Strong coupling expansions in pure lattice Yang-Mills theory at finite temperature

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August 16, 2007

overview

- expansions of SU(2) free energy density in lattice coupling β
- Wilson lattice gauge action
- compactified time direction
- search for phase transition at β_c

starting point

- details: Montvay/Münster chapter 3.4
- Wilson's action

$$S = \sum_p \beta \left(1 - \frac{1}{N} \mathrm{Re}\,\mathrm{Tr}\, U \right) = \sum_p S_p$$

- partition function

$$Z = \int \prod_b dU(b)\, \mathrm{e}^{-S}$$

- free energy density

$$f = -\frac{T}{V} \ln Z$$

further steps

- strong coupling character expansion

$$\mathrm{e}^{-S_p} = \sum_r d_r c_r(\beta)\chi_r(\beta)$$

$$\mathrm{e}^{-S} = \prod_p c_0(\beta)\left[1 + \sum_{r=\frac{1}{2},1,\dots} d_r\, a_r(\beta)\chi_r(U_P)\right]$$

$$c_0(\beta) = I_1(\beta) \qquad a_r(\beta) = \frac{c_r}{c_0} = \frac{I_{2r+1}(\beta)}{I_1(\beta)}$$

further steps

- strong coupling character expansion

$$
\begin{aligned}
\mathrm{e}^{-S_p} &= \sum_r d_r c_r(\beta)\chi_r(\beta) \\
\mathrm{e}^{-S} &= \prod_p c_0(\beta)\left[1 + \sum_{r=\frac{1}{2},1,\ldots} d_r\, a_r(\beta)\chi_r(U_P)\right] \\
c_0(\beta) &= I_1(\beta) \qquad\qquad a_r(\beta) = \frac{c_r}{c_0} = \frac{I_{2r+1}(\beta)}{I_1(\beta)}
\end{aligned}
$$

- grouping the plaquettes in graphs

$$
\begin{aligned}
Z &= c_0^{6\Omega} \sum_{\mathcal{G}} \Phi(\mathcal{G}) \qquad\qquad \Omega = VN_t \\
\Phi(\mathcal{G}) &= \int \prod_b dU(b) \prod_{p\in\mathcal{G}} d_{r_p}\, a_{r_p}(\beta)\chi_{r_p}(U_p)
\end{aligned}
$$

further steps

- divide graphs into disjoint, connected parts

$$\mathcal{G} = X_1 \cup \dots \cup X_n \qquad \Phi(\mathcal{G}) = \prod_i \Phi(X_i)$$

further steps

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$$\mathcal{G} = X_1 \cup \cdots \cup X_n \qquad \Phi(\mathcal{G}) = \prod_i \Phi(X_i)$$

- free energy density

$$f = -\frac{T}{V} \ln Z \qquad T = \frac{1}{N_t}$$

$$= -6 \ln c_0 - \frac{1}{\Omega} \sum_{\{X_i\}} \prod_i \Phi(X_i)$$

finite temperature

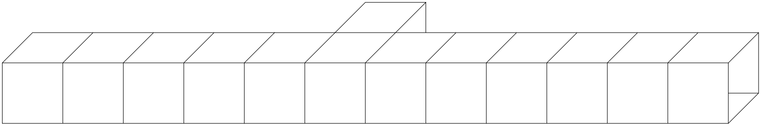
- compactify the temporal lattice extension according to

$$T = \frac{1}{N_t}$$

- consider $f(N_T < \infty) - f(N_t = \infty)$

- only graphs of length $L \geq N_t$ contribute (others cancel in subtraction)

some graphs



calculation

- series:

$$f(N_t, u) \quad = \quad \frac{3}{N_t} u^{4N_t} \sum_{n=0}^{4} P_n(N_t)\, u^{2n}$$

- expansion parameter

$$u \equiv a_{1/2}(\beta) = \frac{1}{4}\beta + \mathcal{O}(\beta^2)$$

calculation

- series:

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- first orders correspond to a glueball gas

$$f(N_t, u) \;=\; -\frac{1}{N_t}\left\{ \mathrm{e}^{-m(A_1^{++})N_t} + 2\,\mathrm{e}^{-m(E^{++})N_t} + \mathcal{O}(u^4) \right\}$$

series analysis

- ratio method

$$\lim_{n\to\infty} \frac{P_{n+1}}{P_n}\bigg|_{N_t\ fixed} = u_c \qquad \longrightarrow \qquad \beta_c$$

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- extrapolation through Padé approximants

$$[L, M](x) \equiv \frac{p_0 + p_1 x + \cdots + p_L x^L}{q_0 + q_1 x + \cdots + q_M x^M}$$

- Estimates of β_c: Zeroes of the denominator

results

- Padé table for SU(2), $N_t = 2$

$[L, M]$	u_c	β_c
$[2, 1]$	$\pm 0.4905i$	
$[1, 2]$	± 0.4143	1.8865
$[0, 3]$	± 0.4675	2.2257

- Monte-Carlo: $\beta_c = 1.8800(30)$

results

- Padé table for SU(2), $N_t = 3$

$[L, M]$	u_c	β_c
$[2, 1]$	$\pm 0.4217i$	
$[1, 2]$	± 0.3467	1.5133
$[0, 3]$	± 0.5009	2.4538
$[3, 1]$	$\pm 0.3550i$	
$[2, 2]$	± 0.4109	1.8665
$[1, 3]$	± 0.4026	1.8187
$[0, 4]$	± 0.4505	2.1089

- Monte-Carlo: $\beta_c = 2.1830(60)$

outlook

- 1-2 more orders realistic
- $\mathcal{O}(u^{14})$ involves approx. 200 graphs
- more sophisticated analyses methods (differential approximants)
- SU(3): straightforward to calculate, but behaviour is worse