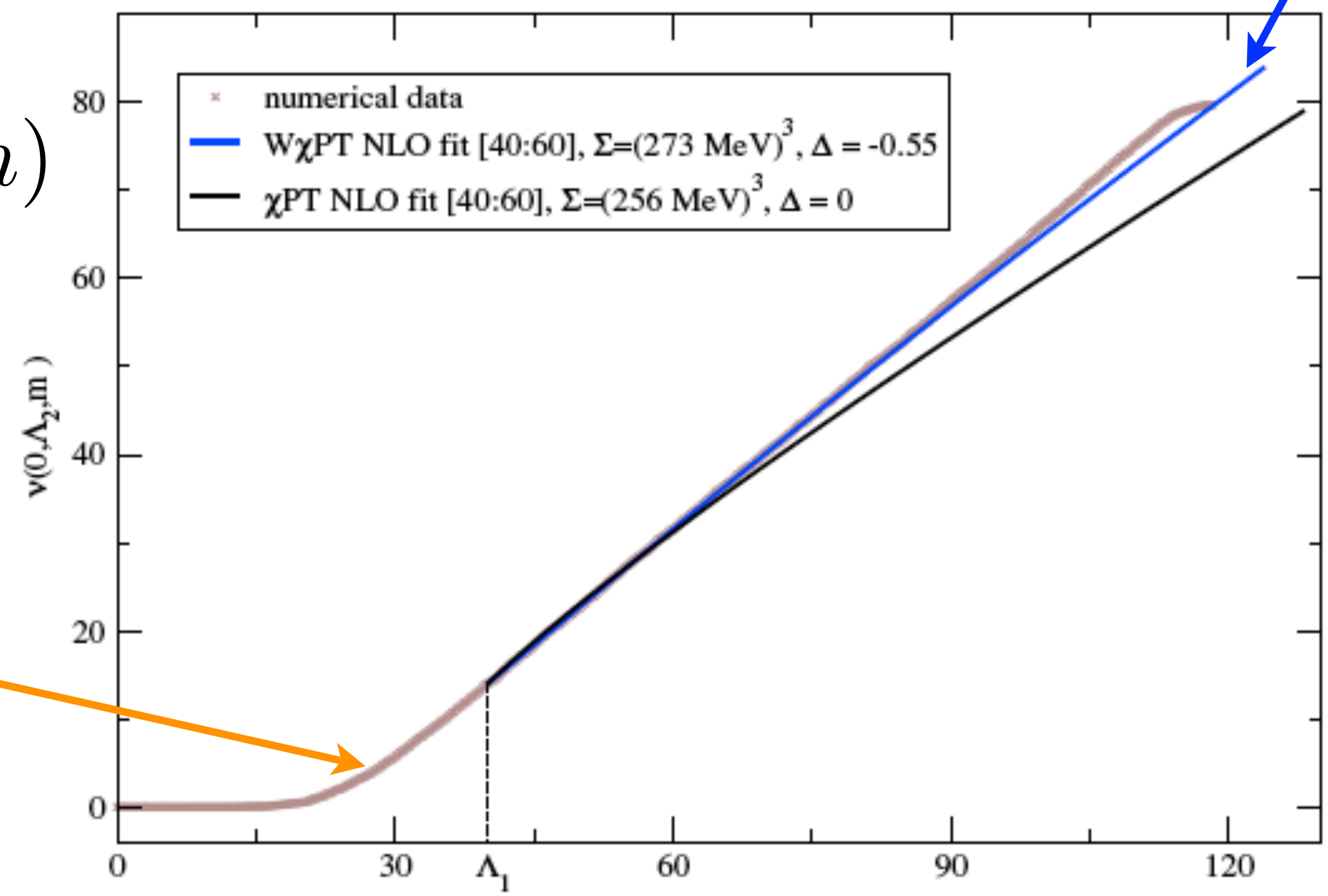
Example of results (2) [Necco & Shindler]

Integrated spectral density of $Q = \gamma_5 D_W = Q^\dagger$

WChPT fit

$\nu(0, \Lambda_2, m)$

Data from [Giusti & Luscher, 08]



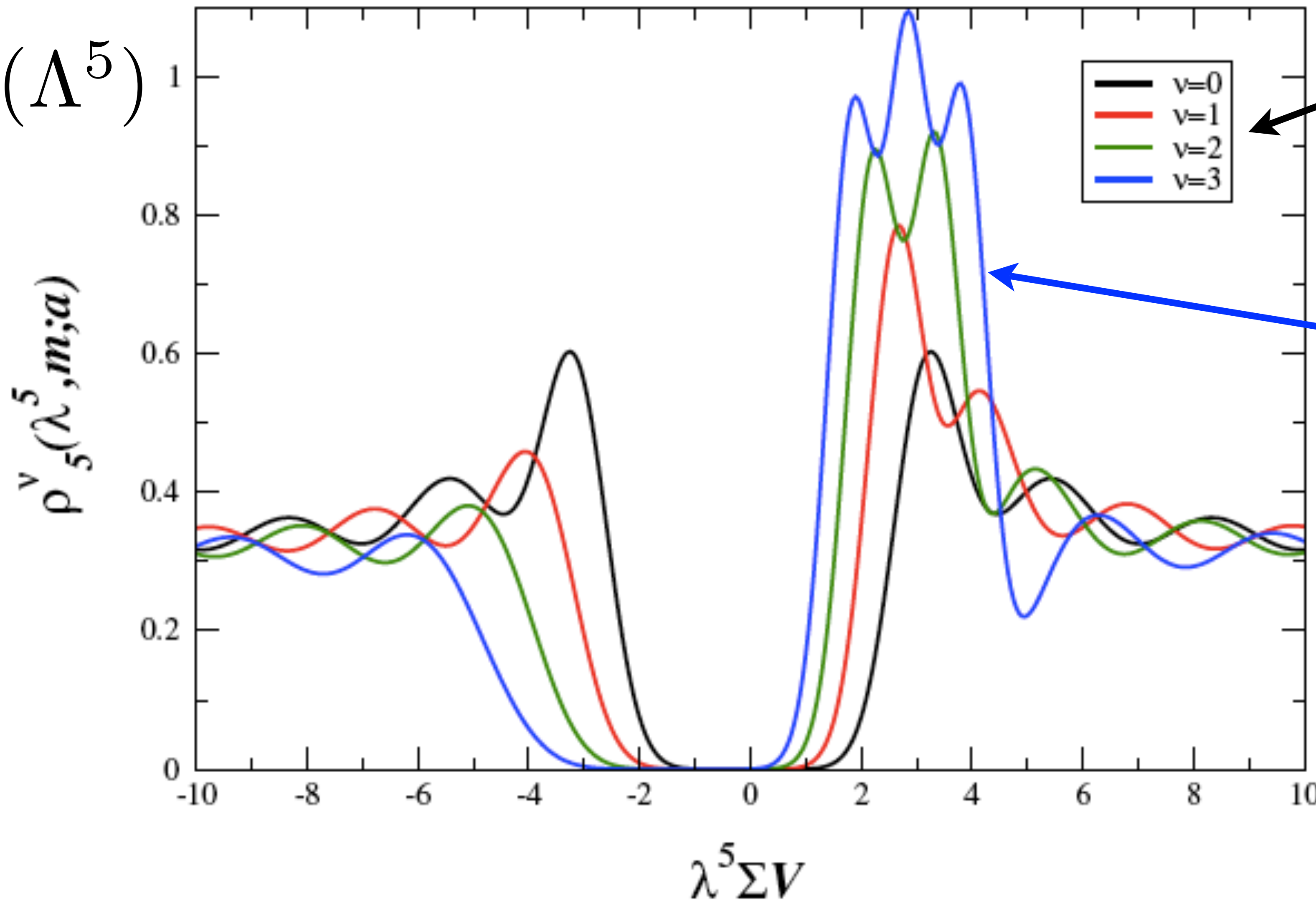
$\Lambda_2(\mathrm{MeV})$

Fit finds $W'_8 > 0$ (~First-order scenario)

Example of results (3) [Akemann et al.]

Microscopic spectral density of $Q = \gamma_5 D_W = Q^\dagger$

$\rho_5(\Lambda^5)$



ν is topological charge

Would-be topological zero-modes

Results for W'$_8$<0, W'$_6$=W'$_7$=0

(In my convention: [Akemann et al.] use opposite signs)

Puzzles & Inconsistencies

- Infinite volume analysis of spectrum gives sensible results only if $W'_8 < 0$ and is independent of W'_7 [SS]
- Large volume, $m \gg a^2$ analysis works for either sign of W'_8, and is independent of W'_7 [Necco & Shindler]
- ε-regime/RMT analysis gives first-principles argument that $W'_8<0$ (assuming $W'_6=W'_7=0$; signs in my convention!) and results depend also on both W'_6 & W'_7 [Damgaard et al.; Akemann et al.]
    - In general, constraint is $W'_8 < W'_6 + W'_7$ (?)
    - "Mean-field" limit should match with other calculations
- Constraints on W'_8 and detailed results do not agree

Argument for W'$_8$<0

[Akemann et al.]

$$\gamma_5 D_W \gamma_5 = D_W \;\Rightarrow\; \det{}^2(D_W) \geq 0$$

$$\Rightarrow Z_{\mathrm{LQCD}} = \int DU e^{-S_g} \det{}^2(D_W) > 0$$

$$\Rightarrow (?) Z_{\nu,\mathrm{LQCD}} > 0$$ (sign indep. of m)

$$Z_{\mathrm{ChPT}} = \int_{SU(N)} d\Sigma \; e^{m\frac{f^2 B_0 V}{2}\langle\Sigma+\Sigma^\dagger\rangle + a^2 W_8' V \langle \Sigma^2 + (\Sigma^\dagger)^2\rangle} > 0$$

$$Z_{\nu,\mathrm{ChPT}} = \int_{U(N)} d\Sigma \; \det(\Sigma)^\nu e^{m\frac{f^2 B_0 V}{2}\langle\Sigma+\Sigma^\dagger\rangle + a^2 W_8' V \langle \Sigma^2 + (\Sigma^\dagger)^2\rangle}$$

has indeterminate sign for odd ν if W'$_8$>0

Implications of $W'_8<0$

$$c_2 = -8\hat{a}^2(2W'_6 + W'_8)$$

- At large N_c: $W'_8 < 0 \ \Rightarrow c_2 > 0$
  - ➡ Only Aoki-phase scenario is allowed!
  - ➡ Appears to contradict numerical evidence of first-order scenario
- First-order scenario is allowed if $W'_6 > |W'_8|/2$
  - Not unreasonable for N_c=3
- Need to determine W'_6, W'_7, W'_8 from simulations
  - One approach is to match eigenvalue properties to WChPT/RMT [Damgaard, Heller & Splittorff]
  - Another is to study PQ pion scattering [Hansen & SS]

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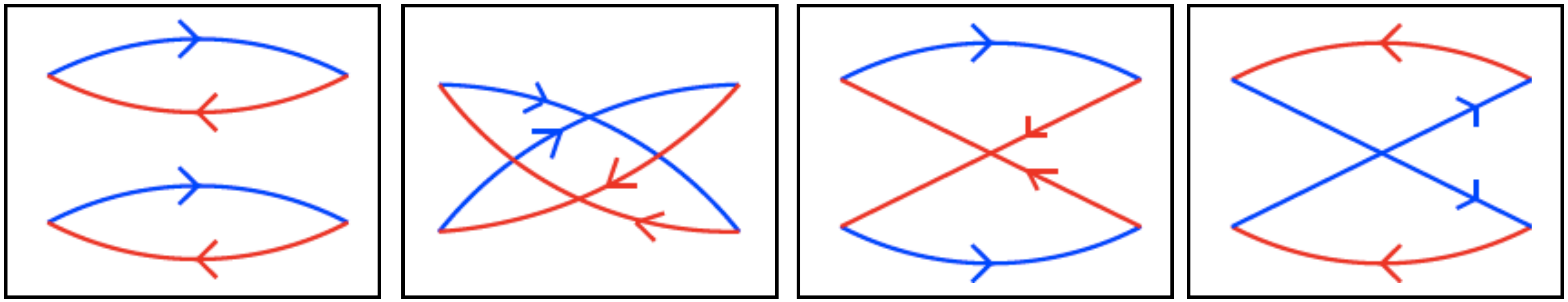
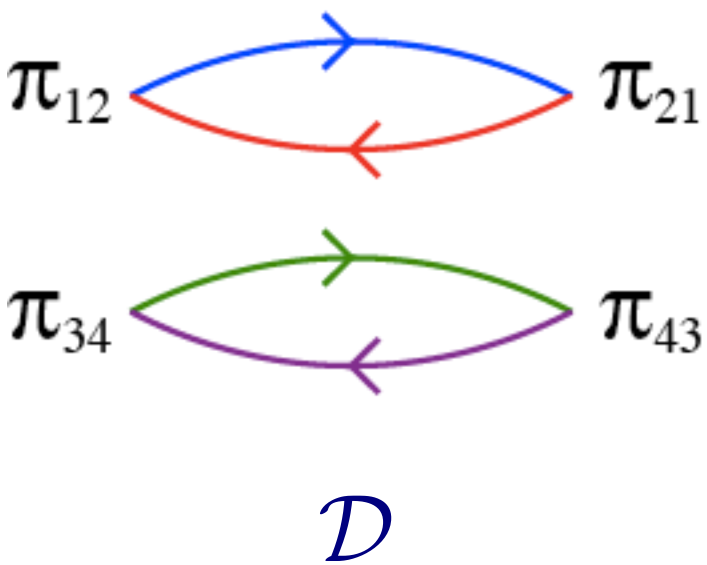
- A proposal for determination of all three $O(a^2)$ partially quenched low-energy coefficients
- Some open questions

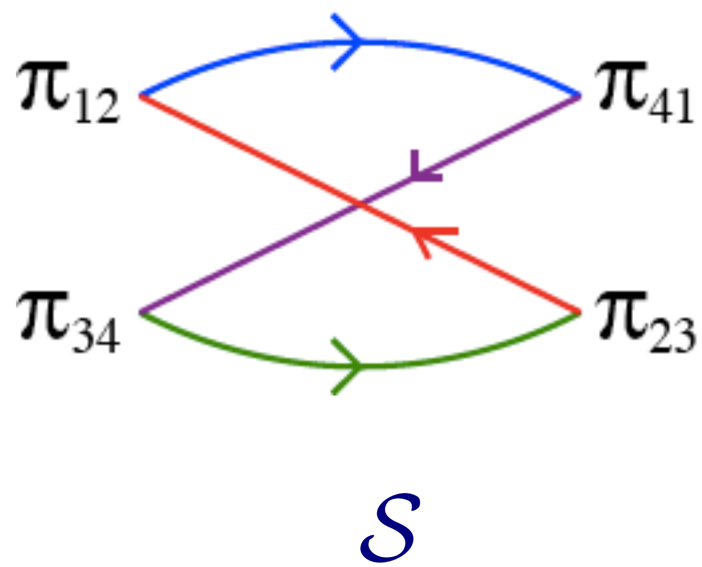
Dismembering $\pi^+\pi^+$ scattering

- Unquenched theory has 4 Wick contractions



- Can separate in PQ theory (and in practice)





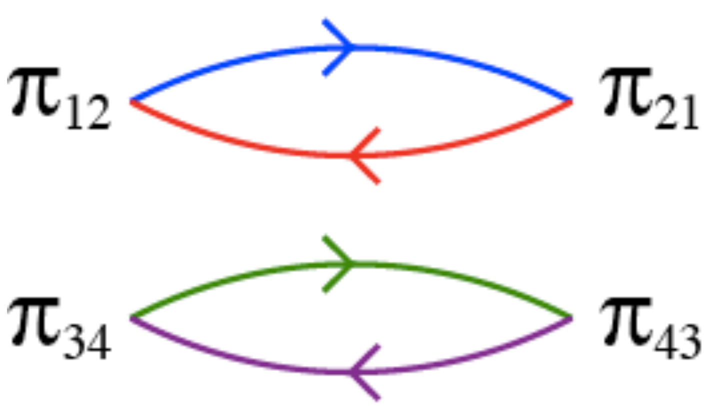
PQ ππ scattering

$$\mathcal{L}_0 = \frac{f^2}{4}\langle \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \rangle - \frac{f^2}{4} 2B_0 \langle M^\dagger \Sigma + \Sigma^\dagger M \rangle$$
$$- \hat{a}^2 W_6' \langle \Sigma + \Sigma^\dagger \rangle^2 - \hat{a}^2 W_7' \langle (\Sigma - \Sigma^\dagger)^2 \rangle - \hat{a}^2 W_8' \langle \Sigma^2 + (\Sigma^\dagger)^2 \rangle$$

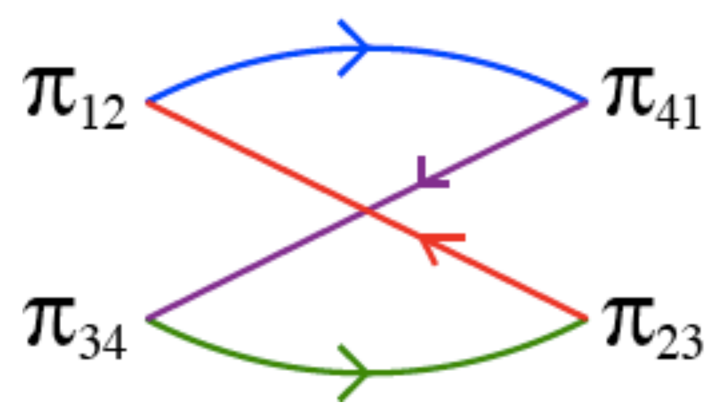
Contributes to *D* and *S* at tree-level

Does not contribute at tree-level

Contributes only to *S* at tree-level



$\mathcal{D}$



$\mathcal{S}$

PQ ππ scattering

- We find [Hansen & SS]

W'$_6$ enters here

W'$_7$ enters here

$$\mathcal{D} = \frac{32\hat{a}^2 W_6'}{f^4} + \text{NLO} + \text{NNLO}$$

$$\mathcal{S} = \frac{16\hat{a}^2 W_8'}{f^4} + \frac{2M_\pi^2 - s}{2f^2} + \text{NLO} + \text{NNLO}$$

- Since PQ amplitude is unphysical (non-unitary), cannot use Luscher's method for calculating scattering lengths and amplitudes from finite volume energy shifts

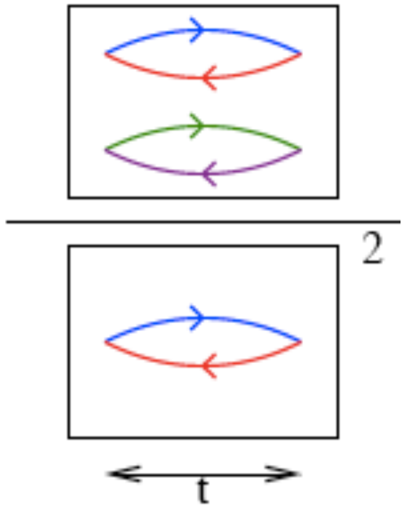
- Instead, we propose simply calculating correlation functions and matching to PQWChPT--- used in quenched approx. by [Bernard & Golterman, 95]

PQ finite-volume correlators

- Up to corrections ~exp(-$M_\pi L$) we find

$$R_D(t) = \frac{\langle \tilde\pi_{12}(\vec 0,0)\tilde\pi_{34}(\vec 0,0)\tilde\pi_{21}(\vec 0,0)\tilde\pi_{43}(\vec 0,0)\rangle}{\langle \tilde\pi_{12}(\vec 0,0)\tilde\pi_{21}(\vec 0,0)\rangle^2} =$$

$$= [1 + O(L^{-3})] + \frac{t}{4M_\pi^2 L^3}\left(\mathcal{D}^{NNLO}_{\rm thresh} + \frac{\text{known NNLO}}{L} + \dots\right) + \dots$$



- Can determine *D* (and thus W'_6) from coefficient of t (and similarly for *S*)
- t^2 and higher order terms do not build up an exponential (unlike for a physical correlator)
- Also possible to determine W'_7 from tree-level 3-pion scattering, but likely very difficult in practice

Outline

- Brief introduction to Wilson fermions & associated discretization errors
- Determining the low-energy effective theory
- Predictions for phase diagram
- Summary of application to eigenvalue spectrum
- A proposal for determination of all three $O(a^2)$ partially quenched low-energy coefficients
- Some open questions

Questions for the workshop

- Is $W'_8 < 0$ required?
- If so, does the argument have wider applicability to EFTs?
- Can we resolve differences between results for spectrum of Q?
- Is it practical to use PQ $\pi\pi$ "amplitudes" to determine W'_6 and W'_8?
- Do we need to use WChPT to fit numerical results now that we are entering the LCE regime?
- How solid is the theoretical footing of PQ ChPT?